Inflation, Real Interest Rates, and the Bond Market: A Study of French Index-Linked Government Bonds*

Olesya V. Grishchenko† Franck Moraux‡ Olga Pakulyak§

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Abstract

We study the evolution of real rates implied by relatively recent and rapidly growing market for inflation-indexed French sovereign bonds (*Obligations Assimilables du Trésor (OAT) indexée sur l'indice des prix à la consommation de la zone euro*). First, we fit the market data to produce the smoothed term structure of real rates following Nelson-Siegel model. Second, we construct the term structure of inflation compensation rate — the compensation rate required by market participants to hold the nominal OAT securities instead of inflation-indexed OAT securities. We backcast inflation compensation to the period prior to the introduction of the French inflation-protected securities. The availability of such series provides invaluable information about inflation expectations and risks as viewed by the OAT market participants. We also conduct the sensitivity analysis of euro-area inflation compensation to French, U.S., and German macroeconomic data releases and we find that euro-area inflation expectations are reasonably well anchored in our sample.

JEL Classification: G12, G13, G14
Keywords: Term structure of real interest rates, French inflation-linked bonds (OAT €i), Nelson-Siegel model, inflation expectations, macroeconomic news, inflation anchoring

*The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve System.

†Division of Monetary Affairs, Federal Reserve Board, Washington, DC 20551; olesya.v.grishchenko@frb.gov

‡Université de Rennes & CREM (UMR 6211) franck.moraux@univ-rennes1.fr
CREM is the Centre de Recherche en Economie et Management.

§Institut Agro Rennes-Angers & SMART (UMR 1302) olga.pakulyak@agrocampus-ouest.fr
SMART is the Structures et Marchés Agricoles, Ressources et Territoires.

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1 Introduction

Central banks around the world have an important mandate to guarantee price stability in the domestic economies. Often, central banks follow inflation targeting rules, explicit or implicit, as part of their monetary policy agenda.[1] Stable prices pave a way for maximizing economic growth, optimal employment, exchange rate, and financial stability in a domestic economy. To that end, it is extremely important to use all available tools to central bank economists and monetary policymakers to monitor inflation expectations. One of such tool is market-implied inflation expectations that can be inferred from comparing nominal (inflation-unadjusted) and real (inflation-adjusted) rates of comparable maturities in an economy in question. Such research is long-standing in the US (e.g., Gürkaynak, Sack, and Wright, 2007, 2010; Christensen, Lopez, and Rudebusch, 2010; Grishchenko and Huang, 2013; D'Amico, Kim, and Wei, 2018; Chang, 2019, to name just a few), UK, Japan, and Euro-area (e.g., Barr and Campbell, 1997; Evans, 1998; Kita and Tortorice, 2018), and, to some extent.

Despite some advances made in the literature about understanding time variation in inflation expectations and driving forces behind them, the literature about the French inflation-linked debt is virtually non-existent, with an exception of a few studies (see, e.g., Pericoli, 2014; Bekaert and Ermolov, 2021).[2] However, nominal outstanding amount of the French inflation-adjusted debt represents about 12 percent of the U.S. Treasury Inflation-Protected Securities (TIPS) whose outstanding nominal amount has been about 1.9 trillion of US dollars as of December 31, 2022.[3] In addition, French inflation-adjusted debt market is unique because the French Treasury issues two

[1]Explicit inflation targeting was first introduced in New Zealand in 1990, then in Canada in 1991, followed by UK in 1992. For example, Japan, the USA and some other countries have inflation rate target of 2 percent. The European Central Bank introduced a famous inflation targeting rule of *below, but close to 2 percent* in the medium term (see, e.g., Lyziak and Paloviita, 2016; Paloviita, Haavio, Jalasjoki, and Kilponen, 2017). The U.S. Federal Reserve targets 2 percent inflation in the long term, according to the Statement on Longer-Run Goals and Monetary Policy Strategy issued in 2008 (see FOMC (2018)). The issue of stable prices and anchored inflation expectations has been an active focus of research among academic researchers and central bankers (see, e.g., Gürkaynak, Levin, Marder, and Swanson, 2007; Beechey, Johannsen, and Levin, 2011; Grishchenko, Mouabbi, and Renne, 2019, among many others).

[2]Pericoli (2014) studies the technical aspects of fitting the spline curve of the OAT€i securities, while Bekaert and Ermolov (2021) study the co-movement between US, UK, and French interest rates.

[3]See the Monthly Statement of the Public Debt by the U.S. Treasury `https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt/summary-of-treasury-securities-outstanding`.

types of such securities that link bond cash payments to two indexes: Euro-area inflation index — *Harmonized Index of Consumer Prices* (HICP); and French domestic inflation index, — *Consumer Price Index* (CPI). Two types of securities are called respectively *Obligations Assimilables du Trésor indexée sur l'indice des prix à la consommation de la zone euro* — OAT€i; and *Obligations Assimilables du Trésor indexée sur l'indice des prix à la consommation en France* — OATi. Neither of the above two papers studies the difference between the two markets.

Our paper fills the gap by focusing on the unique French sovereign inflation-adjusted debt market with the goal of understanding time variation in inflation expectations in France and Euro area using French inflation-protected securities.[4]

First, we construct the term structure of *real* — rates implied by inflation-adjusted securities — interest rates implied by the French sovereign indexed debt securities (OAT€i). We follow the methodology outlined for nominal French securities (OATs) in Grishchenko, Moraux, and Pakulyak (GMR, 2020), the first comprehensive study of the French nominal OAT market, to our knowledge. We provide a comprehensive description of the OAT€i market and characterise the real yield curve implied by OAT€i.

Second, we derive OAT€i-implied inflation compensation measure at various maturities — a difference between nominal and real yields at comparable maturities that reflects compensation for bearing inflation risk implicitly requested by market participants of the nominal OATs. To that end, inflation compensation provides a gauge, albeit imperfect, of inflation expectations. The reason is that inflation compensation, besides inflation expectations, contains inflation risk premium component, that is relatively tedious to disentangle from inflation expectations, this is the focus of our future research. See, e.g., Grishchenko and Huang (2013) for a model-free methodology to achieve this. Another component that is likely to affect inflation compensation is the presence of illiquidity premium that investors demand to investor in a relatively recent OAT€i market. Similarly, the illiqidity premium has been studied by issue has been studied by (Grishchenko and

[4]In this version of the paper we focus only on the inflation-adjusted debt that is linked to the euro-area inflation index HICP, but we plan to post the updated version within a few months that provides results for the debt that is linked to the French CPI.

Huang, 2013; D'Amico, Kim, and Wei, 2018; Andreasen, Christensen, and Riddell, 2021, to name just a few). Authors conclude that liquidity was relatively scarce at the onset of the TIPS market and was, particularly, impaired during the Global Financial Crisis (GFC).[5]

Third, we study sensitivity of variation in inflation compensation, similar to Beechey, Johannsen, and Levin (2011) to French, U.S., and German macroeconomic news. Understanding how sensitive inflation expectations to the macroeconomic news provides important insights for policymakers whether inflation expectations are well anchored and thus relatively irresponsive, especially on the long horizons, for example, at 5-year to 5 years ahead horizon, or macroeconomic releases have a material impact. Such information helps understanding whether monetary policies set by central banks are credible.

We have four main results in the paper. First, using the smoothed term structure of real rates implied by the Nelson and Siegel (1987) model, we look at the basic properties of the real rates. We observe that real rates were declining since the global financial crisis and that they were on average very low, namely, lower than 1 percent. One average, the real rates were negative at maturities up to 5 years. The term structure of real rates was upward sloping, Second, we have shown that the breakeven inflation rates are driven by three latent factors (principal components), where the second and the third factor are materially more important than for either OAT nominal or OAT€i real rates. Third, We have backcast forward inflation compensation to the period prior to the introduction of the French inflation-protected securities (before 2004). We have shown that far forward inflation compensation was declining since the global financial crisis. Fourth, we explored the sensitivity of forward measures of inflation compensation to international macroeconomic news and concluded that inflation expectations have been relatively well anchored.

The rest of the paper is organized as follows. In Section 2 we describe the the French inflation-protected government debt and the data we use in the study. In Section 3 we describe the estimation methodology and the data of the OAT€i securities. in Section 4 we report the results of the fitted term structure of interest rates and discuss various issues about its fit. In Section 5 we

[5]We leave these issue in application to the OAT€i market for future research.

describe the backcasting procedure and implied breakeven inflation before and after the introduction of French inflation-protected debt. In Section 6 we analyze sensitivity of the OAT€i-based inflation compensation to international macroeconomic news. In section 7 we provide concluding remarks and define directions for ongoing and future research.

2 French Market for Inflation-Protected Debt

Agence France Trésor (the AFT, the French Treasury Department) has first issued inflation-protected debt in September 1998 linked to the French CPI, followed by the inflation-protected debt linked to the HICP. The coupons and face value of such bonds are adjusted to a relevant inflation index.[6] Both OATi and OAT€i make inflation-adjusted annual coupon payments, that can go up or down in nominal terms. The value of paid coupons is the product of the reference coupon rate, the reference face value, and the indexation coefficient (we call the index factor). Reference coupon rate and face value are mentioned in the bond contract. The indexation coefficient is the ratio of the current inflation level to the reference inflation level. The current inflation level is interpolated between the index value 3 months and 2 months ago. And reference level of inflation is the inflation level of a given year. Currently, the reference year is 2015 for both CPI and HICP. Similarly to the TIPS, all French inflation-linked bonds are protected against deflation at maturity (but for coupons) and the reimbursement at maturity cannot be smaller than the bond's face value.[7]

There are 18 OAT€i bonds issued since 2004. Table 1 provides the ISIN, the issue date, the coupon rate, the expiration date of the security (maturity), the term-to-maturity of the bond at the issuance, and the total number of available daily observations for the security. We collect bond characteristics from the AFT, cross-check them with Bloomberg, and collect daily bid prices from Bloomberg. In total we have 29,801 daily observations of OAT€i securities.

[6] The term "harmonized" reflects the fact that all the countries in the European Union follow the same methodology. This ensures a consistent comparison across different economies.

[7] We provide more details on the calculation of the inflation-adjusted coupons and the inflation-adjusted face value in Section 3.2.

As Table 1 reports, at the market onset in early 2000s, the bonds' coupon rates were around 3 percent and have been declining since then, likely reflecting declining interest rates and bonds' profitability globally, as the central banks around the world slashed interest rates to nearly zero following the aftermath of the GFC. OAT€i bonds have coupon rates of 0.1 percent since 2016. Concerning the term-to-maturity of the bond at the issuance, we have several observations. The average maturity of the OAT€i bonds is about 15.5 years, and these bonds have a wide spectrum of maturities. The shortest maturity bond (ISIN FR0108664055) of 3.8 years was issued in October 2006, and the longest maturity bond (ISIN FR0010447367) of 33.4 years was issued in March 2007. The ranges of time-to-maturities for other bonds available for estimation over our sample period are plotted in Figure 1. Each line represents one security. The date is shown on the horizontal axis and the remaining time-to-maturity is shown on the vertical axis in years. The upper-left and lower-right points of each line correspond to the issue date and to the bond expiration date, respectively.

Figure 2 plots the year-end notional outstanding amount and the number of securities of the French OAT€i securities. As Figure 2 shows, the OAT€i market enjoyed the steady growth since its first issuance in 2001. At the end of 2022, the OAT€i market had a notional outstanding amount of about €148 billions (about 160 billions of US Dollars as of January 31, 2023) representing about 8.5 percent of the U.S. TIPS market. Of note, the market of OAT securities linked to the domestic French CPI had a notional outstanding amount of about 66 €billions (about 72 billions of US Dollars as of January 2023) representing about 3.8 percent of the U.S. TIPS market. So, overall, OAT securities adjusted to HICP and to domestic CPI represent two-thirds and one-third of the inflation-indexed French debt market. So, albeit still small, the OAT€i debt market segment evolves rapidly and definitely presents a wealth of information that would potentially help monetary policy makers, central banking economists, and interested researchers to assess inflation expectations and risks around them.

Figure 3 plots historical levels of inflation in France and European Union. Panel A shows the long history of inflation in France, as measured for its domestic CPI. Similarly to the U.S., France had an era of hyperinflation in 1970s and 1980s, followed by "great moderation" period, when average inflation was in general lower than 5 percent. In the very late part of our sample, in 2021-2022, inflation in France was in the rise, due to a convolution of factors, such as the COVID-imposed supply-chain restrictions, and later in 2022, the energy crisis in Europe. Panel B shows the time variation in the CPI domestic rates more closely, since 1990. Inflation like in the end of our sample period, 2021-2022, is the highest in the last 30 years. The second biggest increase in inflation has occurred around 2007-2008, when realized inflation in France reached about 4 percent, which was much lower than most recent realized inflation that breached the 6 percent level in 2022. Panel C shows the HICP euro-area-based inflation rates since 1990. The HICP and French domestic CPI inflation rates closely track each other in the last 30 years. The correlation between the two time series is 99.36 percent since 1999.

3 Methodology

This section discusses the basic concepts and pricing of the OAT€i securities, Nelson-Siegel methodology of fitting the zero-coupon yield curves, and estimation methodology.

3.1 Basic concepts

The first and most basic concept for pricing any fixed-income asset is the discount function or the price of a zero-coupon bond that represents the value at time t of paying €1 at a future point of time T. We denote this bond price as $B(t,T)$, and it is worth introducing the continuously compounded zero-coupon yield on this bond denoted by $y(t,T)$. The zero-coupon bond price and this zero-coupon bond yield are linked via the relationship

$$B(t,T) = \exp\left[-y(t,T) \times (T-t)\right]. \tag{1}$$

or equivalently

$$y(t,T) = -\frac{1}{T-t} \ln B(t,T). \tag{2}$$

We price any coupon-bearing bond by the no-arbitrage argument. The difference between the nominal bonds and inflation-adjusted bonds is that the former The time $t-$price of a coupon bond maturing in $T-t$ years, promising $N_{c,t}$ coupon payments $\tilde{c} = c \times IF_t$, and paying inflation-adjusted principal $e1 \times IF_T$ in $T-t$ years, is given by

$$P(c,t,T) = \sum_{i=1}^{N_{c,t}} c \times IF_t \times B(t,t_i) + B(t,T) \times \max[IF_T; 1], \tag{3}$$

where t_i stands for the $i-th$ coupon payment date and t_{N_t} is the last payment date. Therefore, $t_{N_{c,t}} = T$.[8] Similarly, to the U.S. TIPS market, OAT€i securities are protected against deflation, so the final redemption amount cannot be less than the nominal face value. Therefore, OAT€i securities have a deflation option at the end of bond's life. Grishchenko, Vanden, and Zhang (2016) have shown that the value of this protection option did not exceed 6 and 1 basis points for 5- and 10-year bonds, respectively, during a brief period of a few months of deflation expectations during the GFC in 2007-2008. Christensen, Lopez, and Rudebusch (2016) similarly find that the value of the average deflation protection was close to zero. Given that euro area did not experience deflation period in our 2004-2022 sample, we leave the treatment of this feature in the OAT€i securities outside of the scope of the paper.

[8]The index factor IF_t that is used to adjust the cash flows of inflation-linked bond is calculated as the ratio between the "reference index" — the index value for a given date t — and the base index for the bond — namely the historical index value. The base index is determined when the bond is issued and it never changes. The reference index is calculated by interpolation of the two- to three-month lagged HICP value depending on the current day of the month,

$$\text{Index factor} = \frac{\text{Reference index}}{\text{Base index}}.$$

The coupon amount on day t_i is calculated as:

$$\text{Coupon amount}_{t_i} = \text{Coupon rate (in \%)} \times IF_{t_i} \times \text{Reference face value}.$$

At maturity, the redemption amount is calculated as:

$$\text{Redemption amount} = \text{Reference face value} \times \max[IF_T; 1].$$

Next, we provide definitions for par yields, forward rates, zero-coupon yields, and modified duration that we use in the curve fitting of the OAT€i securities.

Par yields: Market participants usually quote bond prices in terms of par yields. The par yield over a certain horizon T is the coupon rate at which a coupon bond security maturing at T will trade at par. To compute it, denote by Y the yield-to-maturity of the coupon bond; Y makes the present value of future (annual) cash flows equal to the coupon bond price. And one has

$$P\left(c,t,T\right)=\sum_{i=1}^{N_{c,t}}\frac{c}{\left(1+Y\right)^{t_i-t}}+\frac{1}{\left(1+Y\right)^{T-t}} \tag{4}$$

Its continuously compounded counterpart $y=\ln\left(1+Y\right)$. Adjusting for inflation indexation and setting the price of the coupon bond in equation (3) to $p(c,t,T)\ln P(c,t,T)=\$1$, we obtain the solution for the coupon rate $c\equiv y^c(t,T)$:

$$y^c(t,T)=\frac{1-B(t,T)}{\sum_{i=1}^{N_t}B(t,t_i)}. \tag{5}$$

Forward rates: The yield curve can also be expressed in terms of forward rates. A forward rate is the rate that an investor is able to lock in some time in the future by trading zero-coupon bonds of different horizons now. For example, if an investor wishes to lock in an $m-$period rate between T and $T+m$ years in the future, this forward rate, denoted as $f(t,T,m)$, can be obtained as follows:

$$f(t,T,m)=-\frac{1}{m}\ln\frac{B(t,T+m)}{B(t,T)}=\frac{1}{m}\left((T+m)y(t,T+m)-Ty(t,T)\right). \tag{6}$$

Taking the limit $m\to 0$, we obtain the instantaneous forward rate $f(t,T,0)$:

$$f(t,T,0)=\lim_{m\to 0}f(t,T,m)=y(t,T)+Ty'(t,T)=-\frac{\partial}{\partial T}\ln P(t,T). \tag{7}$$

Equation (7) essentially means that if the forward rate is above (below) the yield at a certain maturity, then the yield curve is upward (downward) sloping at that maturity.

Zero-coupon yields: The zero-coupon yield over time $T - t$ can be thought of as a continuous roll-over of the instantaneous forward rate investments and therefore can be expressed as the average of the forward rates over the horizon $T - t$:

$$y(t,T) = \frac{1}{T-t}\int_t^T f(t,x,0)dx. \tag{8}$$

It is useful to think of the forward rates rather than yields themselves as describing the yield curves. For example, the 30-year OAT yield can be represented as the average of the one-year forward rates over 30 years. While forward rates at shorter horizons might be influenced by cyclical factors (such as monetary policy expectations), at longer horizons forward rates appear to be reflecting more fundamental factors like changes in the risk attitudes of investors. Zero-coupon yields combine information about these two types of factors in one number, while forward rates disentangle this information.

Modified duration: Finally, we use the concept of the modified duration used in our yield curve estimation:

$$D = \frac{D_{Mac}}{1+Y}, \tag{9}$$

where Y stands for the yield-to-maturity and D_{Mac} is the Macaulay duration, where the latter is computed as the weighted average of the time (in years) that the investor must wait to receive the cash flows of a coupon bond:

$$D_{Mac} = \frac{1}{p(c,t,T)}\sum_{i=1}^{N_{c,t}}(t_i - t)\times c \times B(t,t_i) + (T-t)\times F \times B(t,T). \tag{10}$$

The modified duration is very popular among participants because it connects more explicitly the change in yields to the change in prices.[9]

[9] See, e.g. Martellini, Priaulet, and Priaulet (2003) for additional information about duration concepts.

3.2 Nelson-Siegel methodology

We broadly follow GSW and GMR approaches to fit the OAT€i-implied (real) yield curve but we use Nelson and Siegel (1987) (NS) functional form due to a limited number of bonds in our sample. The NS curve fitting approach describes the dynamics of the instantaneous forward rates $f(t, m, 0)$ m periods ahead at time t as follows:

$$f\left(t, m; \Theta\right) = \beta_0 + \beta_1 \exp\left[-\frac{m}{\tau_1}\right] + \beta_2 \frac{m}{\tau_1} \exp\left[-\frac{m}{\tau_1}\right], \tag{11}$$

where $\Theta = \{\beta_0,\ \beta_1,\ \beta_2,\ \tau_1\}$ are four Nelson-Siegel parameters. This methodology is quite effective at capturing the general shape of the OAT€i-implied yield curve, while smoothing through idiosyncratic variation in the yields of individual inflation-protected securities. $\beta_0 + \beta_1$ and β_0 have a natural interpretation of the short rates at the short and long end of the yield curve, respectively. The third term of the NS functional form identifies the location and the size of the hump in the term structure of interest rates.

Zero-coupon m-period continuously compounded zero-coupon yield at time t is obtained by integrating $f\left(t, m; \Theta\right)$ over the interest rate horizon $[t, t+m]$ using (8) and (11):

$$y\left(t, t+m; \Theta\right) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} + \beta_2 \left[\frac{1 - e^{-\frac{m}{\tau_1}}}{\frac{m}{\tau_1}} - e^{-\frac{m}{\tau_1}}\right] \tag{12}$$

where $\Theta = \{\beta_0, \beta_1, \beta_2, \tau_1\}$ are four parameters to be estimated. We have also fit Svensson (1994) functional form that has extra 2 parameters to allow for the second hump but the results are broadly similar. While this feature is important to capture the shape of the nominal yield curve, we find that the second hump presence is virtually nonexistent in the case of OAT€i yield curve. On average, Svensson fit yielded only 1 basis point gain in the mean absolute fitting error, relative to NS fit. However, there are significantly fewer OAT€i securities than nominal OAT securities — 18 (see Table 1) vs about 200, as of late May 2022— so the cost of fitting extra two parameters (for Svensson model) overcomes the benefit.[10]

Using observed OAT€i securities' bid prices on a daily basis, we estimate the NS model, by minimizing the sum of squared deviations between observed bond prices and model predicted values, the deviations being weighted by the inverse **modified** duration of the considered bond.[11] Specifically, we solve

$$\widehat{\Theta}_t = \arg\min_{\Theta_t} \sum_{i=1}^{N_t} \frac{1}{D_i} \left(\widehat{p}\left(t, t+m_i\right) - p(c, t+m_i; \Theta_t)\right)^2 \tag{13}$$

where $\widehat{p}$ is the observed price and N_t indicates the number of available bond prices at time t.

3.3 Estimation

We collect at time t a set of observed bond prices $P^o\left(c_k, t, T_k\right)$, $k = 1, ..., N_t$ where c_k and T_k are the coupon and maturity of the bond k, respectively, and N_t is the number of bond prices available on day t. Observed and model bond prices are related via the following relationship:

$$P^o\left(c_k, t, T_k\right) = P\left(c_k, t, T_k; \Theta_t\right) + \varepsilon_{t,k}, \tag{14}$$

where the vector of error terms $\varepsilon' = \left(\varepsilon_{t,1}, ..., \varepsilon_{t,N_t}\right)$ has a zero mean and a diagonal covariance matrix with possibly different variances on the diagonal. We exclude from the estimation securities with duration shorter than 1 year, following GSW and GMR papers. Excluding such securities prevents particular institutional details, unrelated to variation that reflects changes in fundamentals, to affect the fit and inference about the yield curve.[12] We do not impose any other filters in our estimation.

The set of parameters Θ_t is estimated by minimizing a weighted sum of squared errors whose weights are the inverses of the squared modified duration D defined in equation (9). More formally,

[10]Results are available upon request.

[11]Some authors use mid quotes (the average of bid and ask quotes). See, e.g., Ermolov 2017.

[12]One example is that some long-term asset (pension or insurance) managers tend to sell off shorter-duration bonds in re-balancing their portfolios.

the solution set satisfies

$$\widehat{\Theta}_t = \arg\min_{\Theta_t} \sum_{k=1}^{N_t} \left[\frac{P^o(c_k, t, T_k) - P(c_k, t, T_k; \Theta_t)}{D_k} \right]^2 \tag{15}$$

where D_k is the modified duration of the bond k. This particular weighting scheme is an appropriate way to deal with the nonlinear relation between yields and prices (see Svensson 1994; GSW; Gauthier and Simonato 2012). As explained by GSW (see their footnote 4 on page 2296), this way to proceed avoids converting bond prices into yields and therefore speeds up the calibration exercise.[13]

Unlike for the nominal curve fit case in GMR, we do not place any restrictions on four parameters in estimation, due to two reasons: a lower number of parameters to estimate for the OAT€i securities relative to the number of parameters used in curve fit for the nominal OAT securities: there are four parameters in the current Nelson-Siegel setting whereas we had 6 Svensson parameters in the case of nominal OATs; (2) a lower number of OAT€i securities relative to the amount of nominal OATs.[14]

We then compute, at a given time t, mean absolute error (MAE) of the model fit for particular maturity bins. $MAE_t(\tau)$ averages the absolute differences between the observed and Nelson-Siegel predicted yield-to-maturity of the bonds within a particular maturity bin τ:

$$MAE_t(\tau) = \frac{1}{N_t(\tau)} \sum_{k=1}^{N_t(\tau)} \left| y^o(c_k, t, T_k) - y\left(c_k, t, T_k; \widehat{\Theta}_t\right) \right|, \tag{16}$$

where $N_t(\tau)$ is the number of bonds within a particular maturity bin τ; $y^o(c_k, t, T_k)$ and $y\left(c_k, t, T_k; \widehat{\Theta}_t\right)$ are the observed and fitted yield-to-maturity of the bond k, respectively. MAE_t represents the mean absolute error across all securities and all maturities on a particular day.

[13]Note that some other authors use more standard durations. For example, HPW use the Macaulay duration in estimating the curve.

[14]We experimented with placing constraints on the parameters and did not find any meaningful differences in statistical fit and economic interpretation relative to when we did not constrain the parameters.

4 Results

In this section we discuss the Nelson-Siegel model fit and the OAT€i-implied term structure of real rates. We discuss the model fit in Section 4.1, then we we discuss the estimated term structure of real interest rates in Section 4.2, and, finally, we discuss how many factors drive the real interest rates and provide insights about predictability of OAT€-i real rates in Section 4.3.

4.1 Model fit

We estimate the Nelson-Siegel model parameters following the methodology discussed in Section 3. Figure 4 plots the daily time series of the mean absolute error MAE_t computed as in (16) across all available securities each day. The times series extends from November 17, 2004 — the first day when we had at least four securities traded to be able to estimate 4 NS parameters — until May 12, 2022 (the last day in our sample). Overall, fit exhibits quite a bit of variation, with errors ranging from under 2 basis points to above 12 basis points during the time of the GFC. the OAT€i fit worsened significantly in time leading to the GFC in early period of 2008, improved by the end of 2010 and was fluctuating around 3 to 5 basis points since then. The fit again worsened noticeably in the beginning of the COVID-19 pandemic, with some short improvement thereafter, and deteriorated again in 2022 at the end of our sample. Generally, the deterioration in fit happens in times of general strained market functioning and scarce liquidity as Hu, Pan, and Wang (2013) argue. They proxy the market illiquidity with the so-called *noise measure*, which is the square root of the average difference between predicted and observed yields on the market. They argue that when the trading capital is scarce it is more difficult to smooth out the arbitrage trades leading to observed yields away from their potential equilibrium values. The noise measure is closely related to the MAE measure shown in Figure 4. GMR computed both the MAE and noise measures for the nominal OAT securities in Figures 3 and 16 in their paper. These nominal OAT fit measures are highly correlated with the OAT€i MAE measure presented in Figure 4, indicating that both nominal and inflation-adjusted debt markets in France experienced strained conditions in similar periods. Figure 5 breaks the total MAE into six separate MAEs that represent the curve fit across different maturity bins for OAT€i securities: 0-to-2 years, 2-to-5 years, 5-to-10 years, 10-to-20 years, 20-to-30 years, and 30-to-50-years. As indicated by this, the curve fit was deteriorated nearly uniformly across maturities during the GFC. However, the deterioration in fit during the sovereign bond crisis in 2011-2012 was led primarily by the longer-term securities, 10-20-year segment (middle right panel) and 20-30-year segment (lower left panel). Most of the recent deterioration in the fit is accounted for by short-term securities (2-to-5-years, top right panel), with a smaller contribution to deterioration accounted by longer-term securities (5-to-10-year, middle left panel, 10-to-20-year, middle right panel, and 10-to-20-year, lower left panel). Of note, 0-to-2-year securities' fit is the worst around 2018 when the fitting error reached about 20 basis points. Table 3 reports the summary statistics for MAE across maturities.

4.2 The term structure of real and breakeven inflation rates

Figure 6 shows the estimated Nelson-Siegel real par yield curve implied by OAT€i securities on three different dates: January 2, 2009 (amidst the GFC), February 18, 2020 (beginning of COVID-19 pandemic), and May 12, 2022 (the end of our sample period). The left-hand side of these figures shows the model-implied par yield curve along with observed (blue round circles) and predicted (red crosses) continuously compounded yields. The predicted yields are computed using parameters that are estimated using bond quotes on the indicated day. The right-hand side of these figures shows security-specific fitting errors computed as differences between observed and predicted yield-to-maturity. As it is visible from the figure, the model fit has improved from 2009 to 2020 as more securities have been added by the AFT to the inflation-indexed debt market and market participants became more familiar with it.

Figure 7 plot the real instantaneous forward rate and implied zero-coupon yield curves on the left hand-side, and breakeven forward rates and zero-coupon yield curves in the right-hand side. Breakeven rate, also known as inflation compensation, is the difference between the nominal and real yields of comparable maturities. It represents a compensation to an investor in the nominal debt market for future inflation uncertainty. The figure shows these objects on three days November 28, 2007, July 27, 2016, and March 17, 2022. These graphs illustrate that the two sorts of curves have had different shapes over our sample period, Most of the time the real forward rates and yield curves are upward sloping, albeit the zero-coupon yield curve was inverted on intermediate maturities in 2016. Interestingly, breakeven yield curves range from inverted upward sloping in 2007 to upward-sloping in 2016 to downward-sloping in 2022 potentially representing different inflation regimes. For example, downward-sloping yield curve in 2022 likely reflects the fact that the short-term expected inflation was running significantly above the ECB target of two percent in large part due to high realized HICP inflation driven by the energy crisis in Europe but that market participants expect expected inflation converge back to the target.

Figure 8 plots the unconditional average real term structure computed as the mean value at each horizon point across the sample period. The term structure of real rates is upward sloping, on average, consistent with the real yield curve graphs in Figure 7 on the left. Our result is consistent with Ermolov (2017) who also documents the upward-sloping real term structure unconditionally that prevails in several countries in the world including France, but in contrast to Ang, Bekaert, and Wei (2007) who document the nearly flat real term structure of interest rates in the U.S. The latter authors do not use the TIPS market yet in their results as this market was at the very onset during the time of their study. We find that in our sample real rates are very low, ranging from -0.47% basis points at the 2-year horizon to -15 basis points at 5-year horizon to 0.37% at the 10-year horizon to 0.83% at the 30-year horizon. These estimates roughly in line with Ermolov (2017) who finds that unconditional liquidity-adjusted real rate at the 5-year horizon is 5 basis points in the 2001-2016 sample. The difference between our results and this study is likely due to a convolution of factors: (1) our sample period is longer and includes the period when nominal interest rates were bound by the zero-lower-bound environment driving real rates even lower; and (2) we did not explicitly adjust our term structure for liquidity premium, as Ermolov (2017) did. So, it is likely, that real liquidity-adjusted rates would be even lower in our sample.

4.3 Factors driving real and breakeven interest rates

In this section we describe the factors that likely affect the real interest rates and contribute to its predictability. Figure 9 plots time series of 2-, 5-, 10- and 30-year zero-coupon real yields in our sample. The figure shows real rates have been declining since the end of the GFC, the pattern that holds for all maturities with reasonably high degree of variation in them.[15] At the end of our sample, the real rates were around negative 1% for the 5- and 10-year maturities.

What drives variation in OAT€i-implied real and breakeven rates? Table 4 reports the volatility (standard deviation across our sample) of forward breakeven rates for 5-, 7-, and 10-year maturities and at one-day, one-month, three-month, and six-month horizons. The volatility of breakeven rates increases with horizon as Figure 4 reports in its second column. We also compute the variance ratio (VR) statistic of Lo and MacKinlay (1988) that tests the predictability of the underlying series. Under the null, there is no predictability in the series. As the third column reports, the VR test rejects the null everywhere except for the 10-year breakeven rate at the six-month holding period, but the corresponding test statistic is not far from the 10% level. Overall, this evidence overall implies that inflation compensation is predictable, to a high degree.

Next, we report the principal component(PC) decomposition of the nominal, real, and breakeven rates in Table 5. The main observation in this Table is that the second PC is much more important for the variation in breakeven rates where it explains about 12 percent of overall variation in breakeven rates, in contrast to variation in nominal and real rates when it explains only 3 to 5 percent of variation in nominal and real rates, respectively.

5 Backcasting inflation

In this section we briefly describe conduct the backcasting exercise to estimate breakeven rates before the introduction of the OAT€i securities.

Section 4.2 reports the time series for breakeven inflation rates that provide a gauge for inflation

[15]The decline in 2-year rates is less visible due to a particularly poor fit around the European sovereign bond crisis.

expectation in France and Europe, in general, and that became available thanks to the introduction of the OAT€i securities in 2004. In this section we conduct backcasting exercise and provide estimated inflation compensation series that go back to January 1999, the onset of the European monetary union.

We are interested in time series for longer-term inflation compensation, namely, 5-year 5-year forward period, in order to be able to assess the stability of inflation expectations in Section 6. To that end, we follow the GSW methodology and regress available 5-year 5-year forward breakeven rates for the 2004-2022 sample period on the three principal components of the nominal yields computed using Svensson model in GMR. We use then fitted regression coefficients and nominal principal components available for a longer period of time to compute the implied inflation compensation.

We find that the R-square of our regression is about 85% and These high enough values indicate an important relationship between the nominal and breakeven rates. Figure 11 plots the backcast of the five-year forward five-year breakeven rate as the orange line and the "observed" breakeven inflation rate as the blue line. First, we observe that when observed breakeven rates are available, the two series come relatively close to each other, suggesting a high degree of relationship indeed between the nominal yields and breakeven rates. Second, backcast inflation compensation series is smoothed through some idiosyncratic variations, likely leaving us with the series that reflect more variation in fundamentals. Third, we observe that backcast 5-year 5-year forward inflation compensation has had an almost monotonic declining trend since the GFC when it reached its minimum of about 1.2 percent at the onset of the COVID-19 pandemic crisis. Of note, the 5-year 5-year forward inflation compensation has been the highest in 2000, when it reached almost 3 percent. Fourth, at the end of our sample in 2002, observed breakeven rate spiked back up to levels around 3 percent leading to an increase to about 2 percent of the backcast inflation compensation, a level has not seen seen since 2015.

6 Sensitivity of inflation compensation to macroeconomic news

In this section we describe our results about sensitivity of inflation compensation measures to macroeconomic news in France, United States, and Germany. We measure macroeconomic news as the surprise component of a data release, similar to Beechey, Johannsen, and Levin (2011). The surprise is calculated as the difference between the actual released value and the median survey expectation of a particular macroeconomic series, standardised by the corresponding standard deviation of the surprise series. We hand collected news data from Bloomberg L.P. Bloomberg's survey that is based on a selection of professional economists who submit their forecasts to Bloomberg before or on the Friday prior to the data release.

Then we regress the one-day changes of the 1-year 1 year forward, 1-year 4 years forward, 1-year 6 years forward, 1-year 9 years forward, and 5-year and 5 years forward breakeven rates on the macro surprise series. These horizons represent a set of horizons that a central banker interested in a stability of inflation expectation would typically look for. For example, the ECB targets the medium-term inflation expectations, and the 1-year 4 years forward inflation compensation would be an appropriate measure to look for when one would like to learn about the stability of inflation expectations in the euro area. As another example, the Federal Reserve targets the longer-term inflation expectations and the 5-year 5 year forward inflation compensation would be an appropriate horizon to study when one would ask how well anchored inflation expectations are in the U.S.

6.1 Evidence from forward measures of inflation compensation

We broadly follow methodology and series in Beechey, Johannsen, and Levin (2011).

For France, we use Consumer Confidence, Business Confidence, and CPI series as measures of macroeconomic activity and compute surprises respectively. Table 6 reports the results. We observe, that in general, inflation compensation series are relatively irresponsive to French macro news with the exception of, possibly, shorter-term inflation compensation (1-year 1 year forward), which is reasonable, as short-term inflation compensation is heavily influenced by the realized inflation prints as well as potentially noisy fit at those horizons as we discussed in Section 4.

For the U.S., we use Capacity Utilization, University of Michigan Consumer Sentiment, Personal Consumer Expenditures Core index, Initial Jobless Claims, ISM Manufacturing, and Conference Board Index as measures of macroeconomic activity. Table 7 reports the results. In general, and similar to results reported in Table 6, the reaction in inflation compensation measures to US macro news appears to be relatively muted at nearly all horizon with perhaps some exceptions at the short horizons.

For Germany, we use Business Climate and Expected Economic Growth indexes. Results reported in Table 8 are broadly similar to those reported in Tables 6 and 7.

Overall, we conclude that based on the above evidence, inflation expectations appear relatively well anchored in our sample, even in the presence of the GFC, COVID-19 pandemic crisis, and European energy crisis, most recently.

7 Conclusion and future research

We study the evolution of real rates implied by relatively recent and rapidly growing market for inflation-indexed French sovereign bonds (*Obligations Assimilables du Trésor (OAT) indexée sur l'indice des prix à la consommation de la zone euro*). We have four main results in the paper. First, using the smoothed term structure of real rates implied by the Nelson and Siegel (1987) model, we look at the basic properties of the real rates. We observe that real rates were declining since the global financial crisis and that they were on average very low, namely, lower than 1 percent. One average, the real rates were negative at maturities up to 5 years. The term structure of real rates was upward sloping, Second, we have shown that the breakeven inflation rates are driven by three latent factors (principal components), where the second and the third factor are materially more important than for either OAT nominal or OAT€i real rates. Third, We have backcast forward inflation compensation to the period prior to the introduction of the French inflation-protected securities (before 2004). We have shown that far forward inflation compensation was declining since the global financial crisis. Fourth, we explored the sensitivity of forward measures of inflation compensation to international macroeconomic news and concluded that inflation expectations have been relatively well anchored.

So far, we left out several important considerations that we are currently working and intend to include in the draft in the very near future. First, we did not include in the study the OATi bonds — securities linked to the domestic French CPI. It is interesting to see whether the information from OAT€i and OATi is complimentary to each other. Our preliminary findings suggest that the answer to this question is "yes". Second, we did not adjust our OAT€i-implied yields for illiquidity considerations, which is likely an important issue for this market. Third, we intend to decompose inflation compensation series into inflation expectations and inflation risk premium components to better understand the drivers behind time variation in inflation compensation. These three main topics present our focus for ongoing research.

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Table 1: Summary of OAT€i securities

ISIN	Type	Issue	Coupon	Maturity	Term	Obs
FR0000188013	OAT€i	10/31/2001	3	07/25/2012	10.73	2793
FR0000188799	OAT€i	10/31/2002	3.15	07/25/2032	29.73	4215
FR0010050559	OAT€i	01/22/2004	2.25	07/25/2020	16.51	3902
FR0010135525	OAT€i	11/23/2004	1.6	07/25/2015	10.67	2787
FR0108664055	OAT€i	10/04/2006	1.25	07/25/2010	3.81	1111
FR0010447367	OAT€i	03/14/2007	1.8	07/25/2040	33.37	3079
FR0010899765	OAT€i	05/25/2010	1.1	07/25/2022	12.17	2250
FR0011008705	OAT€i	02/16/2011	1.85	07/25/2027	16.44	2057
FR0011237643	OAT€i	07/25/2011	0.25	07/25/2018	7.00	1636
FR0011427848	OAT€i	02/26/2013	0.25	07/25/2024	11.41	1530
FR0011982776	OAT€i	06/18/2014	0.7	07/25/2030	16.10	1188
FR0013140035	OAT€i	03/21/2016	0.1	03/01/2021	4.94	730
FR0013209871	OAT€i	10/05/2016	0.1	07/25/2047	30.80	588
FR0013327491	OAT€i	04/06/2018	0.1	07/25/2036	18.30	198
FR0013410552	OAT€i	03/25/2019	0.1	03/01/2029	9.94	822
FR0013519253	OAT€i	06/22/2020	0.1	03/01/2026	5.69	497
FR0014001N38	OAT€i	01/25/2021	0.1	07/25/2031	10.49	342
FR0014008181	OAT€i	02/01/2022	0.1	07/25/2053	31.48	76

This table reports our sample of the *Obligation Assimilables du Trésor ndexée sur l'indice des prix à la consommation de la zone euro* (OAT€i) issued between October 31, 2001 and May 31, 2022. Source: *Agence France Trésor* and Bloomberg.

Table 2: Summary statistics of fitted OAT€i yields

	2yr	5yr	7yr	10yr	20yr	30yr
Panel A: Zero-coupon real rates						
Mean	-0.4777	-0.1546	0.1096	0.3666	0.7159	0.8286
Max	18.6449	2.7327	2.8898	2.9722	2.8698	2.9332
Min	-4.7358	-3.1122	-2.5846	-2.1377	-1.5858	-1.4108
Std. Dev.	1.8019	1.2310	1.2137	1.1855	1.1167	1.0798
Skewness	2.2728	0.3113	0.1399	-0.0153	-0.2130	-0.2766
Kurtosis	12.8115	-0.9760	-1.2214	-1.3245	-1.3041	-1.2487
AR(1)	0.9332	0.9989	0.9992	0.9992	0.9992	0.9961
Panel B: Forward real rates						
Mean	-1.2378	0.6261	0.8795	1.0252	1.0669	1.0422
Max	8.7744	3.2822	3.2566	3.0594	3.4327	3.7972
Min	-103.3574	-1.7129	-1.4255	-1.2726	-1.1830	-1.1820
Std. Dev.	4.7164	1.3017	1.2143	1.1304	1.0325	1.0118
Skewness	-9.7174	-0.1095	-0.2824	-0.3791	-0.4008	-0.3534
Kurtosis	153.2756	-1.5479	-1.4022	-1.2538	-1.1161	-1.0837
AR(1)	0.7938	0.9990	0.9989	0.9987	0.9983	0.9942

This table reports summary statistics of the Nelson and Siegel (1987) fitted zero-coupon yields (Panel A) and instantaneous forward rates (Panel B) for 2-, 5-, 7-, 10-, 20- and 30-year maturities by our sample of the inflation linked OAT€i securities. All statistics are reported in the annualized percent. Sample: November 17, 2004 to May 12, 2022. Frequency: daily.

Table 3: Summary statistics of fitting errors

	0-2yr	2-5yr	5-10yr	10-20yr	20-30yr	30-50yr	total
Mean	0.8059	2.1694	2.5042	2.6397	2.6937	1.2287	2.5694
Max	21.0462	17.5410	20.7871	16.7530	16.4227	16.6121	12.6540
Min	0.0001	0.0001	0.0014	0.0012	0.0001	0.0022	0.0028
SD	3.6655	2.3180	2.0722	2.4158	2.6236	3.6883	1.4728

This table reports descriptive statistics of the daily fitting errors for OAT€i securities in the indicated maturity bins. The fitting errors are defined as the mean absolute errors between observed and predicted yields according to Nelson and Siegel (1987) model. The sample period is from November 17, 2004 to May 12, 2022. The errors are reported in basis points.

Table 4: Volatility of changes in 5-, 7-, and 10-year forward inflation compensation at selected horizons

Horizons	Std. Dev.	Variance Ratio
Panel A: instantaneous 5-year forward rates		
One day	5.0971	
One month	15.7091	-2.8675***
Three month	24.2569	-2.2629**
Six months	26.9963	-2.2356**
Panel B: instantaneous 7-year forward rates		
One day	4.7015	
One month	14.4088	-3.3209***
Three month	21.8624	-2.4741**
Six months	26.3411	-2.1068**
Panel C: instantaneous 10-year forward rates		
One day	4.8239	
One month	13.9302	-2.5733***
Three month	20.4276	-1.9564**
Six months	26.3862	-1.5583

This table reports the standard deviation of one-day and one-, three- and six-month changes in the 5-, 7-, 10-year instantaneous forward rates of inflation compensation. They are computed assuming 22 days per month. The variance ratio statistic is the heteroskedasticity robust test statistic of Lo and MacKinlay (1988) and has a standard normal asymptotic distribution. *, ** and *** denote significance at the 10, 5 and 1 percent significance levels respectively.

Table 5: Principal Component Decomposition

PC	Nominal	RealEU	Breakeven
PC1	0.9679	0.9427	0.8524
PC2	0.0281	0.0472	0.1181
PC3	0.0031	0.0092	0.0269

This table reports the percent of variance in Svensson (1994) fitted nominal zero-coupon yields, Nelson and Siegel (1987) fitted real zero-coupon yields, and implied breakeven rates explained by the first three principal components. The sample period for nominal rates is from November 17, 2004 to May 12, 2022. Frequency: monthly.

Table 6: OAT€i inflation compensation and French macro news

	Horizon				
	1y1y	1y4y	1y6y	1y9y	5y5y
Consumer Confidence	0.635	0.003	0.003	0.003	0.003
	(0.298)	(0.007)	(0.005)	(0.004)	(0.004)
Business Confidence	-0.011	-0.006	-0.003	-0.000	-0.002
	(0.007)	(0.004)	(0.005)	(0.003)	(0.004)
CPI	2.690	-0.002	0.000	-0.001	-0.000
	(4.211)	(0.004)	(0.002)	(0.002)	(0.002)

This table reports the sensitivity of OAT-based inflation compensation to several French macroeconomic news. Results are reported for several horizons: 1-year 1 year ahead, 1-year 4 years ahead, 1-year 6 years ahead, 1-year 9 years ahead, and 5-year 5 years ahead inflation compensation. The sensitivity is estimated by β using the methodology described in Section 6. Frequency: monthly.

Table 7: OAT€i inflation compensation and U.S. macro news

	Horizon				
	1y1y	1y4y	1y6y	1y9y	5y5y
Capacity Utilization	-2.289	0.002	0.000	0.002	0.001
	(2.047)	(0.004)	(0.002)	(0.003)	(0.002)
UMich Consumer Sentiment	-2.449	-0.009	-0.003	-0.003	-0.003
	(2.383)	(0.008)	(0.004)	(0.004)	(0.003)
PCE Core	-1.896	0.005	-0.001	0.003	0.001
	(2.065)	(0.006)	(0.003)	(0.002)	(0.002)
Initial Jobless Claims	0.000	0.000	0.000	0.000	0.000
	(-0.341)	(0.002)	(0.002)	(0.002)	(0.002)
ISM Manufacturing	0.000	0.000	0.000	0.000	0.000
	(0.043)	(0.000)	(0.010)	(0.006)	(0.008)
Conference Board Index	0.000	0.000	0.000	0.000	0.000
	(-1.375)	(0.003)	(-0.001)	(-0.003)	(-0.002)

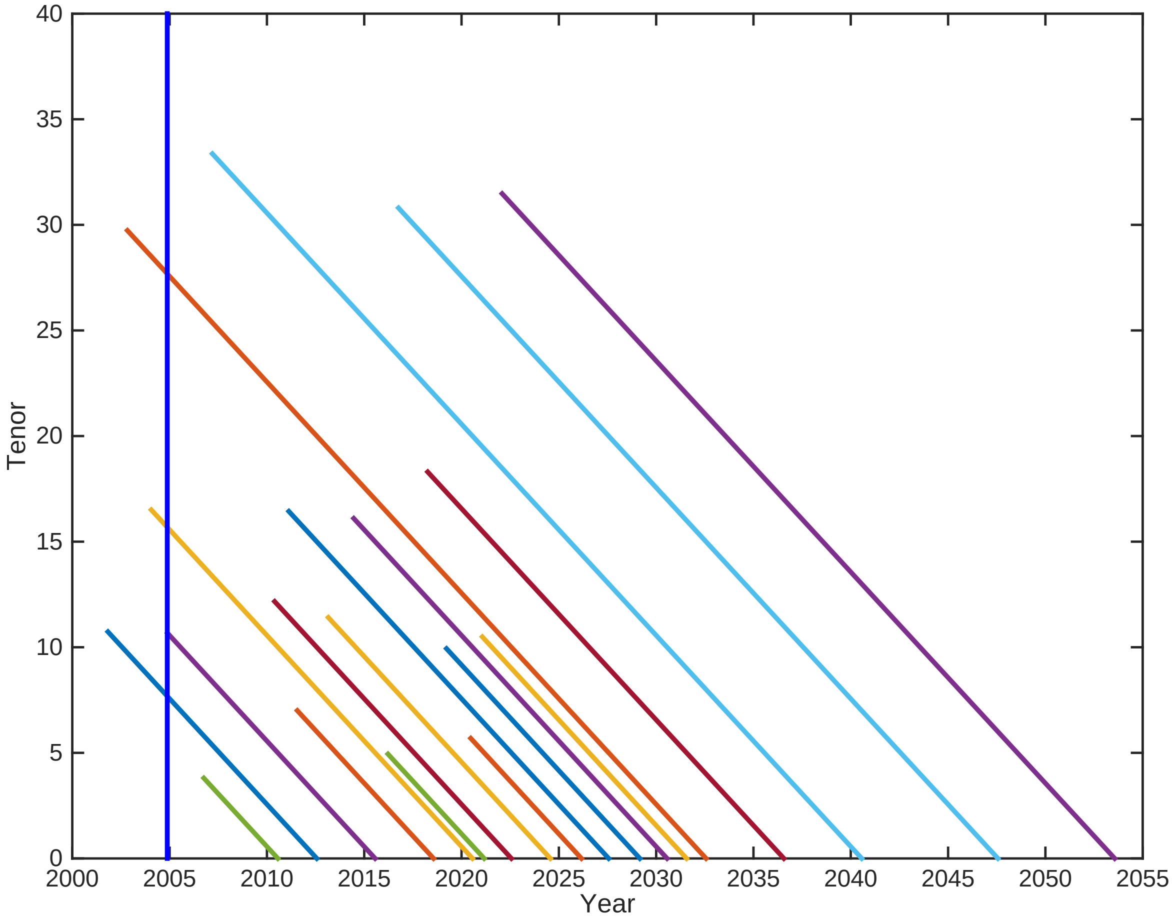
This table reports the sensitivity of OAT-based inflation compensation to several US macroeconomic news. Results are reported for several horizons: 1-year 1 year ahead, 1-year 4 years ahead, 1-year 6 years ahead, 1-year 9 years ahead, and 5-year 5 years ahead inflation compensation. The sensitivity is estimated by β using the methodology described in Section 6. Frequency: monthly.

Table 8: OAT€i inflation compensation and German macro news

	Horizon				
	1y1y	1y4y	1y6y	1y9y	5y5y
Business Climate	0.000	0.000	0.000	0.000	0.000
	(-2.010)	(0.003)	(0.010)	(0.003)	(0.007)
Expected Economic Growth	0.000	0.000	0.000	0.000	0.000
	(1.281)	(-0.002)	(-0.001)	(0.001)	(0.000)

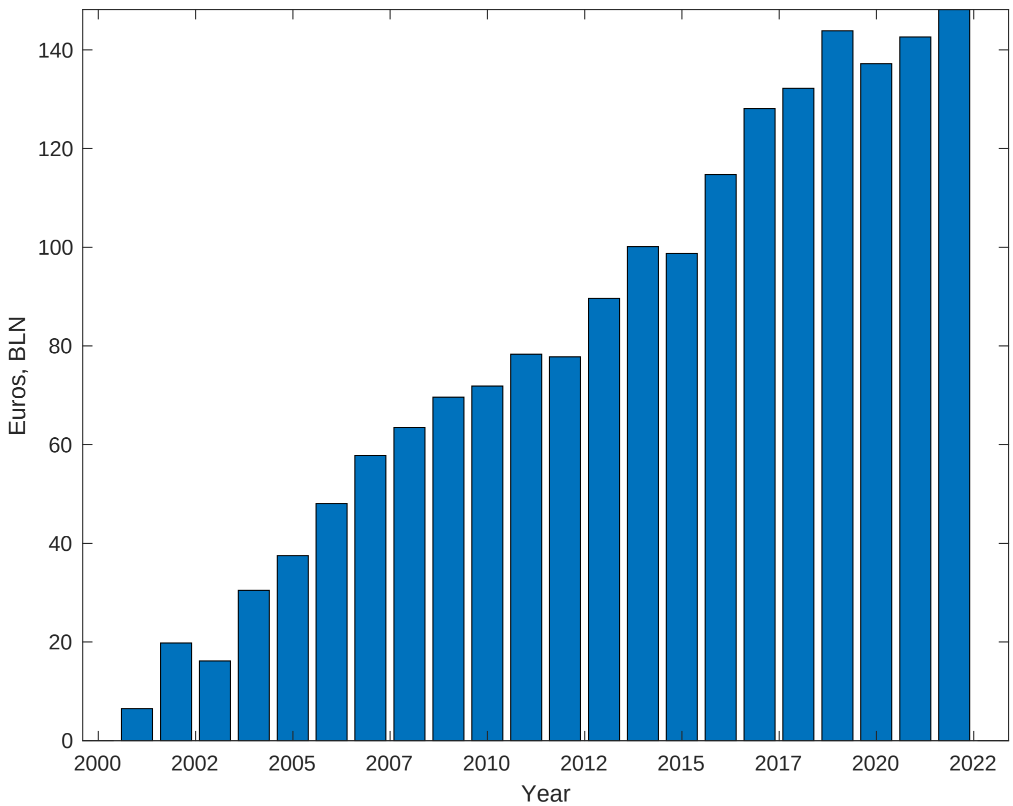
This table reports the sensitivity of OAT-based inflation compensation to several German macroeconomic news. Results are reported for several horizons: 1-year 1 year ahead, 1-year 4 years ahead, 1-year 6 years ahead, 1-year 9 years ahead, and 5-year 5 years ahead inflation compensation. The sensitivity is estimated by β using the methodology described in Section 6. Frequency: monthly.

Figure 1: Maturity Distribution of the French OAT€i debt



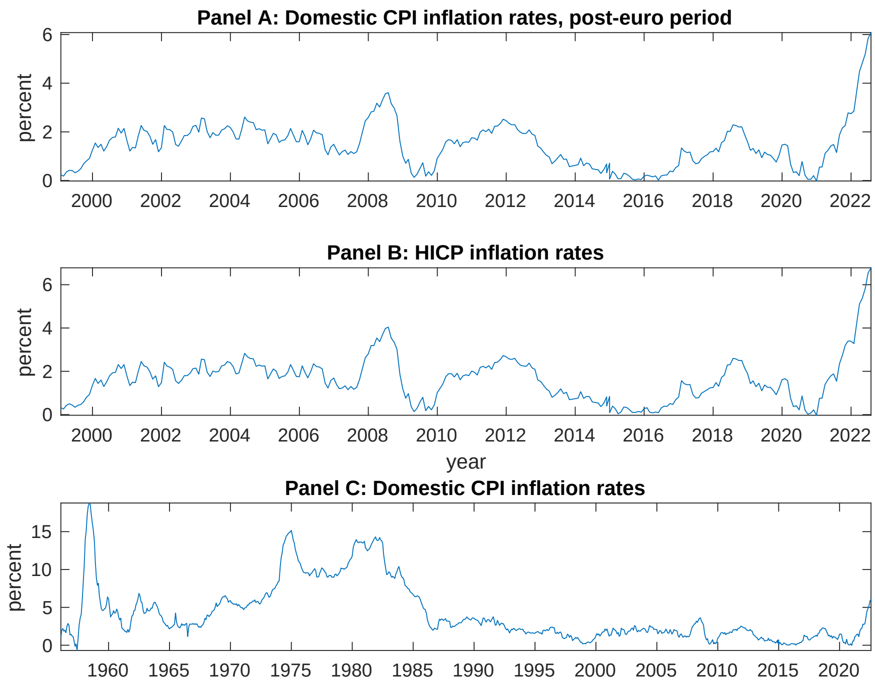
This figure shows the maturity structure of the French OAT€i securities issued from October 31, 2001 to May 31, 2022. Source: Bloomberg.

Figure 2: Notional amount of the French OAT€i debt



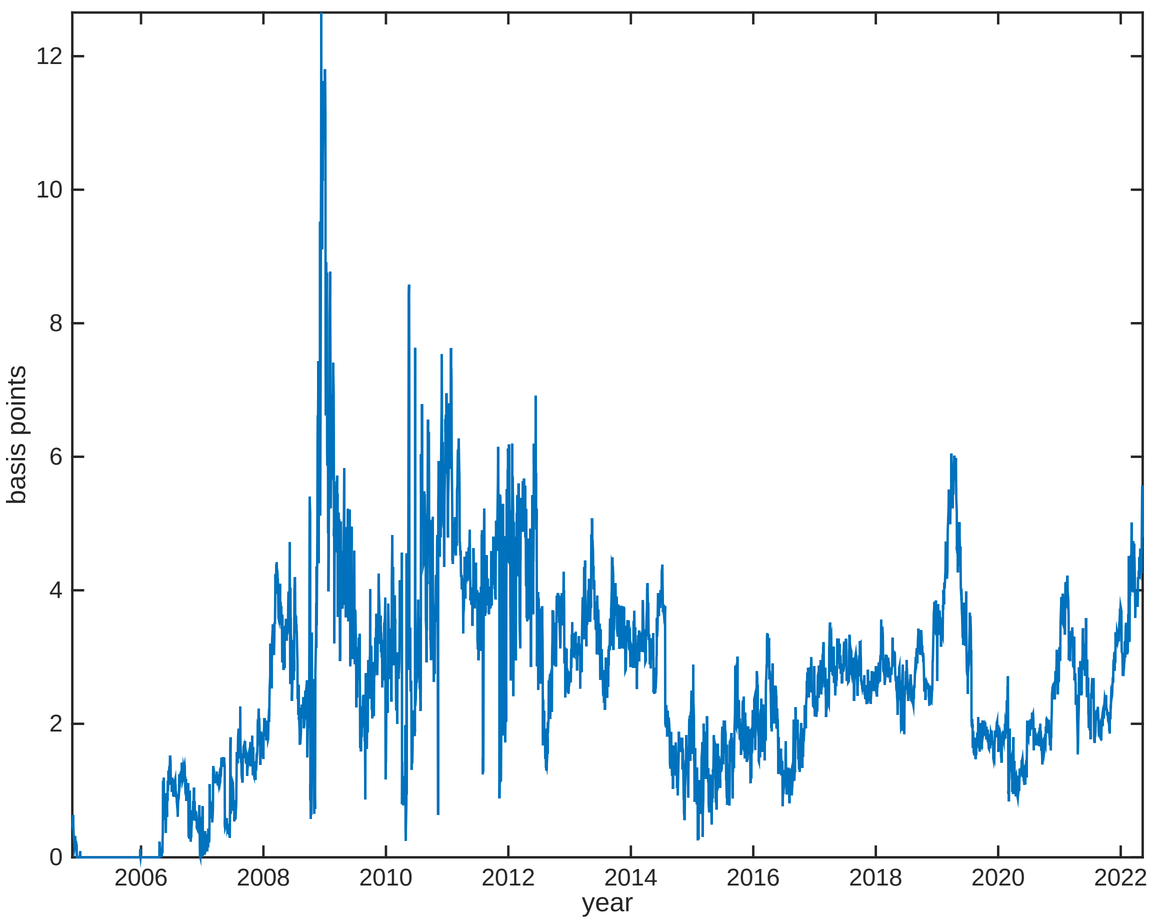
This figure shows the outstanding amount of the French OAT€i debt as of December 31, 2021. Data are hand-collected and merged from the monthly newsletters of *Agence France Trésor*.

Figure 3: Time series of French realized inflation rates



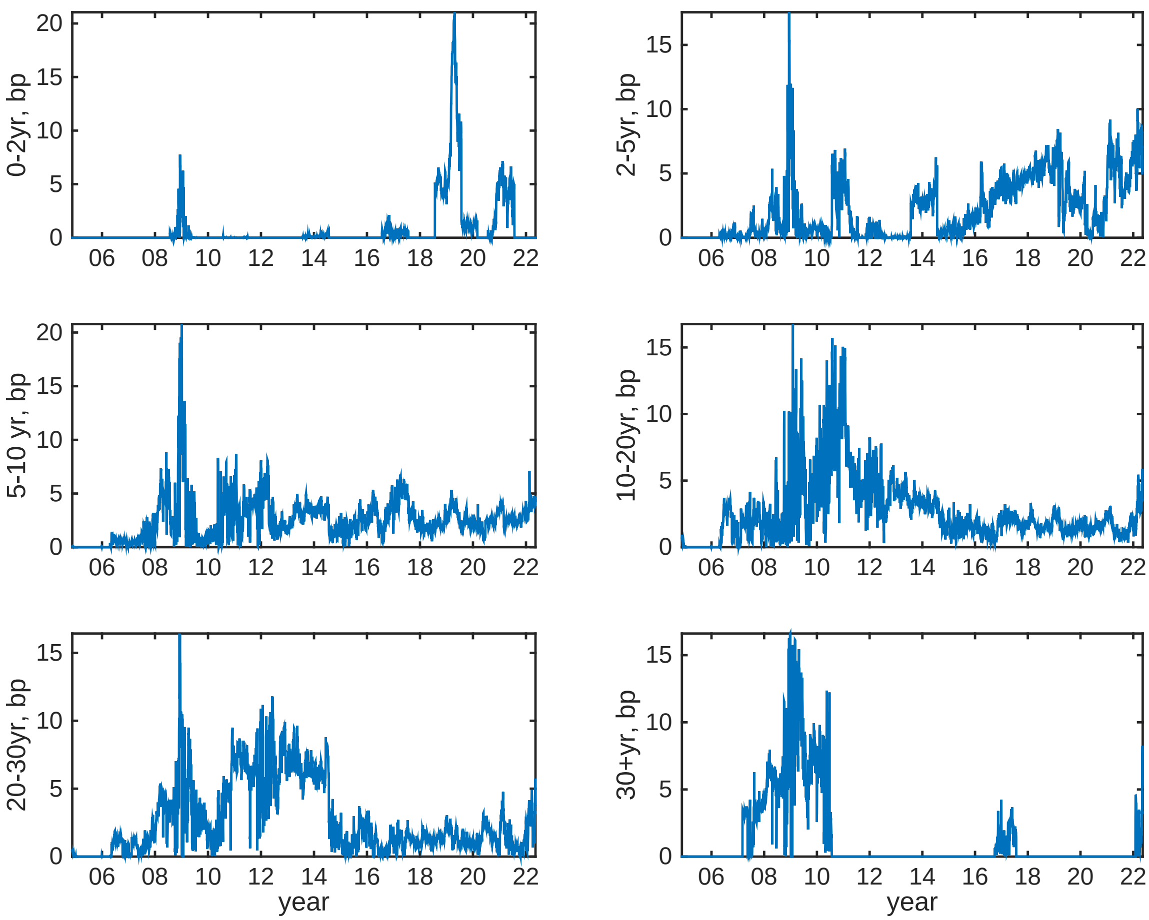
Panels A and B report French CPI and euro-area HICP annualized inflation rates starting from January 1999, respectively, until July 2022. Panel C plots a longer history of the French domestic CPI annualized inflation rates (HICP) from January 1956 to July 2022. Frequency: monthly. Source: Worldwide Inflation data, inflation.eu.

Figure 4: Fitting Errors for OAT€i sample



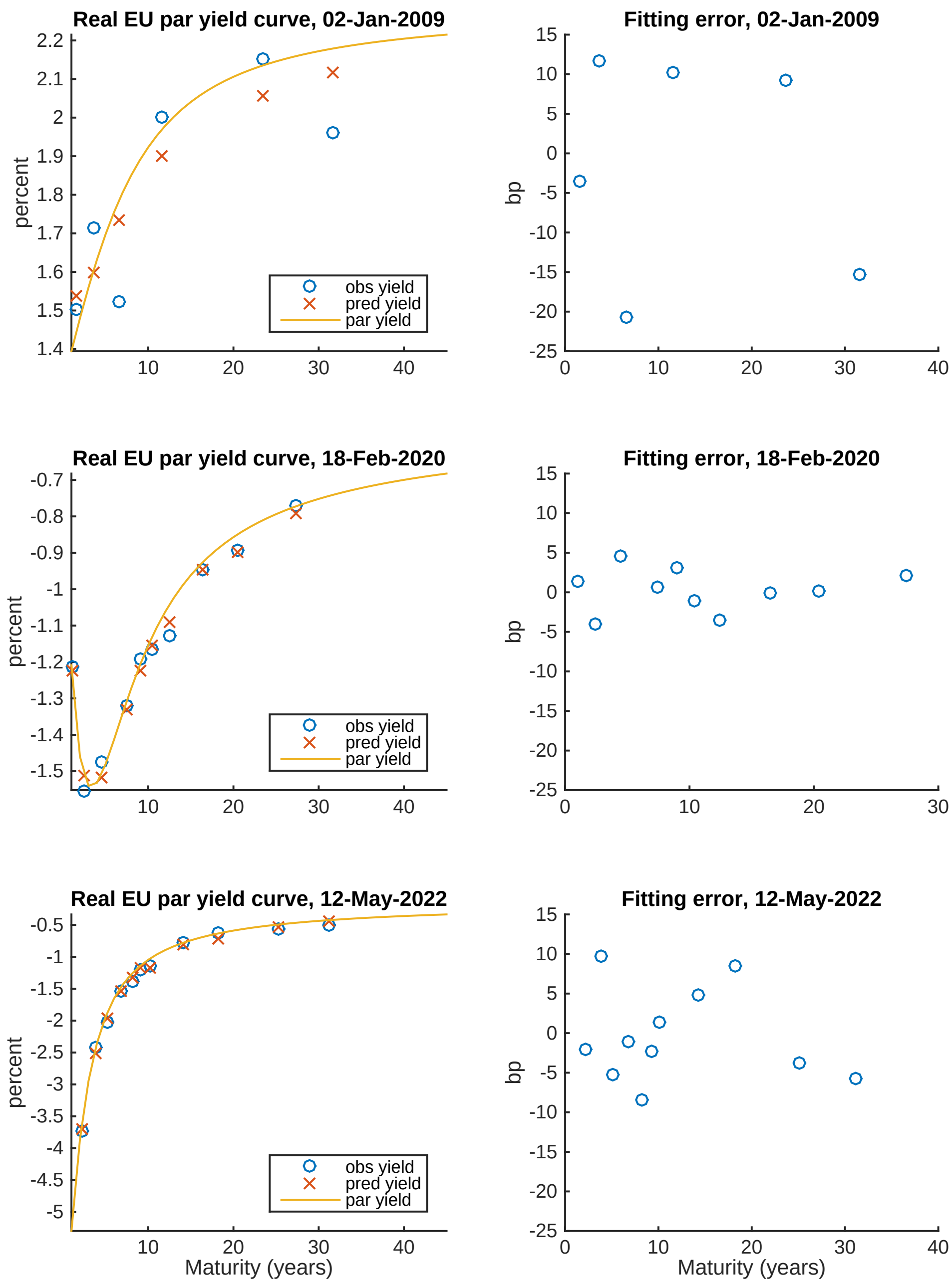
This figure shows the total fitting error implied by the Nelson and Siegel (1987) model. The fitting error is computed as the mean absolute error between the predicted and the observed yields across all available OAT€i securities on a particular day. The fitting errors are shown in basis points. Sample period: November 17, 2004, to May 12, 2022. Frequency: Daily.

Figure 5: Maturity-specific Fitting Errors for OAT€i sample



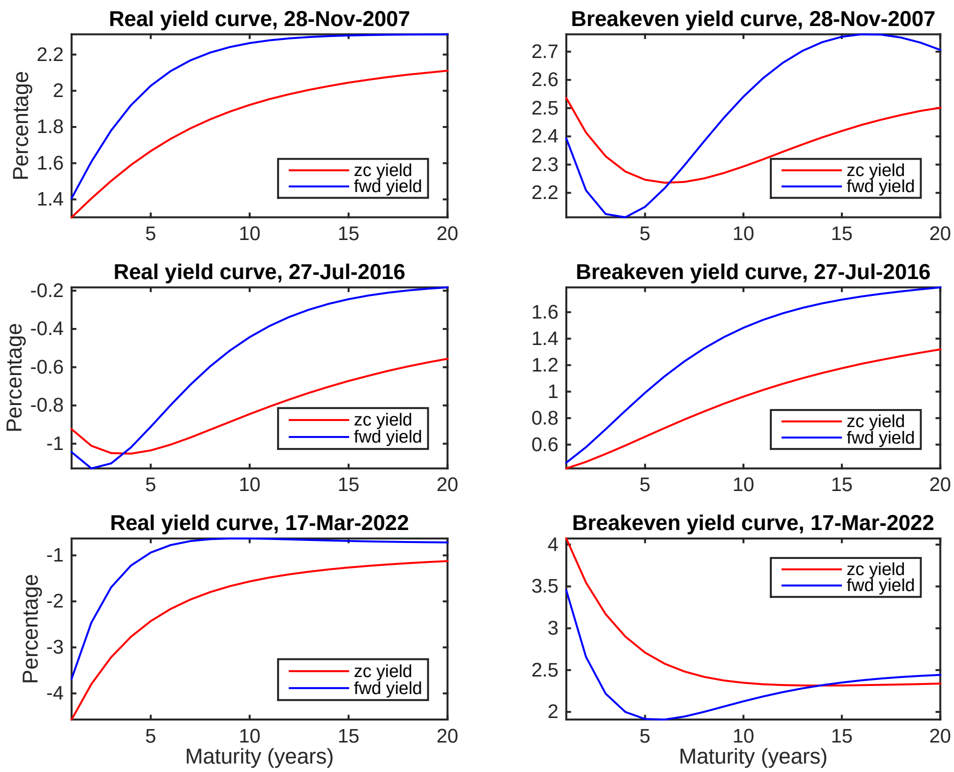
This figure shows the fitting errors of the Nelson and Siegel (1987) model implied by the OAT€i securities. The fitting error is computed as the mean absolute error between the predicted and the market prices in a certain maturity bin. We report the errors for four maturity bins: 0-2-year, 2-5-year, 5-10-year, 10-20-year, 20-30-year, 30+ year bin. The fitting errors are shown in basis points. Sample period: November 17, 2004, to May 12, 2022. Frequency: Daily.

Figure 6: Par Yield Curve for OAT€i sample



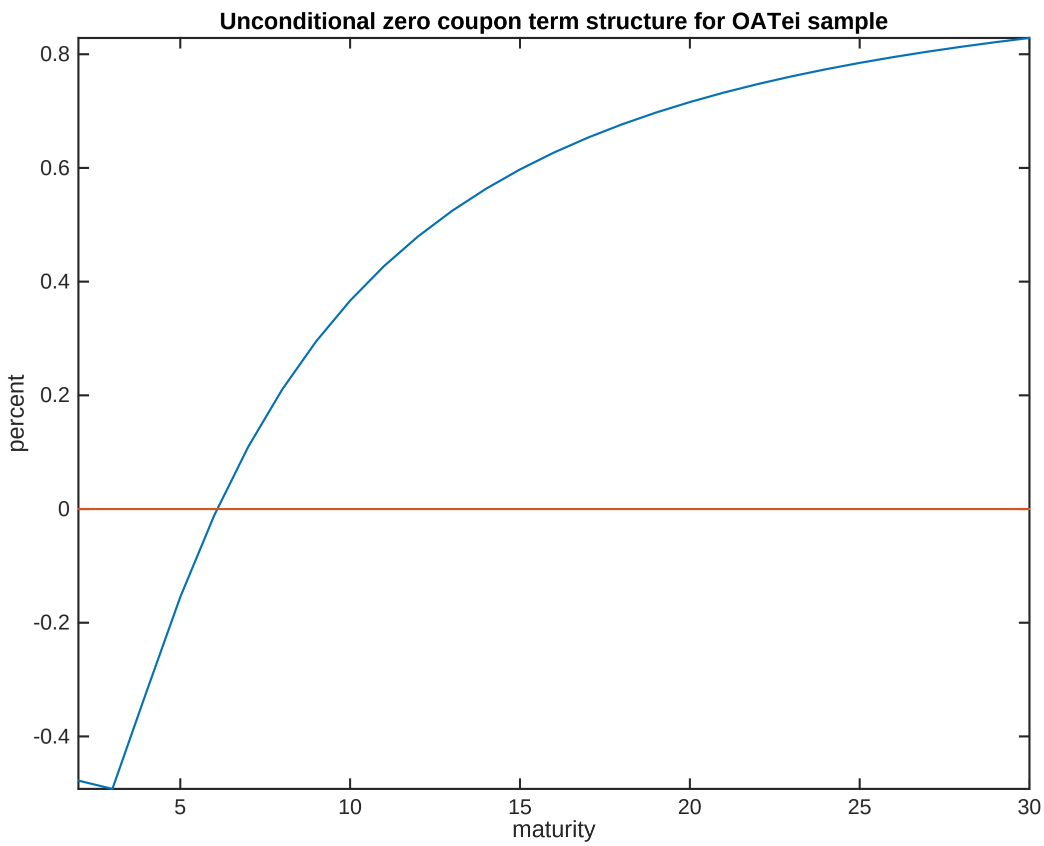
This figure shows the par yield curve and the fit of individual OAT€i securities (left-hand side charts) along with security-specific fitting errors (right-hand side charts) in three days across the sample period: January 2, 2009, February 18, 2020, and May 12, 2022. The fitted real yields are reported in annualized percent, the fitting errors are reported in basis points.

Figure 7: Zero-Coupon and Forward rates for OAT€i sample



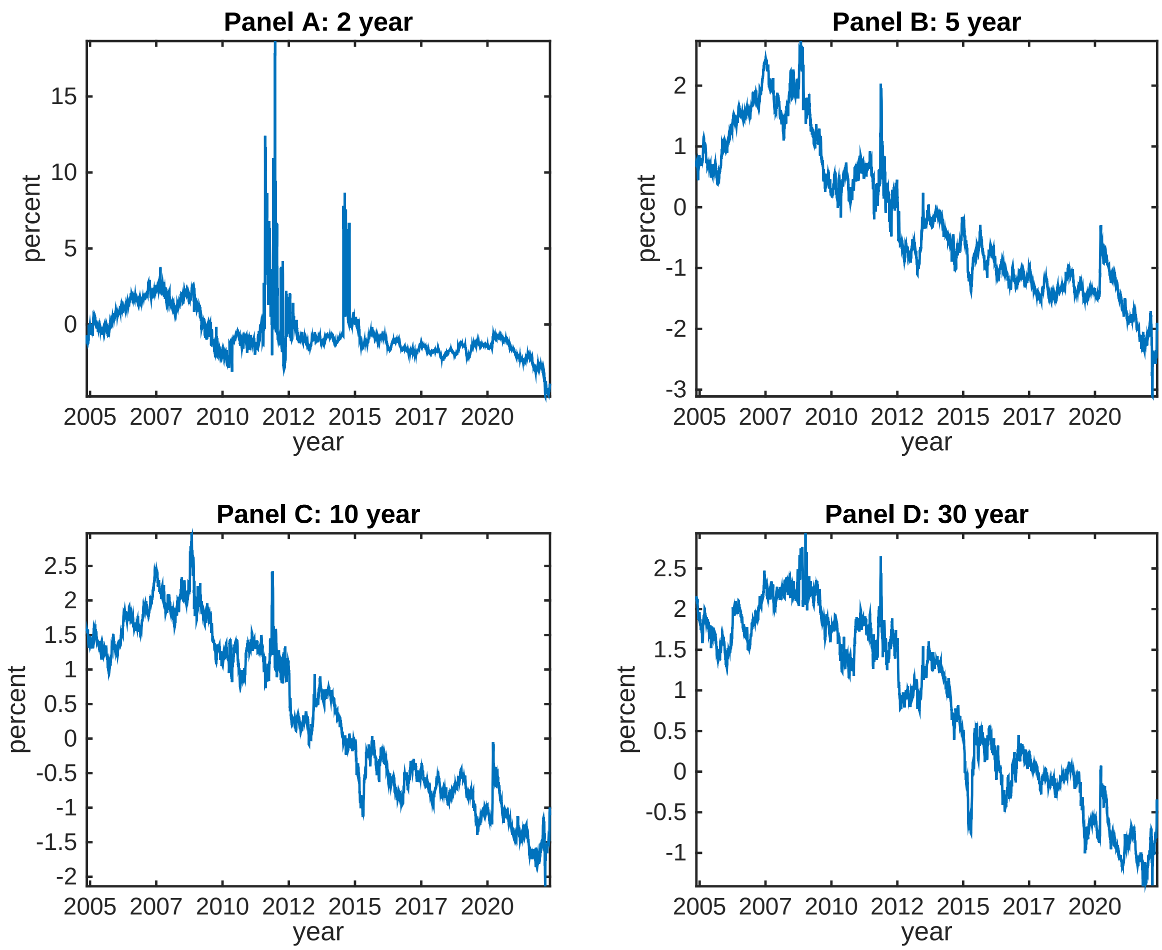
This figure shows zero-coupon and forward real yields implied by the price quotes of OAT€i securities (left-hand side charts) along with inflation compensation (right-hand side charts) in three days across the sample period: November 28, 2007, July 27, 2016, and March 17, 2022. The fitted real and breakeven yields are reported in annualized percent.

Figure 8: Unconditional Term Structure of Zero-Coupon Real Yields



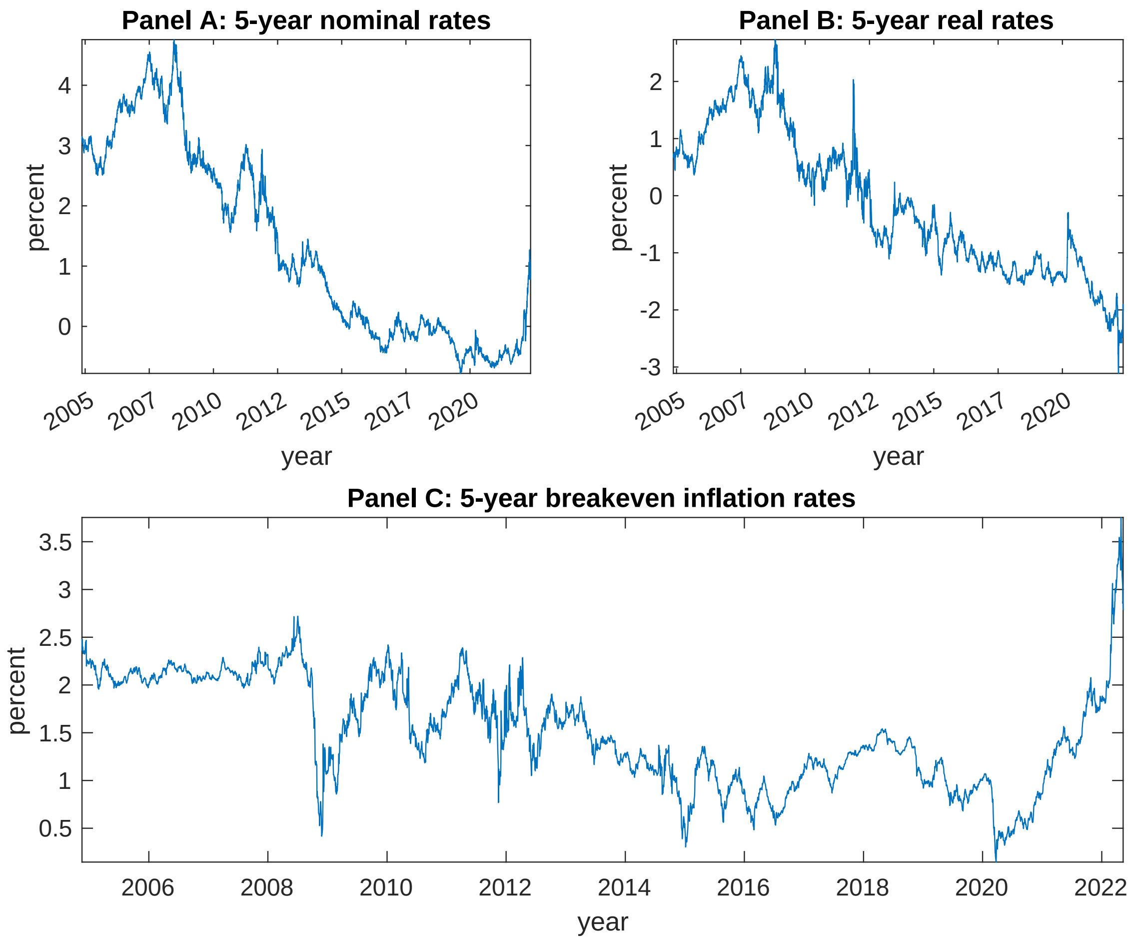
This figure shows the unconditional zero-coupon term structure of real rates implied by the OAT€i quotes. The unconditional sample mean is computed using sample from November 17, 2004 to May 12, 2022 and shows the term structure for Nelson and Siegel (1987) fitted zero-coupon yields of maturities between 2 and 30 years.

Figure 9: Time Series of Zero-Coupon Real Yields for OAT€i sample



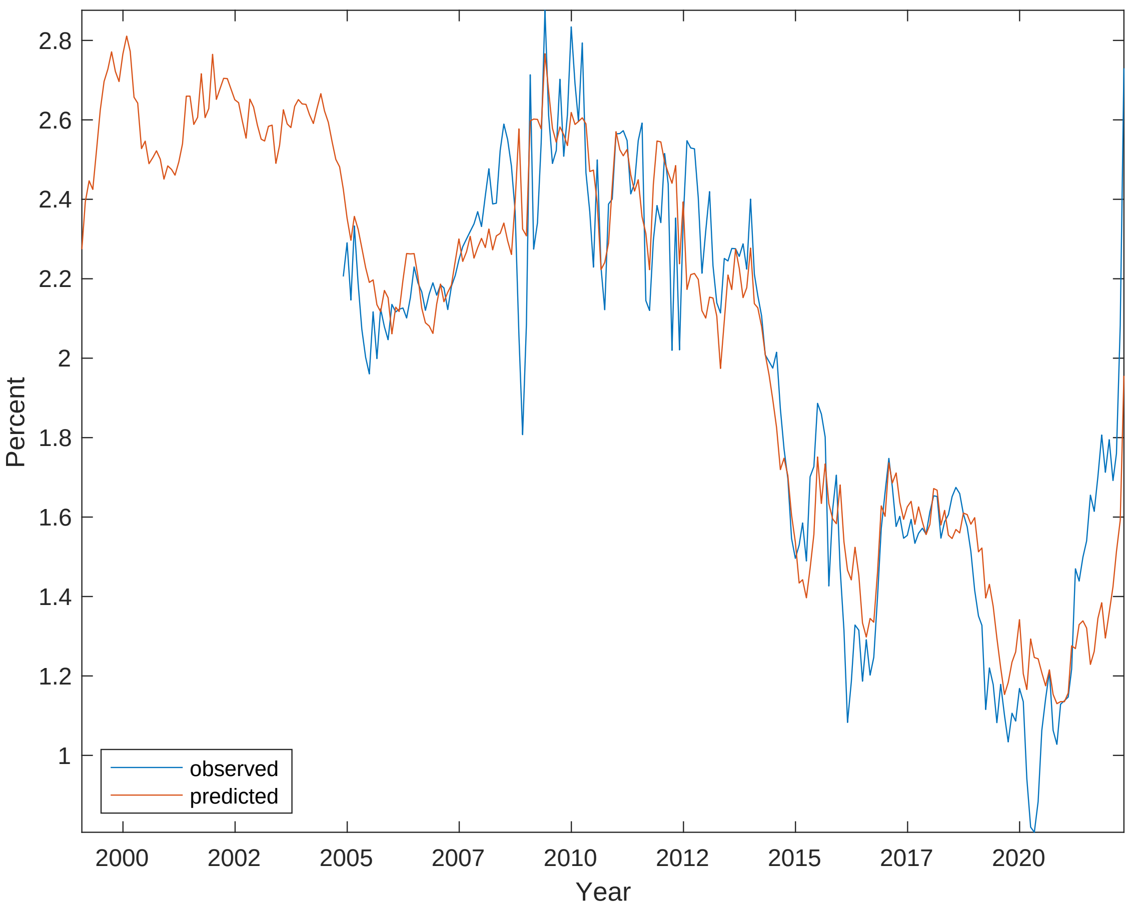
This figure shows the time series of the Nelson and Siegel (1987) fitted 2-, 5-, 10-, and 30-year zero-coupon real yields implied by the price quotes of OAT€i securities from November 17, 2004, to May 12, 2022, at daily frequency.

Figure 10: Time series of 5-year nominal, real, and breakeven inflation rates for OAT€i sample



This figure shows the time series of the Svensson (1994) and Nelson and Siegel (1987) fitted 5-year zero-coupon nominal and real rates, respectively. Nominal and real fitted yields are implied by OAT and OAT€i quotes, respectively. Panel C shows 5-year breakeven inflation rates. Sample period is November 17, 2004, to May 12, 2022, at daily frequency.

Figure 11: Actual and Fitted Five-to-Ten Year Forward Inflation Compensation



This figure shows the actual and fitted five-year forward five-year inflation compensation rate. We follow the Gürkaynak, Sack, and Wright (2010) methodology to conduct the backcasting investigation. Figure reports the results for the OAT€i sample for the euro-area sample period from January 31, 1999, to April 30, 2022.