# On the correlation of systemic dimensions


**Abstract**

We propose an analysis of the correlation between three systemic risk measures: $\Delta CoVaR$, Granger causalities, and ILLIQ. We interpret the measures above as proxies for Systemic Dimensions, respectively, Losses, Connectedness, and Illiquidity. Our study shows that systemic dimensions are uncorrelated in stable periods and lose orthogonality in crisis periods. This result confirms that systemic risk is a composition of different types of risk that occur concomitantly. Moreover, we show that, by applying a specific lag for each systemic dimension, we improve the identification of systemic crisis periods and reduce the noise of the measure. This constitutes evidence of a specific order in the occurrence of Systemic Dimensions, hinting that a chronology of systemic events might exist.


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*JEL Classification:* G01, G21, G22, G23, G28.

## 1. Introduction

A financial crisis is a moment when all goes badly. This adage illustrates the most common view of a financial crisis. Numerous academics have tried to define and study these abnormal events but never agreed on a specific definition. Hence, the sentence above comes the closest to the truth. It is possible that a financial crisis is not defined by the amount lost or the number of firms' failures but simply by the fact that 'all go badly.' When looking at the history of financial crises, it provides a relevant, although imprecise, description (Kindleberger, 1978; Atkinson et al., 2013). It is even more accurate during a systemic crisis, which, most would agree, is worse than a financial crisis. A systemic crisis combines different risks occurring simultaneously. This article builds upon this insight and poses the question of the existing correlations between risk measures. However, there is extensive literature on correlation risk[1], there are very few studies on how different risks are correlated. We postulate that since Systemic Risk (SR) is a combination of different risks, a good proxy for SR measurement is the correlation between different risk indicators.

The first challenge of the article is to find and show that there exist different SR measures[2], thus defining how two measures can be different. The following challenge is straightforward. In the case where there exist more than two 'different' measures of SR, how can we find a measure of correlation for more than two variables? We base our initial thinking on the assumption that SR is a combination of various risks called Systemic Dimensions. We propose to study specifically three of them: Losses, Connectedness and Illiquidity. The measures taken to proxy these Systemic Dimensions are the following. First, the $\Delta$CoVaR that comes from the work of Adrian and Brunnermeier (2016) and proxies losses, the DGC (Billio et al., 2012) for connectedness, and ILLIQ, from Amihud (2002), for illiquidity. This chapter shows that the Systemic

[1] Correlation risk refers to the fact that financial returns of different firms are highly correlated, hence providing evidence of herding behavior and/or common exposure on a specific risk.

[2] There exist various types of SR measures. Here, 'different' refers to the specificity of a measure. Systemic Dimensions are, by definition, different types of SR.

Dimensions are orthogonal in stable times and become correlated in crisis periods. The tests are performed on two distinct samples. The first spawns from June 2000 to June 2020; and is based on the daily stock returns of 75 American firms (25 banks, 25 insurers, and 25 investment firms). For robustness, we add a second sample based on European data. The second sample comprises the daily stock returns of 100 European financial firms (25 banks, 25 brokers, 25 insurers, and 25 investment firms). We use a Principal Component Analysis (PCA) as a correlation measure for our Systemic Dimensions. We apply the PCA over a rolling forward window and find that the first component's inertia shows clearly the period of systemic crises in our sample. By showing that the correlation of different (systemic) risk measures is enough to find systemic crises in our sample, we argue that the very nature of SR lies in these correlations.

The contributions of this chapter are twofold. First, our study provides an innovative point of view on the nature of SR. Up to now, most of the existing literature had the underlying assumption that SR was univariate so that a single indicator could reflect SR. To answer that assumption, one can find numerous studies that provide a single measure to describe SR. We assume that there exist multiple types of SR and that the 'real' SR is measurable by the correlation between these different types of SR[3]. This is the first study, to our knowledge, to provide such an approach. We further assume that SR is also a succession of occurrences of Systemic Dimensions. Its measurement remains a challenge, especially because of that temporal aspect. This chapter provides an initial answer to that aspect. We propose to apply a specific lag procedure to the Systemic Dimensions in order to capture this temporal effect[4]. Assuming that a Systemic Event (SE) is a succession of occurrences in Systemic Dimensions, the stages of increase and decrease in the underlying measures are offset from one another. By re-aligning the Systemic Dimensions before computing their correlation, we allow for a reduction in the noise of the measure. We are the first to present such a procedure and put forward empirical evidence

[3]The different types of SR relate directly to the Systemic Dimensions, which are, by definition, different type of risks that are subsequent to SR.

[4]For a graphical representation of the lag procedure, see Figure 4.

of a chronology in a SE, in both the European and U.S. markets. Furthermore, we compare the evolution of SR between the European and American financial systems. Our results show the robustness and applicability of our approach.

The structure of this chapter is as follows. The following section exposes the existing literature. It first presents the underlying SR theory and its relationship with banking models. In a second time, it covers the existing measures and concludes with the concept of dependence. The following section presents our sample data and the descriptive statistics. It also provides the measures used to proxy the Systemic Dimensions and the use of a PCA for computing the correlation among the Systemic Dimensions. We will next expose our results for the American and European cases and, in a second time, present a focus on each type of firm considered in this research. The last section concludes the chapter.

## 2. Literature Review

This section exposes the current literature on SR. It first introduces the theoretical models that have pioneered the research on SR. In a second time, the section presents famous measurements proposed by academics over the last two decades. Eventually, it concludes the section with a discussion on the measures of dependency and correlations in finance.

### *2.1. Theory of Systemic Risk*

The study of SR directly stems from banking crises. One can easily argue that the globalization of finance was due to banking activity. Banking evolved and allowed businesses to invest, grow, and spread through the years. This fact is observable by the extent of banking regulation that took place over the years (Glass-Steagall Act (1993), Dodd-Frank Act (2010), Basel Accords (1988, 2004, 2010), etc.). The initial endeavors were to discover the natural properties of such phenomena. Among the most known papers, we note the work of Bryant (1980), Diamond and Dybvig (1986), and Kiyotaki and Moore (1997) on banking crises[5]. Such papers highlight the risk associated with banking activity and the solutions to regulate it. Kiyotaki and Moore

[5] For a general overview on bank regulation, see Dewatripont and Tirole (1994).

(1997) particularly focused on the importance of credit cycles in a shock's transmission mechanism. They make an explicit link between banking activity and financial crises. Moreover, the risk of a bank run was one of the major issues tackled in the early research, and the creation of deposit insurance directly stemmed from it. Although, the total efficiency of deposit insurance as a solution to bank runs remains arguable (Aglietta, 1993).

It is important to note that research on SR also started with the study of financial crises. The pioneering work of Kindleberger (1978), Minsky (1991) and Mishkin (1992) have laid a solid foundation for our understanding of the underlying process of financial crises. Kindleberger (1978) and Minsky (1991) share a common analysis of how financial crises build up. In their conception, a financial crisis is inherent to capitalism and regularly develops according to a given pattern: A change, called 'displacement', occurs and causes a shift in the behavior of financial agents. Afterward, firms working appropriately (hedge units) need to borrow to pay the interest of their loans (speculative units). Firms that were already risky (speculative units) are forced to continuously borrow to survive (Ponzi units). This creation and increase in financial fragility lead to a point where most of the financial system is composed of Ponzi units. A 'not-so-unusual'[6] event happens, pushing the first firms into bankruptcy. In Minsky (1991) and Kindleberger (1978)'s view, the crisis is created from irrational behavior[7]. On the other hand, Mishkin believes that financial crises root in information asymmetry. Mishkin (1992) presents the different causes of a financial crisis and a chronology of how they entangle together. Information asymmetry creates adverse selection and moral hazard issues that worsen the crisis.

Due to the crises which spawned at the beginning of our millennium (Dot-Com bubble, Subprime crisis, European debt crisis, etc.), we have observed a rise in SR interest. Leading theoretical work such as, and among others, Acharya (2009), Freixas and Rochet (2010), Brun-

[6]In Minsky's view, the first failures of a crisis start because of a given macroeconomic event. It is called 'not-so-unusual' because such an event would not have incurred any failure in regular times. Because of the large degree of financial fragility, failures occur.

[7]See Blanchard and Watson (1982) for a study on how bubbles can develop under rational expectations.

nermeier and Oehmke (2013) and Chen et al. (2013), have demonstrated the general interest in providing a theoretical framework for SR. As said by Brunnermeier and Oehmke (2013), in systemic assessment, theory should lead the operational measurement. The traditional view of treating the banking sector as the only systemic institution slowly became less and less relevant. Recent theories also include insurers and hedge funds in their scope. The logic behind this adaption of scope is that the mechanisms linking all financial agents together constitute the roots of SR.

### 2.2. *Measures of Systemic Risk*

The subprime crisis created a relentless stream of research on measuring SR. The impact of the crisis was so unexpectedly large that it forced both academics and regulators to tackle the issue of measurement. We present in this section the most famous measures[8]. Existing measures can be separated into two main categories. The ones that try to provide a global index of SR and the others that propose a firm-level contribution to SR. As explained in Borio (2003) and Bisias et al. (2012), both types of measure are complementary.

On the one hand, measures providing a global index are essential for macroprudential supervision. Among these, the most famous are the following: The Composite Indicator of Systemic Stress (CISS), created by Hollo et al. (2012), focuses on how much of the financial system is at risk at any time point. The indicator is created from various financial stress indicators. It implies that SR is higher when many markets are at risk, taking into account various types of risk. In the same vein, Kritzman et al. (2011) propose a systemic indicator called the Absorption Ration (AR) based on a PCA on financial returns. Hu et al. (2010) propose a measure of noise based on bond price. The rationale of the measure is that, in stable times, the Treasury market should be almost noiseless. Although, in crisis periods, the amount of capital in the bond market is reduced, resulting in more noise. Their measure gives an indicator of liquidity crises and liquidity risk over time. All these measures allow for an analysis of how general SR

[8]For a more comprehensive view of the current literature, see Bisias et al. (2012) and Benoit et al. (2017) on SR measurement, and Smaga (2014) on the definition and concept of SR.

evolves. They also permit regulators to evaluate the performance of their actions.

On the other hand, we also need individual risk measures to identify which agent is causing the most risk in the financial market. This type of study makes, from far, the largest part of the current literature on SR. Their objective is to display the risks which can cause an institution to fail. We find first the measures linked to financial losses: the $\Delta CoVaR$ of Adrian and Brunnermeier (2016), Systemic Expected Shortfall (SES) of Acharya et al. (2017) and Co-risk created by Chan Lau et al. (2009). Such articles try to display the size of losses associated with an unusually risky period. Another stream of SR measures focuses on the connectedness of financial institutions. Freixas et al. (2000) represent the interbank system as a network and show the effect of one (or more) bank insolvency over the whole network[9]. Giesecke and Kim (2011) propose a model timing banks' default. The model is made to display systemic linkages in an economy. Billio et al. (2012) show, through Granger causality regressions and PCA, the systemic importance of connectedness. The PCA shows the general level of correlation among financial returns, while Granger causalities identify the critical linkages between financial firms.

### *2.3. Dependency measures*

The measures presented above cover a wide range of variety in the measurement approach or underlying definitions of SR. We set ourselves apart from these studies by assuming that SR is multidimensional. In other words, SR is a combination of various risks. Consequently, we provide in this chapter a new method for assessing SR. The methods assume that (existing) SR measures assess different types of SR and that the 'real' SR lies in their correlation. To that end, this section introduces the concept of correlation and dependency. Lehmann (1966) describes three distinct types of dependency:

1. *Quadrant Dependency:* $P(X \leqslant x, Y \leqslant y) \geqslant P(X \leqslant x)P(Y \leqslant y)$. In such case, there is de-

[9]For an update on the network models of SR, see Caccioli et al. (2018).

pendency if the joint probability of an event is greater the product of the marginal probabilities.

*2. Regression Dependency:* $P(Y \leqslant y|X \leqslant x) \geqslant P(Y \leqslant y)$. Such dependency exists when the probability of an event is greater when considering that the other event already occurred.

*3. Likelihood Ratio Dependence:* $f(x,y')f(x',y) \leqslant f(x,y)f(x',y')$, for $x' < x$ and $y' < y$, with $f(x,y)$ being the joint density of $X$ and $Y$. The dependence here lies in the fact that probability of observing large (small) values of the couple $(x,y)$ is greater than the one of observing disparate (one large associated with one small) values in the couple $(x,y)$.

The appropriate approach will differ significantly depending on the chosen case. In this study, we present the correlations between our Systemic Dimensions. Hence, our approach would tend to fit the first definition of dependence above. Although, results in the chronological order of occurrence of Systemic Dimensions would advocate for using dependency measures associated with the second definition. We leave this gap for future research. Unfortunately, the current literature regarding correlation in financial risks is almost nonexistent. However, there is a burgeoning one on correlation risk, that is, the risk associated with the correlation between financial returns. Among the most famous works, we find:

- *Copulas:* A copula is a concept coming from probability theory and whose development is based on Sklar's theorem. It corresponds to a joint cumulative distribution function of a random vector with uniform marginals. It is used to study the dependence between random variables. As an introduction, Embrechts (2009) provides solid and thorough explanations, as a must-read section that allows one to quickly get a comprehensive view on copulas[10]. In finance, Malevergne and Sornette (2003) tests whether the existing dependencies between pairs of financial assets can be modeled convincingly by the Gaussian (and Student) Copula. Finally, Kole et al. (2007) shows the importance of a copula's

[10] For a detailed description of the most important area of application, see the work of Genest et al. (2009).

choice for modeling dependence.

- *GARCH models:* Generalized Auto-Regressive Conditional Heretoskedastic (GARCH) models are one of the most used tools to model dynamic variance and correlations. An initial method was the BEKK model[11] based on the work of Engle and Kroner (1995). Bollerslev (1990) constructed a model for modeling conditional correlation, called Constant Conditional Correlation (CCC). Engle (2002) built upon the work of Bollerslev (1990) in order to make a model which can model a Dynamic Conditional Correlation (DCC), that is to say, model time-varying conditional correlation.

- *PCA:* Additionally, the papers cited above that applied PCA directly to SR (Billio et al. 2012; Kritzman et al. 2011) have been using it for measuring commonality among financial assets. Avellaneda and Serur (2020) used a Hierarchical PCA to expose clusters of assets based on their principal components. Fenn et al. (2011) shows the time-varying behavior of correlations and principal components for financial returns. Shapira et al. (2009) study the correlations among stocks taking into account the market's index.

Over the methods presented above, this chapter uses the PCA for measuring the correlation in Systemic Dimensions. This choice has multiple justifications. First of all, the PCA is a method of dimensionality reduction, hence providing a single measure[12] for the correlation between our Systemic Dimensions. Moreover, the procedure is computationally cheap and can be easily adapted to specific needs. Finally, we can perform a rolling-forward procedure to obtain a dynamic indicator of SR.

[11] The acronym BEKK is based on the names of the authors: Baba, Engle, Kraft, and Kroner.

[12] In this chapter, the PCA is used as a correlation measure. More precisely, it is performed over our measures (Systemic Dimensions) in order to extract the inertia of the first principal component. The underlying logic behind that choice is that the main (common) component of any set of measures built to assess SR should be SR.

## 3. Data & Methodology

This section presents the data used for constructing the SR measures. In a second time, we detail the mathematical formulation of each measure. Finally, we present the methodology of the PCA used as a correlation indicator among our Systemic Dimensions.

### *3.1. Data & Summary Statistics*

This research uses two distinct sets of data. The first is composed of American firms. More specifically, it includes the daily returns of 75 of the largest banks, insurance/reinsurance companies, and asset management firms in the U.S. from June 2000 to June 2020. The firms were first selected on the Osiris database depending on their market capitalization. We have extracted daily prices and market capitalization[13] from the Eikon database. In addition, we extracted daily prices, daily volumes, and market capitalization for the underlying stock of the S&P100, which will be used for constructing a proxy for market liquidity. The summary statistics are provided in the Appendix 6.3 (see Table 1). Additionally, the financial system, as used in the $\Delta CoVaR$ of Adrian and Brunnermeier (2016), is simply an index constructed from the 75 financial firms in our sample, which are market capitalization weighted. Formally:

$$R_{system}(t) = \frac{1}{MV_{tot}} \sum_{i=1}^{N} R_i(t)\ MV_i \tag{1}$$

In order to add robustness to our results, we perform the analysis over a second sample composed of the daily stock returns of 100 of the largest banks, insurance companies, brokerage companies, and asset management firms. The data starts in January 2005 and ends in December 2017. The construction of the financial system via a market capitalization-weighted index is the same as for our American sample (see Equation 1). The daily returns, volumes, and capitalization of the Eurostoxx 50 were also retrieved to compute the market liquidity proxy. The data for the European sample was retrieved on Datastream. The summary statistics are presented in the Appendix 6.3, Table 2.

[13]The market capitalization are used in order to derive the financial system needed for the computation of the ΔCoVaR.

### 3.2. *Systemic Risk measures*

#### 3.2.1. *CoVaR*

The Conditional Value-at-Risk (CoVaR) results from the work of Adrian and Brunnermeier (2016). Simply put, it represents the q% worst loss possible of a firm $i$, given that a firm $j$ is already at its p% worst loss. Originally, the authors use quantile regressions, but we decided on using the bivariate Gaussian model in order to have daily observations for our $\Delta CoVaR$s. By definition, $CoVaR$ is a $VaR$ of an institution $j$ conditional to some event $\mathbb{C}(X_i)$ on firm $i$'s returns. Thus, we define $CoVaR^{j\,|\,i}(q)$ as,

$$Pr(X_j \leqslant CoVaR^{j\,|\,i}(q) \mid \mathbb{C}(X_i)) = q \tag{2}$$

The formulation of the $CoVaR$ is useful when it comes to changing the condition terms, which generates multiple values and multiple interpretations of the measurement. In this chapter, we propose that the event $\mathbb{C}(X_i)$ consists in the fact that the $i^{th}$ firm's returns are at their $VaR$, then,

$$CoVaR^{j\,|\,i}(q,p) \equiv Pr(X_j \leqslant CoVaR^{j\,|\,i}(q,p) \mid X_i = VaR_i(p)) = q \tag{3}$$

In order to construct the $\Delta CoVaR$, we need to subtract the 50% level $CoVaR$ to the $q\%$ level $CoVaR$.This specification describes differences in $CoVaR$s between when the conditioning event is that firm $i$ is either distressed and when firm $i$ is 'stable' or at its usual level. Formally,

$$\Delta CoVaR^{j\,|\,i}(q,p) = CoVaR^{j\,|\,i}(q,p) - CoVaR^{j\,|\,i}(q,50\%) \tag{4}$$

As presented in the appendices of Adrian and Brunnermeier (2016), we use the Gaussian model by assuming that the returns of firms $i$ and $j$ follow a normal bivariate distribution. The mathematical development is made available in the Appendix 6.1. We then obtain the following expression for $CoVaR$.

$$CoVaR^{j\,|\,i}(q,p,t) = N^{-1}(q)\sqrt{1-\rho_t^2}\sigma_{j,t} + N^{-1}(p)\sigma_{j,t}\rho_t \tag{5}$$

With $\rho_t$ denoting the correlation between firms $i$ and $j$. Like Adrian and Brunnermeier

(2016), we focus our study of the *CoVaR* not on the relationships between two firms but rather on the analysis of one firm and the system. As for the *VaR* computation, we decided to implement time-varying correlations in our *CoVaR*. One obstacle, when trying to compute the *CoVaR*, as described above, is estimating the dynamic conditional correlation (DCC). In our computation, we follow the work of Engle (2002). We propose $R(t)$ as,

$$R(t) = \begin{pmatrix} R_i(t) \\ R_j(t) \end{pmatrix} \tag{6}$$

With $R_i(t)$ and $R_j(t)$ being the returns of firms $i$ and $j$. We assume that $R(t)$ follows a bivariate normal distribution with mean equal to zero and with the conditional covariance matrix $H(t)$.

$$R(t)|\,\mathbb{F}(t) \sim N_{\mathbb{R}^2}(0, H(t)) \tag{7}$$

Where:

$$H(t) = D(t)\;\Lambda(t)\;D(t) \tag{8}$$

With $D(t)$ being a diagonal vector of conditional standard deviations $\sqrt{h_i(t)}$ and $\Lambda(t)$ being the conditional correlation matrix. Generally, the $h_i(t)$ term is the volatility extracted from a univariate GARCH model; in our case it comes from the EGARCH performed for the computation of *VaR*s. We specify,

$$\varepsilon(t) = D(t)^{-1}R(t) \tag{9}$$

As all variances need to have positive values, $\Lambda(t)$ must be positive definite. A matrix is positive definite if it has non-negative eigenvalues. In order to respect that constraint, Engle gives the following proxy[14] which is developed to be a matrix version of a GARCH model.

$$Q(t) = S\,(1-\alpha-\beta) + \alpha(\varepsilon(t-1)\varepsilon(t-1)') + \beta Q(t-1) \tag{10}$$

Where $\alpha$ and $\beta$ are non-negative parameters and, so the process is mean-reverting, $\alpha+\beta < 1$.

[14]The development is available either in the original article of Engle (2002), or in the documentation for rmgarch models (Ghalanos, 2019).

$S$ is the unconditional correlation matrix of $R(t)$, obtained simply by computing the following.

$$S = \frac{\sigma_{R_i,R_j}}{\sigma_{R_i}\,\sigma_{R_j}} \tag{11}$$

Where $\sigma_{R_i,R_j}$ is the covariance of $R_i$ and $R_j$. We find our final DCC,

$$\Lambda(t) = diag\{Q(t)\}^{-1}\,Q(t)\,diag\{Q(t)\}^{-1} \tag{12}$$

As all the $CoVaR$s that we compute are either on the $i^{th}$ firm conditionally to the system; or the opposite, we only need to compute the time-varying correlations of each firm against the system. By allowing our measure to be time-varying, we can refine our analysis and study the behavior of the $\Delta CoVaR$ values throughout our whole sample.

*3.2.2. Interconnectedness*

We base our methodology on that of Billio et al. (2012) and develop Granger causality tests (Granger, 1969) to study the interconnectedness of firms in our sample. We perform linear Granger regressions on a rolling window of 200 days in order to get the evolution of the Granger adjacency matrix over the sample. Granger causality regressions are presented as in the original methodology of Granger (1969). The measure consists in performing two regressions, on two random variables and their lagged values. In other words, for the first regression, we regress the returns of the $i^{th}$ firm against the lagged value of the $j^{th}$ firm's returns and its own lagged value. The second regression is identical but inverts $i$ and $j$ in the regression. Formally,

$$\begin{aligned} R_i(t+1) &= \alpha_i R_i(t) + \beta_{ij} R_j(t) + e_i(t+1) \\ R_j(t+1) &= \alpha_j R_j(t) + \beta_{ji} R_i(t) + e_j(t+1) \end{aligned} \tag{13}$$

Where $R_i(t)$ is the return for the $i^{th}$ firm and $R_j(t)$ the return for the $j^{th}$ firm, $e_j(t+1)$ and $e_i(t+1)$ are two uncorrelated white noises. The lagged value prevents the $\beta$ to be biased by autocorrelation effects in the timeseries. We accept that the returns of firm $i$ have caused firm $j$'s returns if $\beta_{ji} \neq 0$; conversely, if $\beta_{ij} \neq 0$ then it is firm $j$ that causes the returns of firm $i$. We denote the fact that firm $i$'s returns cause firm $j$'s returns by the following.

$$\begin{aligned} (i \rightarrow j) \quad if \quad b_{ji} \neq 0 \\ (j \rightarrow i) \quad if \quad b_{ij} \neq 0 \end{aligned} \tag{14}$$

All causality, as defined above, is computed to be significant at the 95% level of confidence. As stated by Billio et al. (2012), if both coefficients are significantly different from 0, there is a feedback relationship between the two asset returns. In a similar fashion of Billio et al. (2012), we develop connectedness indicators from causality regressions: IN, OUT, and DGC.

- The 'IN' measure counts the number of firms whose returns caused the $j^{th}$ firm's returns, in the Granger sense.

$$\mathrm{IN} : (S \rightarrow j) = \frac{1}{N-1} \sum_{i \neq j} (i \rightarrow j)$$

- In contrast to the 'IN' measure, 'OUT' gives the number of Granger connections the $j^{th}$ firm caused.

$$\mathrm{OUT} : (j \rightarrow S) = \frac{1}{N-1} \sum_{i \neq j} (j \rightarrow i)$$

- The DGC or Degree of Granger Causality is an index on how many causal relationships exist in a given system. Operationally, we sum up the IN and OUT measure and divide by the maximal possible amount of connections (74 given our sample). The resulting index gives an idea of how much the system is connected.

### *3.2.3. Market illiquidity*

To gain an idea of how sectors' returns interact with market illiquidity, we follow the methodology of Amihud (2002) in developing AILLIQ. We use AILLIQ as an indicator of market illiquidity. By regressing this proxy against the financial returns of the firms in our sample, we assess their exposure to market illiquidity. To avoid endogeneity, we use the S&P 100, which contains only a few financial firms. The measure can be interpreted as the daily volume adjusted price variation or the price response against one dollar of the trading volume.

Although ILLIQ can also be considered a price impact measure, it remains an attractive illiquidity proxy due to its simplicity and the availability of its components. We begin by computing the ratio between the absolute value of daily returns on one firm's returns and the daily euro trading volume for the same firm. When computing the mean of this ratio (i.e., by dividing by the number of trading days), we get the ILLIQ measure for the firm.

$$ILLIQ_i(y) = 1/D_i(y) \sum_{i=1}^{D_i(y)} |R_i(y,d)|/VOLE_i(y,d) \tag{15}$$

With $R_i(y,d)$ being the return for firm $i$ on day $d$ and in year $y$. Similarly, $VOLE_i(y,d)$ represents the euro volume for firm $i$ on day $d$ and in year $y$. Also, $D_i(y)$ is the number of trading days for the year $y$. The ratio follows the definition of illiquidity as defined by Kyle (1985), which is to say the percentage of price change per unit of daily trading volume, here, in euro. Given our methodology, we need an indicator of market illiquidity rather than one of the illiquidity of each stock of the index. Therefore, we use the AILLIQ measure, as it is the mean of all the $ILLIQ_i$. Formally,

$$AILLIQ(y) = 1/N_i(y) \sum_{i=1}^{N_i(y)} ILLIQ_i(y) \tag{16}$$

We construct a rolling-forward linear regression between the firm's returns and the AILLIQ indicator, taking a 200-day window. By doing so, we obtain a value of $\beta_i(t)$ for each firm $i$ and each day $t$. Formally,

$$AILLIQ(t) = \alpha_i(t) + \beta_i(t)R_i(t) + e_i(t) \tag{17}$$

By posing $\alpha = 1,2,3$ and $M_\alpha$ as the length of the sample in sector $\alpha$, we construct the sector's average beta:

$$\beta_\alpha(t) = 1/M_\alpha \sum_{i=1}^{M_\alpha} \beta_i(t) \tag{18}$$

In the work of Amihud (2002), such regressions were made with inverse dependent and independent variables. Nevertheless, the rationale remains identical: the measure assesses the relationship between market illiquidity and each sector's returns. Amihud (2002) did observe that the regressions give a positive $\beta$ if illiquidity is expected. Therefore, the measure can show us when the illiquidity is not expected, hence, theoretically, when the risk associated with illiquidity is high. We consider the $\beta_\alpha(t)$ as a measure of SR for the following reasons. In a general sense, a market with high illiquidity is prone to greater variations in stock prices. Since excess asset returns will increase the variance, using ILLIQ's structure, they will also increase the illiquidity measurement. It is then straightforward to link the high volatility of asset returns with its high illiquidity, *ceteris paribus*. In the context of SR management, we have to recognize that high market illiquidity has substantial potential in amplifying a fire sale type of event. The measure allows for an additional specification that only a negative $\beta$ will indicate significant SR, mostly because unpredicted illiquidity is more dangerous than expected illiquidity.

### *3.3. Principal Component Analysis*

PCA is a statistical technique allowing to decompose the covariance matrix ($\Sigma$) of the m x n matrix ($X$), into a diagonal matrix of eigenvalues ($\Lambda$). Such as,

$$H^T \Sigma H = \Lambda \equiv diag(\lambda_1, ..., \lambda_m) \tag{19}$$

With $H = (h_1, ..., h_m)$ being an orthogonal matrix of size m x m, and $h_i$ is an eigenvector corresponding to $\lambda_i$. Principal components are then computed as follows.

$$U = H^T X = (U_1, ..., U_m) \tag{20}$$

In more details, the first component consists in $U_1 = h_1 X$, with variance $\lambda_1$. In a similar fashion, the second $U_2 = h_2 X$, with variance $\lambda_2$. Necessarily, all components must be orthogonal, hence $Cov(U_i, U_j) = 0$ for all $i \neq j$. In our case, the PCA serves as an alternative measure for correlation. This article aims to study the correlation structure of SR measures through time. By applying the PCA on our SR measures on a rolling forward window, we expose the changes in their common uncorrelated factors. Additionally, we use the quartimax rotation on our PCA. Generally, the unrotated PCA tries to maximize the variances of each factor and forces orthogonality on the principal components. Such variances of principal components should be strictly decreasing. A rotation can simplify the output by making it more understandable. Specifically, the quartimax method is an orthogonal rotation, resulting in fewer explaining factors. It does so by making large loading on a specific factor larger and conversely on small loadings, resulting in fewer large principal components.

Our results regard the evolution of the proportion of explained variance of the three first principal components. The rationale behind our approach is straightforward. Let us consider that each measure of SR is a noisy signal of the 'real' SR. In other words, the measure assesses the actual risk during a crisis but also shows increases during stable times due to the specificity of the measure. In essence, there exists an implied noise-to-signal ratio for each measure. In stable times, the ratio increases for each measure, making them uncorrelated because noises are uncorrelated from each other. However, during a crisis, the ratio decreases as all the measures indicate an increase in SR. Hence, the measures' correlation increases. In our case, we aim to study that evolution in correlation by applying a rolling-forward PCA to our Systemic Dimensions. By doing so, we interpret that an increase in SR is only relevant when multiple SR measures increase simultaneously.

## 4. Results & Discussion

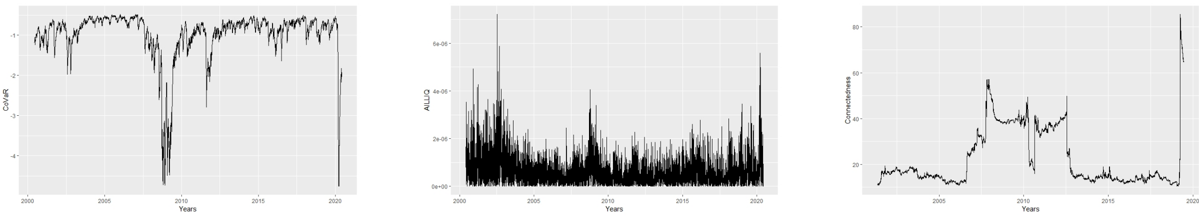
First, we introduce results for the whole U.S. financial system. We study the existing correlations between the three Systemic Dimensions (Loss, Connectedness, and Liquidity). We add an analysis of an identical sector-specific analysis to observe potential differences in SR inside the financial sector. Eventually, we propose an improvement of systemic crises' period identification, based on lagging appropriately Systemic Dimensions before performing PCA. We find that the lagging procedure improves the identification of systemic crises and discuss the reasons for such an improvement.

### *4.1. Othorgonality of Systemic Dimensions*

We postulate the following: Measures of SR typically represent a noisy signal for the actual risk. Since they focus on different types of risk, the noise among each of them is uncorrelated. However, as they are SR measures, the signal is correlated. Logically, the stable periods, where

SR is at the lowest, are characterized by a predominance of noise. Since noise is uncorrelated, measures are uncorrelated accordingly. However, during crisis periods, the proportion of noise to risk decreases. The signal for actual risk increases; hence, so are the measures. The general rationale starts with Systemic Dimensions: Losses, Connectedness, Liquidity. Systemic Dimensions constitute the main risks involved in a systemic crisis. The assumption made in this study is that the orthogonality of the Systemic Dimensions is assured in stable periods and breaks in crisis times. Thus, this research aims to validate this assumption by, first, proving that Systemic Dimensions are orthogonal in stable periods and by identifying crisis periods via the time-varying correlation of Systemic Dimensions. Intuitively, we use the PCA as a proxy for the correlation among the three dimensions. We expect the first component to increase in crisis periods and become the driving factor of all risks.

Figure 1: **Systemic Dimensions - U.S. Sample**
*All average Systemic Dimensions of all firms in our sample from June 2000 to June 2020. Graph (a) displays the* $\Delta CoVaR$ *of Adrian and Brunnermeier (2016). Chart (b) the market illiquidity indicator (AILLIQ) of Amihud (2002). Graph (c) shows the average Degree of Granger Causality (DGC) present in our sample.*




(a) $\Delta CoVaR$ (b) AILLIQ (c) DGC

As Figure 1 tells, all three Systemic Dimensions show significant spikes during crisis periods. We can identify residual stress from the dot-com bubble (2000-2002), the subprime crisis (2007-2009), the European debt crisis (2011-2012) and, more recently, the Covid situation (2020).

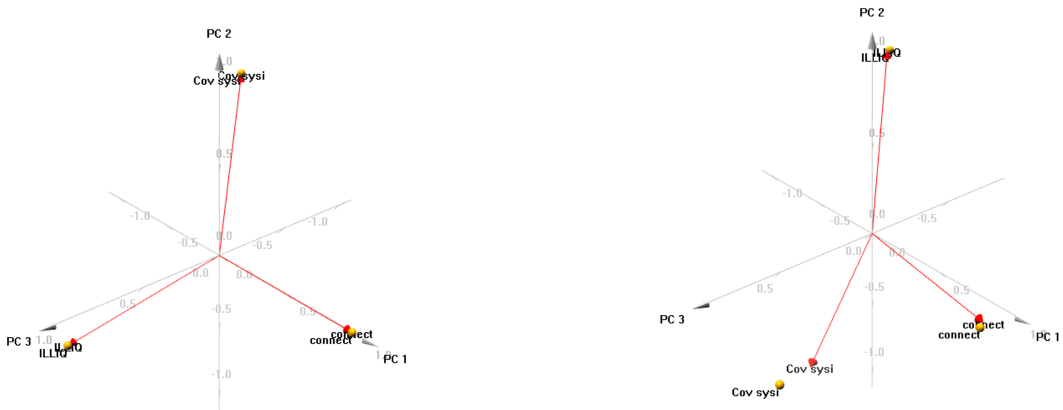
A valid remark would be about the reason to use different measures if they all show clearly SR. The answer is twofold. We probably could find various indicators that show spikes during crisis periods. However, it does not prove that such measures assess SR. Such relationships might be spurious. A second reason regards to noise. Even though each of the measures above serves as a decent indicator of SR, one could argue that they show noise; an increase in these indicators could mean something other than an increase in SR. An underlying reason why we chose to determine three Systemic Dimensions comes from that last point. We argue that an increase in SR could only be ‘significant’ if there is a common increase in all three dimensions. Thus, we can get an accurate view of SR by studying the time-varying correlation between the dimensions.

The orthogonality of the Systemic Dimensions shown in Figure 2 constitutes a simple heuristic to understand the nature of SR. In times of financial stability[15] all Systemic Dimensions are relatively close to being perfectly orthogonal, that is to say, uncorrelated. While, in crisis periods, they appear to be less orthogonal. The result is clearer for the connectedness and losses measures. The conclusion is straightforward: all dimensions measure different risks in stable times, which can increase/decrease ephemerally (i.e., noise). However, all risks occur simultaneously during crises, which translates statistically to an increase in the first component's inertia. The results for our European sample are similar, thus bringing additional robustness (see Figure 12).

A weak point of our argumentation comes from the arbitrary choice of the stable and crisis periods. To prove the robustness of the method and its usefulness, we compute the PCA on a rolling forward window. The time-varying inertia of the first component is shown below (see Figure 3). The variations of inertia of the first component identify the crisis periods. We can see that the realization of a systemic crisis happens when the increase in the explained variance of the first component simultaneously decreases the inertia of the other two components. In other words, a systemic crisis happens when all of the Systemic Dimensions stop being orthogonal. Moreover, the results computed over our European sample show similar dynamics of the inertia of principal components (see Appendix 6.2, Figure 10). More specifically, the SR arises when the increase of the first components is fueled by the decrease in the two other components.

[15]Here, financial stability is intended as outside of financial instability.

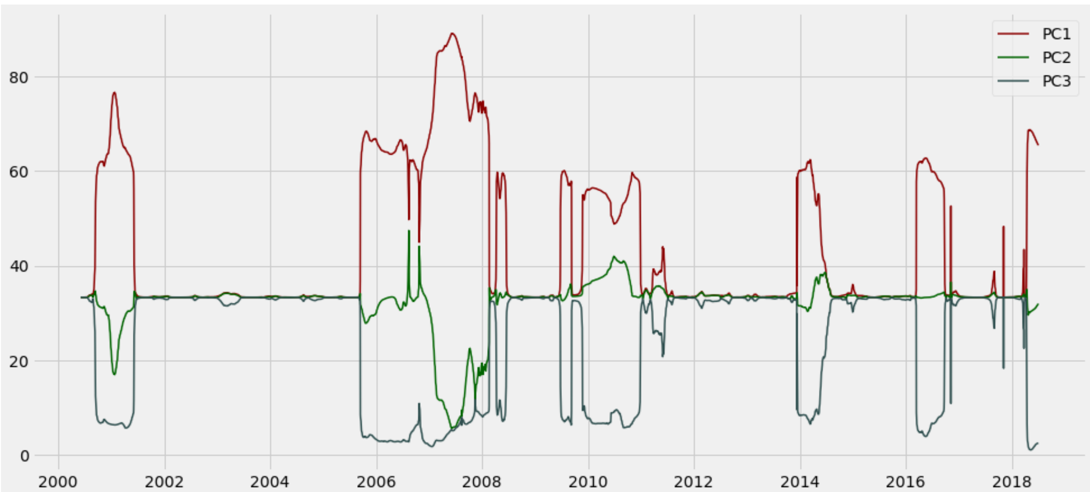
Figure 2: **Orthogonality of Systemic Dimensions - U.S. Sample**
*The chart shows the Systemic Dimensions on the first three principal components dimensions. The PCA is computed using an orthogonal rotation quartimax. 'ILLIQ' corresponds to the measure of Amihud (2002) that approximates the illiquidity dimension. 'connect' refers to the DGC of Billio et al. (2012) for systemic connectedness. 'Cov sysi' correspond to the $\Delta$CoVaR of Adrian and Brunnermeier (2016) for systemic losses. All measures are performed on the sample of 75 U.S. financial firms. Chart (a) shows nearly perfect orthogonality of all risk measures during the timespan, excluding financial crises. Graph (b) shows the increase in Systemic Dimensions' correlation occurring in financial crisis periods.*




(a) Out of crisis

(b) In crisis

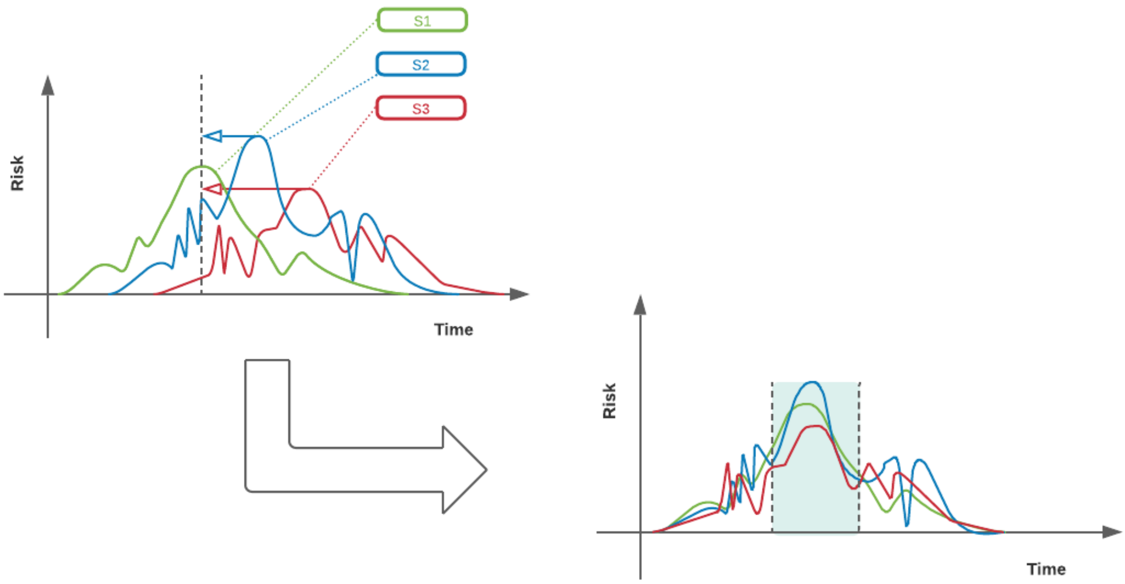
Figure 3: **Inertia of the three first components - U.S. Sample**
*The chart shows the explained variance of the first three principal components arising from a rolling-forward PCA performed with the quartimax rotation. The procedure is performed on the U.S. sample of 75 financial firms over the 2001-2020 period. The first principal component is in black, the second in red, and the third in green. An increase in the first component at the profit of the second and third means an increase in common correlation between Systemic Dimensions. The PCA is performed using a 300 days window. The results shown are centered ([-150; + 150])*



*4.2. Lagged Systemic Dimensions*

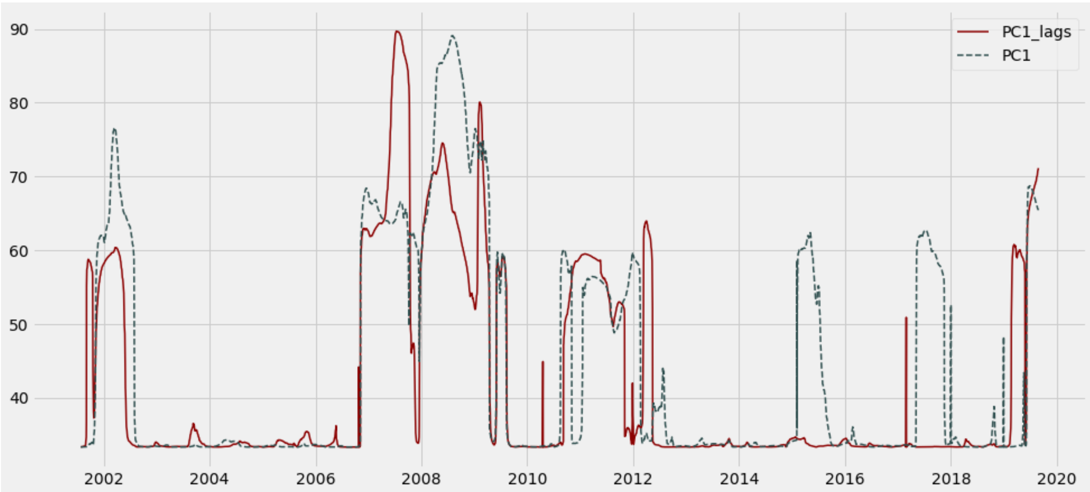
The measure is still yet to be optimal. We argued that noise was one of the reasons why we needed a new measure. However, the inertia clearly shows transitory spikes between 2012 and 2020. There were no notable financial crises in the American market over this period. One could argue that the spike is noise or a concomitant increase in Systemic Dimensions that was no evidence of a crisis. This particular issue comes from the temporal aspect of SR. We assumed above that all Systemic Dimensions were highly correlated *simultaneously* during crisis times. It turns out not to be precisely accurate. We displayed evidence of a particular succession of events in a systemic crisis. There is an order in which things get progressively worst. From this assumption, it would seem reasonable to assume further that Systemic Dimensions evolve at different times, in a given order. Theoretically, the measurement would improve if we could find the appropriate lag. Practically, we lag our variables according to the supposed order, as Figure 4 shows.

Figure 4: **Lagging Systemic Dimensions**.
*Schematic representation of the effect of lagging Systemic Dimensions. The transition, from the upper right to lower left charts, is made by lagging Systemic Dimensions (S2, S3). The lag procedure allows to increase the precision of the estimation of crisis periods, thus showing evidence of a chronology in Systemic Events.*




Following this procedure, we obtain the following results.

Figure 5: **Comparison of inertia of first components - U.S. Sample**
*This graph shows the explained variance of the first component for two rolling-forward PCA performed with the quartimax rotation. The first dashed series in black shows the original rolling-forward PCA of the Systemic Dimensions. The red series shows the rolling-forward PCA for which we have lagged the Systemic Dimensions. The DGC is naturally forward lagged (due to the rolling-forward procedure) by 150 days. We lagged the illiquidity indicator back for 90 days. The lagged series (in red) shows an improvement in crisis identification by not incorporating the temporary increases between 2015 and 2020.*

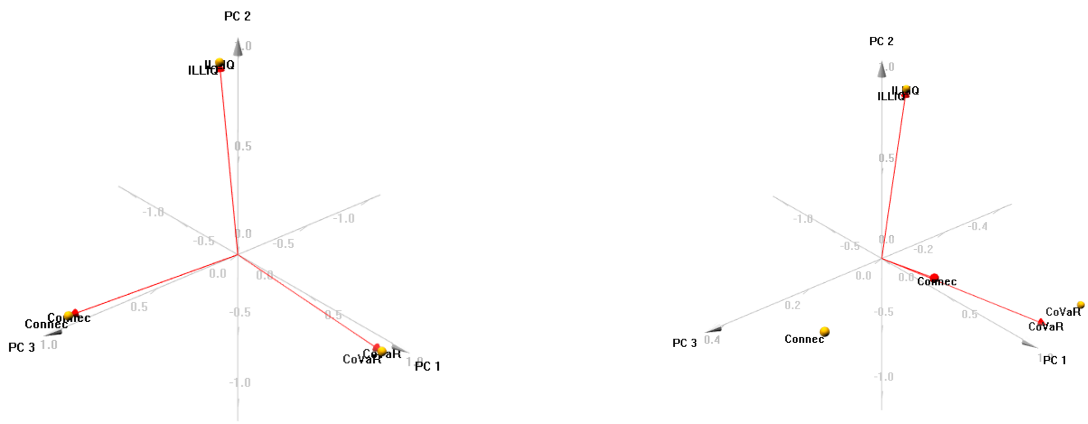



We propose the following process of a systemic event: first, the build of common exposure should happen early in the timeline, followed by losses and, finally, by the amplification via market illiquidity[16]. Since Granger causalities are performed on a rolling window, the measure is naturally delayed forward. We have tried different lags to find an appropriate fit for the illiquidity measure. As Figure 4 displays, we can get rid of the transitory spikes by lagging our Systemic Dimensions.

Such results put forward evidence of an order of occurrence in Systemic Dimensions. As the measure conforms to both the theoretical idea of a systemic event and empirical periods of systemic crisis, we argue that it constitutes an initial proof that a systemic event is composed of various risks (major ones being denoted as Systemic Dimensions) occurring in a specific order. The results are also visible on the European market between 2005 and 2017. The orthogonality of Systemic Dimensions is also showed on Figure 6.

[16]The amplification is usually accompanied by contagion. One could pose that Granger causalities constitute a proxy for contagion. We argue that Granger causalities display a potential for contagion due to a common exposure. Hence, it is a better proxy for the buildup of financial fragility.

Figure 6: **Orthogonality of Lagged Systemic Dimensions - U.S. Sample**
*The chart shows the lagged Systemic Dimensions on the first three principal components dimensions. The PCA is computed using an orthogonal rotation quartimax. 'ILLIQ' corresponds to the measure of Amihud (2002) that approximates the illiquidty dimension. 'connect' refers to the DGC of Billio et al. (2012) for systemic connectedness. 'Cov sysi' correspond to the $\Delta$CoVaR of Adrian and Brunnermeier (2016) for systemic losses. All measure are performed on the sample of 75 U.S. financial firms. Chart (a) shows a nearly perfect orthogonality of all lagged risk measures during the timespan excluding financial crises. Graph (b) shows the increase in lagged Systemic Dimensions' correlation occurring in financial crises periods.*




(a) Out of crisis

(b) In crisis

The loss of orthogonality during the crisis is more apparent for lagged risk measures. In our view, the importance of this result is central. Up to now, most of the existing literature on SR measurement assumed that only one dimension mattered[17]. Our result shows first that different measures of SR are orthogonal in stable times. Each measure assesses drastically different concepts and only measures (together) SR in crisis periods. Hence, considering a single measure blocks the analysis from capturing the complete picture of SR. Secondly, there exists a chronological order in systemic crises. This second fact confirms the importance of the first. Indeed, each dimension should be considered as they all participate in a systemic event. Our results show that an increase in interconnectedness arises first for both the U.S. and the European financial system. The rise in connectedness can be interpreted in two ways: First, it represents the potential for a widespread shock by saying that temporal cross-correlations proxy common exposures. Second, it represents the potential for contagion. The spike in financial losses oc-

[17] Each paper essentially chose a given dimension among the following: Losses, Liquidity, Connectedness, Contagion. The papers assumed that these measurements were proxies of SR.

curs in a second time, captured by ΔCoVaR. Eventually, we observe the increase in exposure to market illiquidity. This sequence of events gives an initial idea of how systemic events develop. Consequently, our results advocate taking into account the multiplicity of Systemic Dimensions and their specific entanglement.

The regulatory implications are significant as well. Even though the SIFI assessment (BCBS, 2013) uses various types of indicators, the constraints posed on SIFIs are standardized. The results of this study preach that: Firstly, SR depends on various risks. Secondly, each type of financial firm has a specific profile of SR. Hence, prudential regulation should identify a specific SR profile[18] per SIFI, which details where the SIFI locates in the systemic event unfolding. The changes are substantial in that the regulation should have to become more flexible to adapt to the profile of the SIFI. The main questions are: What firms participate in the increase of common exposure and how to restrict it? How can we prevent financial losses from occurring? How to prevent a contagion/amplification of these losses? Such questions have already been asked separately. Our results advise such questions to be considered conjointly.

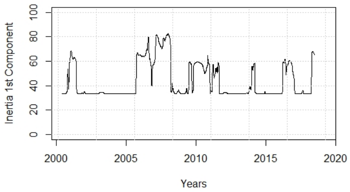
### *4.3. Sectorial Systemic Dimensions correlation*

When applying the PCA on the Systemic Dimensions only computed for a specific type of financial institution, we can see how much this particular sector was at risk. In a general sense, the method applied in this article to identify systemic crises is one for assessing the risk of a system considered. In this section, we thus propose to focus specifically on each type of financial firm to determine the 'sectorial-systemic risk'.

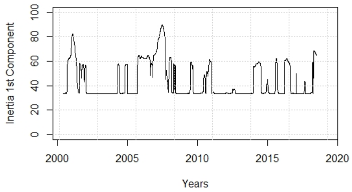
Figure 7 emphasizes the evolution of inertia for the first components of the PCA for types of financial firms. The measure clearly shows spikes during the crisis period in our U.S. sample, although it remains noisier than the global one (Figure 3). This fact appears as logical. When considering a larger system, its inherent risk is determined by when most of its components

[18]We intend by SR profile, a detailed rapport of what risks are more relevant to this firm (or type of firm) in particular.

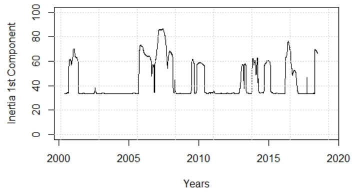
Figure 7: **Sector-specific inertia - U.S. Sample**
*The graphs show the inertia of the first component arising from a rolling-forward PCA computed using the quartimax rotation over a 300 days window on the U.S. sample of 75 financial firms. An increase in the first component denotes an apparition/increase in sectorial Systemic Risk.*




(a) Banks




(b) Insurers

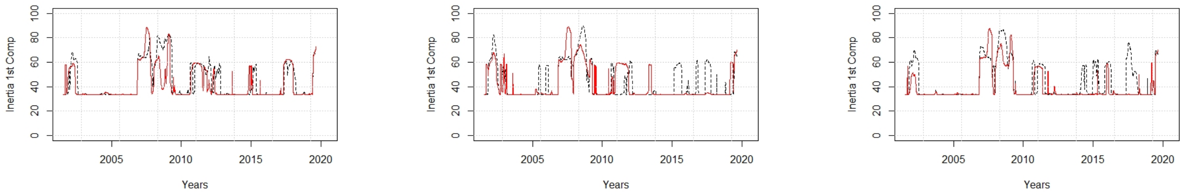



(c) Investment Firms

are at risk[19]. For instance, when only banks are at risk but insurers are not, will not be a high SR period globally. The noise of the measure then logically grows stronger as the system considered goes smaller. An important insight from this approach lies in identifying the period where a specific type of institution is risky. For instance, we can see that all types of institutions were at risk during the subprime crisis, hinting at the systemic aspect of the crisis. Let us then confirm that there exists a chronology in Systemic Dimensions. We apply the (identical) lags on the Systemic Dimensions and compare the accuracy of the method below (see Figure 8).

[19]This is the underlying assumption for our approach.

Figure 8: **Comparison sector-specific inertia - U.S. Sample**
*The graphs show the comparison of inertia of the first components arising from a rolling-forward PCA computed on sectorial Systemic Dimensions with the quartimax rotation over a 300 days window, and the rolling-forward PCA computed on the lagged sectorial Systemic Dimensions, with the same calibration. The procedure is performed on the sample of 75 U.S. financial firms. The red line shows the explained variance for the first component of the lagged sectorial Systemic Dimensions, the dashed black line shows the explained variance of the first component for the sectorial Systemic Dimensions.*




(a) Banks (b) Insurers (c) Investment Firms

Without a doubt, the lagging procedure reduces the noise for all types of firms (even though slightly less for banks). Moreover, it shows more clearly the systemically risky periods in our samples. We argue that the results provide a solid initial evidence of a specific order of occurrence in Systemic Dimensions.

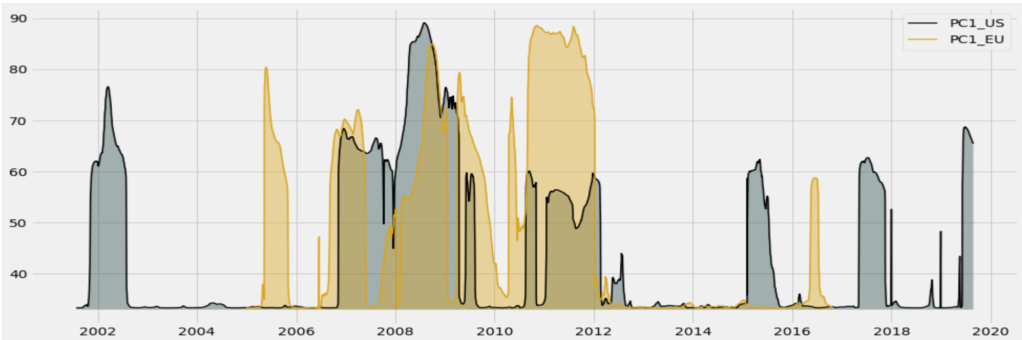
*4.4. Comparing the European and American financial systems*

This section presents the differences between the European and American cases. The first result that stands out is the common reduction in the noise of the measure for both the European and American markets. The transition from Figure 9a to 9b allows for a reduction in transitory spikes for both samples. This constitutes evidence that the lagging procedure (presented in Figure 4, p. 21) allows for improving the identification of crisis periods. Thus, further validating the chronology of SEs.

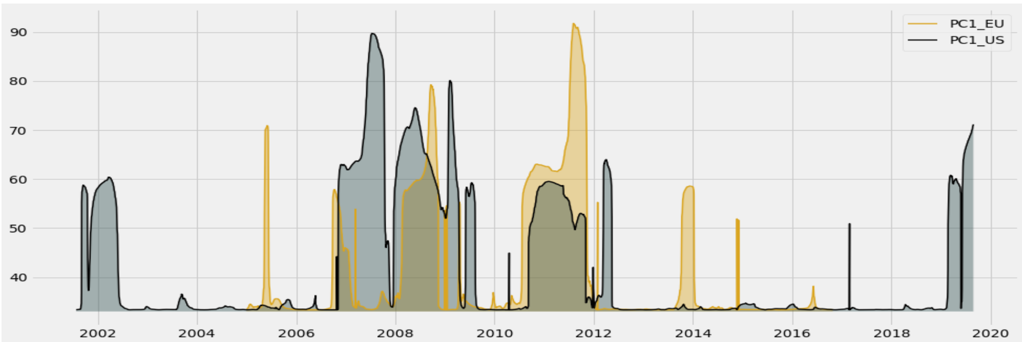
Moreover, the Figure 9b presents notable features. It displays the delayed impact of the Subprimes crisis on the European market. Indeed, the explained variance of the first component for the U.S. sample spike in advance, hence, showing the loss of orthogonality in the U.S. Systemic Dimensions. The increase in the first component's inertia of the European sample only occurs in delay after 2008. The result is coherent with historical events as the crisis started to unravel in the U.S. first before spreading out to the rest of the world. Furthermore, our measure identifies the extent of SR in a country. As Figure 9 depicts, the SR was larger in the

U.S. during the Subprimes crisis. However, the European sample shows a higher level of SR during the European Sovereign Debt crisis. Again, the result is historically coherent. European countries have suffered more financial stress than the U.S. financial firms have during that particular time.

Figure 9: **Comparison of the $1^{st}$ component's inertia - U.S. vs. Europe.**
*Graph (a) displays the inertia of the first component of a PCA computed on the Systemic Dimensions of our European sample compared to the one computed on our American sample. Graph (b) shows the inertia of the first components and the lagged Systemic Dimensions of both samples. The Systemic Dimensions are the $\Delta$CoVaR, the exposure to market illiquidity, and the DGC. The European sample starts in 2005 and ends in 2017, while the American sample starts in 2001 and ends in 2020. PC1_EU and PC1_US refer to, respectively, the inertia of the first component for the European and American samples.*




(a) Inertia of the first components




(b) Inertia of the lagged first components

In essence, our approach[20] studies SR in any given system, whether it is a country, a sector, or the whole financial system. The underlying assumption is straightforward. The method assumes that the SR related to a given system depends on the concomitant occurrence of various risks. In this view, the systemic importance of risks depends on their propensity to be accompanied by other risks. Furthermore, the procedure to follow is simple. The first step is to identify

[20]We define our approach as the assessment of SR via the examination of the correlation of Systemic Dimensions over time.

to Systemic Dimensions of the system. In order to assure the variety of risks, they should be orthogonal in stable periods. The next step is to assess the time-varying correlation between Systemic Dimensions. Using such a procedure, we assess the evolution of SR in Europe and the U.S. between 2001 and 2020 (see Figure 9). This analysis provides additional evidence of a specific evolution of SR depending on the country/region examined.

## 5. Conclusion

SR measurement has been one of the most challenging research fields since the subprime crisis. The difficulties are numerous. As crises come in all sorts and flavors, it is complex to choose the variables that matter the most. Even though a model fits existing crises, it is not evident that such a model will perform well in the next. Furthermore, as finance is mainly driven by human behavior and investors' expectations, few, if none, of our available variables, have stable distributions over time. Our study decided not to choose what indicator matters the most by looking at the correlation between the indicators instead.

We provide a correlation analysis on different SR measures computed on two samples: a European sample composed of 100 financial firms, spawning from 2005 to 2018, and an American sample of 75 firms, starting in 2000 and ending in 2020. Because each measure corresponds to a specific type of risk and, as our results show, is usually orthogonal, we denote them as Systemic Dimensions. 'A moment when all goes bad' is a common thought on the nature of financial crises. Building upon this insight, we study whether it can be applied to SR. In such a case, a systemic crisis consists in the occurrence of all its subsequent risks, hence when Systemic Dimensions lose orthogonality. We propose a rolling-forward PCA analysis on the Systemic Dimensions. We find that Systemic Dimensions are orthogonal outside of crises and lose their orthogonality to become correlated during crisis periods. Moreover, we see that realized systemic crises are identified as moments when the explained variance of the first principal component increases due to the fall of the two others. This constitutes solid evidence that SR occurs when multiple risks happen concomitantly, i.e., when all Systemic Dimensions become correlated. The results are similar for the European market. Additionally, we show that we allow for clearer identification of systemic crisis periods by lagging the Systemic Dimensions in time. The result constitutes evidence of a specific order in a systemic event. From our results, an increase in connectedness happens first. A spike in financial losses follows it. Finally, the exposure to market illiquidity rockets. We present the first research in our knowledge to provide empirical evidence of a specific order in the occurrence of risks during a systemic event. These results open research opportunities in SR measurement and forecasting, as well as in prudential regulation. Furthermore, we compare the evolution of SR for the European and American financial systems. Our method depicts the transmission of SR from the U.S. to Europe during the Subprimes crisis and shows the differences in SR levels during financial crises. In particular, we show that, logically, the European financial system was more at-risk than the American during the European Sovereign Debt crisis.

As time passes by, the financial world keeps getting more complex than ever. There is no certitude as if a proper SR model will ever exist. Mostly due to Lucas' critique (Lucas, 1976). Indeed, if a perfect model to predict SR existed, it is probable that financial agents would act so that the model would become obsolete. However, flexible models, in the sense that they do not rely on precise indicators, have a chance to guide prudential regulation to financial resilience.

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# 6. Appendix

## 6.1. Mathematical Developments

### 6.1.1. Gaussian model for the estimation of CoVaR

We assume that two random variables $X_i$ and $X_j$ follow a bivariate Gaussian distribution. We note:

$$(X_i(t), X_j(t)) \sim \Phi\left(0, \begin{pmatrix} \sigma_i^2(t) & \rho(t)\sigma_i(t)\sigma_j(t) \\ \rho(t)\sigma_j(t)\sigma_i(t) & \sigma_j^2(t) \end{pmatrix}\right) \tag{21}$$

Since both firms $i$ and $j$ follow a bivariate normal distribution, we can write the conditional distribution for firm $i$ as:

$$X_j(t) \,|X_i(t) \sim \Phi\left(\frac{X_i(t)\sigma_j(t)\rho(t)}{\sigma_i(t)}, (1-\rho^2(t))\sigma_j^2(t)\right) \tag{22}$$

We define the standardized value of our random variable $X_j$ as $Z_j$:

$$Z_j(t) = \frac{X_j(t) - X_i(t)\sigma_j(t)\rho(t)/\sigma_i(t)}{\sqrt{1-\rho^2(t)}\sigma_j(t)} \tag{23}$$

And, as such :

$$Z_j(t) \sim \Phi(0,1) \tag{24}$$

We then come back to the definition of the Conditional Value-at-Risk:

$$Pr(X_j \leqslant CoVaR^{j\,|\,i}(q,p) \mid X_i = VaR_i(p)) = q \tag{25}$$

Which we can rearrange by standardizing $X_j$ into $Z_j$:

$$Pr\left( Z_j(t) < \frac{CoVaR^{j\,|i} - X_i(t)\sigma_j(t)\rho(t)/\sigma_i(t)}{\sqrt{1-\rho^2(t)}\sigma_j(t)} |X_i(t) = VaR_i(p,t)\right) = q \tag{26}$$

And, finally, we can develop this into its final form:

$$CoVaR^{j\,|i}(q,p,t) = \Phi^{-1}(q)\sigma_j(t)\sqrt{1-\rho^2(t)} + \Phi^{-1}(p)\rho(t)\sigma_j(t) \tag{27}$$

### 6.2. Figures

Figure 10: **Inertia of the three first components for the European sample**
*The chart shows the explained variance of the first three principal components arising from a rolling-forward PCA performed with the quartimax rotation on our European Sample of 25 Banks, 25 Insurers, 25 Investment Firms and 25 Brokers over the 2005 to 2018 period. First principal component is in black, the second in red, and the third in green. An increase in the first component at the profit of the second and third means an increase in common correlation between Systemic Dimensions. The PCA is performed using a 300 days windows, the results shown are centered ([-150; + 150])*

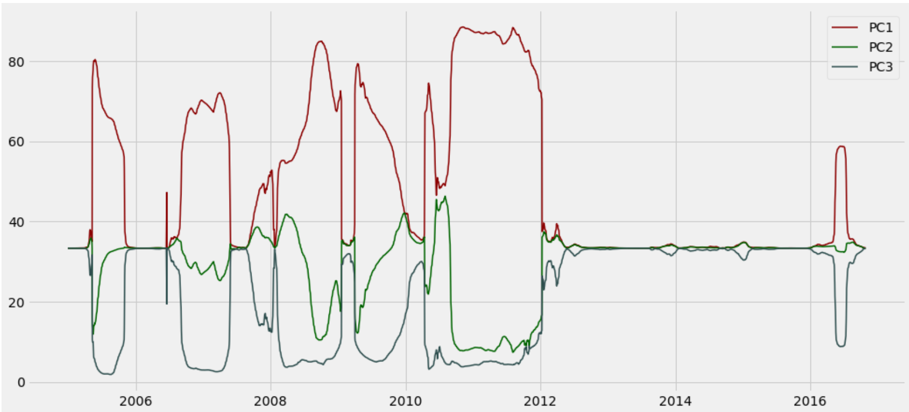



Figure 11: **Comparaison of inertia of first components for the European sample**
*This graph shows the explained variance of the first component for two rolling-forward PCA performed with the quartimax rotation on our European sample of 25 Banks, 25 Insurers, 25 Investment Firms and 25 Brokers over the 2005 to 2018 period. The first dashed series in black shows the original rolling-forward PCA of the Systemic Dimensions. The red series shows the rolling-forward PCA for which we have lagged the Systemic Dimensions. The DGC is naturally forward lagged (due to the rolling-forward procedure) of 150 days. We lagged the illiquidity indicator back for 90 days.*

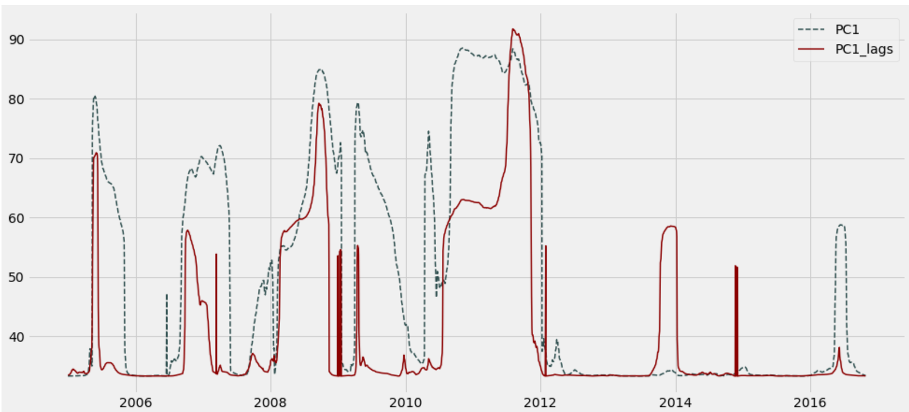
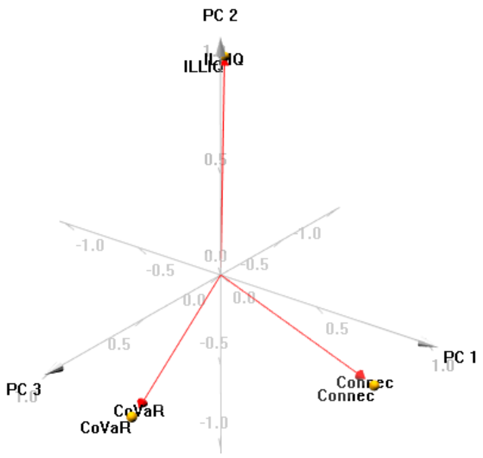
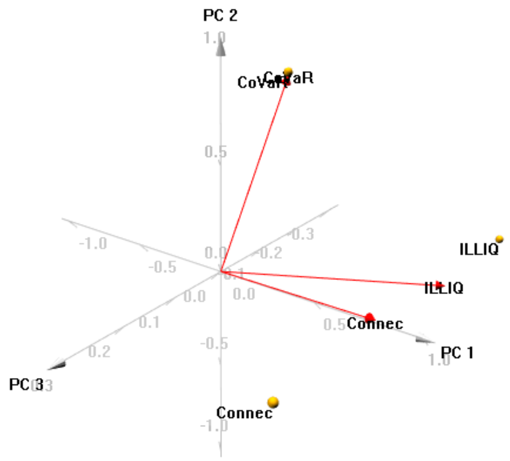


Figure 12: **Orthogonality of Systemic Dimensions - European Sample**

*The chart shows the Systemic Dimensions on the first three principal components dimension. The PCA is computed using an orthogonal rotation quartimax. 'ILLIQ' corresponds to the measure of Amihud (2002) that approximates the illiquidty dimensions. 'connect' refers to the DGC of Billio et al. (2012) for systemic connectedness. 'Cov sysi' correspond to the ΔCoVaR of Adrian and Brunnermeier (2016) for systemic losses. All measure are performed on the sample of 100 European financial firms. Chart (a) shows a nearly perfect orthogonality of all risk measures during the timespan excluding financial crises. Graph (b) shows the increase in Systemic Dimensions' correlation occuring in financial crises periods.*




(a) Out of crisis




(b) In crisis

## *6.3. Tables*

Table 1: **Summary Statistics - U.S. Sample**
*Summary statistics for daily returns of the 75 firms of our U.S. sample: June 2000 to June 2020. Of the 75 firms, 25 are banks, 25 insurers, and 25 investment firms. We include the annualized mean, annualized standard deviation, minimum, maximum, skewness, and kurtosis of the annualized daily returns.*

| Sector | count | mean | sd | max | min | median | skewness | kurtosis | auto-corr |
|---|---|---|---|---|---|---|---|---|---|
| ***Full Sample*** | | | | | | | | | |
| **Banks** | 25 | 0.003 | 0.381 | 0.725 | -0.893 | 0.0 | -0.116 | 59.689 | -0.077 |
| **Insurance** | 25 | 0.038 | 0.351 | 1.982 | -2.000 | 0.0 | -0.375 | 94.063 | -0.087 |
| **Asset Management** | 25 | 0.015 | 0.417 | 0.663 | -1.024 | 0.0 | -0.366 | 20.927 | -0.045 |
| ***During Crises*** | | | | | | | | | |
| **Banks** | 25 | -0.332 | 0.568 | 0.668 | -0.893 | -0.0 | 0.373 | 48.023 | -0.084 |
| **Insurance** | 25 | -0.259 | 0.494 | 0.705 | -0.936 | -0.0 | -0.539 | 17.377 | -0.094 |
| **Asset Management** | 25 | -0.293 | 0.598 | 0.663 | -0.687 | -0.0 | -0.378 | 14.887 | -0.080 |
| ***Outside of Crises*** | | | | | | | | | |
| **Banks** | 25 | 0.173 | 0.279 | 0.725 | -0.663 | 0.0 | 0.451 | 44.525 | -0.075 |
| **Insurance** | 25 | 0.187 | 0.272 | 1.982 | -2.000 | 0.0 | 0.455 | 84.495 | -0.068 |
| **Asset Management** | 25 | 0.175 | 0.321 | 0.605 | -1.024 | 0.0 | -0.000 | 22.493 | -0.011 |

Table 2: **Summary Statistics - European Sample**
*Summary statistics for daily returns of the 100 firms of our European sample: June 2000 to June 2020. The sample includes 25 banks, 25 brokers, 25 insurers, and 25 investment firms. We include the annualized mean, annualized standard deviation, minimum, maximum, skewness, and kurtosis of the annualized daily returns.*

| Sector | count | mean | sd | max | min | median | skewness | kurtosis | auto-corr |
|---|---|---|---|---|---|---|---|---|---|
| ***Full Sample*** | | | | | | | | | |
| **Banks** | 25 | -0.024 | 0.441 | 0.413 | -1.157 | 0.0 | -0.445 | 21.202 | 0.043 |
| **Brokers** | 25 | 0.010 | 0.521 | 1.006 | -0.939 | 0.0 | 0.454 | 27.681 | -0.053 |
| **Insurance** | 25 | 0.079 | 0.341 | 0.788 | -1.495 | 0.0 | -0.882 | 48.047 | 0.002 |
| **Asset Management** | 25 | 0.063 | 0.414 | 0.517 | -1.181 | -0.0 | -0.239 | 28.101 | -0.008 |
| ***During Crises*** | | | | | | | | | |
| **Banks** | 25 | -0.466 | 0.574 | 0.413 | -0.877 | -0.002 | 0.137 | 7.394 | 0.046 |
| **Brokers** | 25 | -0.246 | 0.561 | 0.647 | -0.698 | -0.000 | -0.411 | 17.037 | -0.070 |
| **Insurance** | 25 | -0.260 | 0.458 | 0.302 | -1.495 | -0.001 | -0.500 | 16.851 | -0.001 |
| **Asset Management** | 25 | -0.357 | 0.480 | 0.511 | -0.678 | -0.001 | -0.248 | 12.567 | 0.004 |
| ***Outside of Crises*** | | | | | | | | | |
| **Banks** | 25 | 0.247 | 0.373 | 0.363 | -1.157 | 0.000 | -0.446 | 18.256 | 0.032 |
| **Brokers** | 25 | 0.140 | 0.497 | 1.006 | -0.939 | 0.000 | 0.884 | 25.037 | -0.053 |
| **Insurance** | 25 | 0.247 | 0.279 | 0.788 | -0.887 | 0.001 | -0.120 | 17.523 | 0.004 |
| **Asset Management** | 25 | 0.316 | 0.378 | 0.517 | -1.181 | 0.000 | 0.012 | 28.431 | -0.033 |