Flooded House or Underwater Mortgage?

The Implications of Climate Change and Adaptation on Housing, Financial Assets & Redistribution

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January 31, 2023

Abstract

I study the implications of climate change and adaptation on housing and financial assets, as well as on redistribution more broadly. I build a redistributive growth model in which households and firms are exposed to extreme weather events, which damage housing capital and the firm's tangible capital. The analysis reveals that climate change is intrinsically redistributive, as it amplifies both wage and wealth inequality. Low-income workers experience a relatively larger decline in income due to their exposure to climate-related damages, while the rate at which households with positive savings accumulate wealth rises. Furthermore, I find that adapting to climate change is more challenging for low-income households who are financially constrained, and the failure to reduce vulnerability to climate impacts exacerbates wealth inequality. Additionally, while houses that face climate risk trade at a discount in the market, I demonstrate that the materialization of climate change risk puts upward pressure on house prices, as the supply of such houses becomes reduced. This general equilibrium effect is propagated and amplified over time.

Keywords— Climate Change, Adaptation, Housing, Financial Assets, Income Inequality, Wealth Inequality, Redistributive Trends
JEL classification codes— E44, G51, 044, Q54

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I am grateful to Enrico Perotti for the many discussions that led to a significant improvement of this paper. For helpful comments and suggestions I thank Rick van der Ploeg, Björn Brügemann, Viral Acharya, Simone Lenzu, Yenan Wang, Fabian Greimel and Stefanos Tyros. I also like to thank Andrea Titton, Daniel Schmidt and Robert Lee for their advice on model simulations. This paper benefited from discussions with participants of the brown bag seminar of Amsterdam School of Business, and of the Dutch Economist Day 2022. All potential errors are my own.

1 Introduction

As the impacts of climate change intensify across the globe and *mitigation* efforts remain inadequate in preventing temperatures from rising above 1.5 degree Celsius (UNFCCC. Secretariat (2022)), the significance of *adaptation* as a global response strategy becomes increasingly evident. Adaptation, the process of adjustment to actual or expected climate change in order to reduce vulnerability to its impacts (IPCC (2021)), not only minimizes economic losses and damages, but also has significant effects on housing and financial asset prices, and redistribution. I build a general equilibrium model to shed light on the broader implications of climate change and adaptation. The analysis reveals that climate change has an intrinsic redistributive effect, exacerbating wage inequality. Additionally, the availability of income and wealth play a significant role in determining our capacity to adapt, and this paper also demonstrates the feedback effects of adaptation on wealth inequality in the presence of financial constraints. To quantify the effects of climate change and adaptation, I run model simulation, based on Dutch data. Moreover, I conduct counterfactual analysis, in which I consider the low (RCP 2.6), medium (RCP 4.5), and high (RCP 8.5) greenhouse gas concentration trajectories from the Intergouvernmental Panel for Climate Change (IPCC (2013)).

I embed climate change and adaptation in a redistributive growth model, which is based on Döttling and Perotti (2017). The economy consists of one region, populated by households and firms. Households have preferences over housing and a non-durable consumption good, which is produced by the firm. Households are either low-skilled or high-skilled and work in the firm when young, and their labor is complementary to tangible and intangible capital, respectively. Households purchase a house when young and use the revenue of selling the house to purchase a non-durable consumption good when old. This is a risky investment, however, as households are exposed to *physical* climate risk. Physical risk emerges due to a rise in the frequency of severe weather events, such as wildfires, droughts, river floods and hurricanes, or due to changes in climate patterns, such as sea-level rise which increases the risk of coastal floods. There is a positive probability that these climate impacts (hereafter referred to as 'extreme weather events') will destroy a fraction of houses in each period. Damaged houses have zero resale value and, as costs are too high for households to repair damages in their lifetime, these houses becomes 'uninhabitable' (see Burzyński et al. (2019)). This reduces the expected revenue from selling the undamaged housing stock, a finding consistent with the empirical literature documenting the existence of a price discount for houses exposed to climate risk (see, e.g., Bernstein et al. (2019); Bosker et al. (2019); Baldauf et al. (2020)). While damages reduce the housing wealth of existing homeowners, climate-related losses reduce the stock of houses permanently. The associated increase in the marginal benefit of owning a house rises as climate risk materializes and the associated losses increase puts *upwards* pressure on the contemporaneous house prices, thereby providing a novel channel via which climate risk affects house prices. This general equilibrium effect is propagated, as the supply of houses is reduced permanently, but also becomes amplified as climate risk increases further. I show in model simulations that this general equilibrium effect dominates and climate risk, causing house prices to grow faster than as compared to the growth path in the absence of climate risk.

Firms produce the non-durable consumption good and employ tangible and intangible capital, as well as low- and high-skilled labour in their production process. Like households, firms are exposed to climate risk as extreme weather events destroy their capital. For simplicity, only damages to tangible capital are considered, and these damages are interpreted as the damages to tangible capital relative to the damages to intangible capital. Income falls with climate-related damages, reducing firm profits and dividends. Additionally, wages fall and firms scale back investments in both tangible and intangible capital. Tangible capital has a higher elasticity to climate-related damages, and firms reduce investments in tangible capital to a larger extent. This leads to a stronger decline in the wage of low-skilled workers, who work with tangible capital, exacerbating wage inequality. Climate-related damages also affects firm's financing costs. While the decline in the productivity of tangible capital and the slow down in income growth, put downwards pressure on firm's financing costs, climate risk makes capital relative more scarce, which pushes rates up. When the degree of substitutability between tangible and intangible inputs is sufficiently low, the increased scarcity of capital dominates the other two effects and firm's financing costs become higher as compared to the growth path in the absence of climate risk. This has two important implications for wealth inequality, as the rise in firm's financing costs lowers equity values. First, the decline in equity values reduces the financial wealth of shareholders. However, the rate at which households with positive savings accumulate wealth increases. This indicates that asset price changes are an equalizing force for wealth inequality, whereas savings form an unequalizing force (Bauluz et al. (2022)).

The focus of this paper is on the optimal response to rising climate risk and its consequences on housing and financial assets, as well as on income and wealth inequality. The analysis considers a closed economy, in which climate change is treated as an exogenous event. As such, the emphasis is on *adaptation* strategies, rather than on *mitigation* measures, aimed at reducing greenhouse gas emissions. The main part of this paper examines how households respond to a rise in climate risk by investing in measures that reduce their vulnerability to the effects of extreme weather events. Investments in adaptation, such as installing storm-proof windows or building seawalls and stilts (Fried (2021)), play a crucial role in limiting loss and damages to housing capital, a crucial component of household wealth. As adaptation shifts the distribution of idiosyncratic losses to the left, households trade-off the benefits of avoided climate change damage with the costs of investment. When climate risk is priced accurately in the market, the privately optimal investment in climate change adaptation is efficient and prevents "tragedy of the horizon" effects. However, any imperfect pricing of climate risk leads to underinvestment in adaptation. Policies that contribute to the accurate pricing of climate risk in housing markets are therefore crucial to encourage households to adapt optimally. Additionally, the choice of discount rates is essential to ensure that the privately optimal outcome is efficient.

Finally, credit constraints limit the size of household debt to the liquidation value of the collateral. As destroyed housing capital has zero liquidation value, credit constraints tighten as the economy's climate risk exposure rises. While adaptation offers a countervailing force by its virtue of preserving housing capital, credit constraints make it increasingly difficult for low-skilled households to adapt. Due to their failure to adapt, constrained households to lose a relatively larger part of their housing wealth, and this exacerbates wealth inequality. As recent analyses (e.g. Havlinova et al. (2022)) have pointed out that a significant number of homeowners face credit constraints that prevent them from financing investments that improve the sustainability of their homes, this finding calls for targeted policies which support investments in adaptation by credit-constrained households, to prevent a further rise in wealth inequality.

Related literature and contribution This paper contributes to the literature on the effects of climate change response strategies for inequality. This literature mainly focuses on the implications of carbon pricing for inequality. Känzig (2021) finds that higher carbon prices come at the cost of a decline in economic activity and that poorer households bear the burden of this decline. As poorer households have a high energy share, they experience a relatively larger reduction in their consumption. This effect also becomes reinforced as these households experience a larger decline in income, as they tend to work in sectors which are more impacted by carbon pricing policies. While this paper also shows that low-income workers face a larger decline in their income, this is a result from the complementarity of low-income worker with capital that has a higher exposure to exposures to climate-related damages. Pedroni et al. (2022) study the effect of inequality on optimal fiscal policy and show that the social costs of carbon are lower when economic inequalities are taken into consideration. Moreover, the authors find that it is optimal to use half of the carbon tax revenue to increase transfers, with the other half used to reduce distortionary taxes. Rather than considering the effects of climate change mitigation policies on inequality, this paper is, to my knowledge, the first to study the relation between climate change adaptation policies and wealth inequality. While climate change reduces income and destroys wealth, this paper demonstrates that income and wealth play a significant role in determining our capacity to adapt. Additionally, this paper shows that there are an important feedback effect of adaptation on wealth inequality.

This paper adds more broadly to the literature on climate change and inequality. Dell et al. (2012) first showed that climate change is a larger problem for countries that are not sufficiently rich, as higher temperatures reduce economic growth rates in poor countries substantially and this effect is persistent. Alvarez and Rossi-Hansberg (2021) formalize this argument and confirm that the economic effects of climate change are heterogeneous across space. The authors show that rising temperatures are associated with a decline in the productivity and demonstrate that the welfare of the developing world is more severely affected as a result of climate change as the developing countries face more extreme changes in temperatures. Burzyński et al. (2019) also highlight that climate change has heterogeneous effects and that climate-related damages do not only reduce productivity, but might also make certain areas uninhabitable. Moreover, the authors confirm that poorer regions are most prone to the adverse affects of rising temperatures and that climate change reinforce inequality across the globe as a result. While the aforementioned papers study the relationship between climate change and inequality in a cross-country setting, this paper studies this relationship in a within-country setting and shows that climate change reinforces trends in income inequality due to the complementarity of different workers with distinct types of capital that differ in their exposures to climate risk.

This paper also contributes to the literature that study climate change adaptation in macro-models as well as efficient adaptation. To my knowledge, only Fried (2021) studies the macroeconomic effects of climate change adaptation. While this study abstracts from the effects of climate change adaptation on house price dynamics and on income and wealth inequality, the author quantifies the interactions between climate change, adaptation and federal disaster policy. Fried (2021) finds that disaster aid policies induce moral hazard in adaptation and this reduces adaptation in the US economy, while federal subsidies for adaptation more than correct for this effect. This paper demonstrates that underinvestment in adaptation also follows in the presence of financial constraints and argues that target investments are required to prevent such underinvestment to translate into a further rise in wealth inequality. Additionally, as Fried (2021) point out that the idiosyncratic risk component of damages caused by climate-related disasters matters for the welfare cost of climate change. This paper highlights that the idiosyncratic risk component is of crucial importance for studying the implications of climate risk for homeowner default.

This paper also relates to the literature on efficient adaptation, Mendelsohn (2006) first emphasized that markets encourage efficient adaptation in sectors whose goods are traded, such as agriculture and energy. Mendelsohn (2006) highlights that government should focus their attention on markets for public good, in which the market outcome is not necessarily efficient, and should refrain from intervening in markets from traded goods, as this undermines private incentives. Anderson et al. (2019) further highlight that especially in land markets markets climate change adaptation is faciliated through price signals. This paper formalizes the argument of Mendelsohn (2006) and Anderson et al. (2019). While I abstract from the role of governments, this paper highlights the conditions under which markets do lead to an efficient outcome.

Finally, this paper contributes to the literature that studies the pricing of climate risk in housing markets. This line of research predominately focused on the sea level rise (SLR) risk. Harrison et al. (2001), Bin et al. (2008); Keenan et al. (2018); Gibson et al. (2017), Ortega and Tapınar (2018), Bernstein et al. (2019), Hino and Burke (2020) and Baldauf et al. (2020) find evidence that SLR risk is - at least, to a certain extent - capitalized into US housing markets in coastal states. However, Murfin and Spiegel (2020) conclude that the price effects of flood risk are limited. While Bernstein et al. (2019) and Baldauf et al. (2020) find that the sophistication of buyers, as well as climate change beliefs are key drivers behind the existence/level of the discount, Murfin and Spiegel (2020) argue that no price effects should be found as climate change believers sort in unexposed neighbourhoods, while deniers sort in exposed neighbourhoods. Indeed, Bakkensen and Barrage (2021) show that heterogeneity in beliefs reconciles the mixed empirical evidence and further argues that flood risk is not fully reflected in house prices due to high degrees of belief heterogeneity in coastal areas. This paper abstracts from heterogeneity in climate-change beliefs. Rather, this paper focuses on heterogeneity in households skill levels, as this drives outcomes for income and wealth and therefore plays a crucial role in determining households' capacity to adapt. Consistent with the aforementioned research, Consistent with the existing literature, houses exposed to climate risk traded at a discount in the market. However, the results reveal a novel channel via which climate risk affects house prices. As a result of climate-related damages, the supply of housing shrinks over time and this puts upwards pressure on house prices. Model simulations show that this general equilibrium effect dominates, causing house prices to rise faster than predicted by the growth path in the absence of climate risk.

As the evolution of house prices matter for mortgage market dynamics, this paper also adds to the literature that studies the relationship between the rise in climate risk and mortgage market dynamics (Bakkensen et al. (2022), Issler et al. (2019) and Ouazad and Kahn (2019)). Bakkensen et al. (2022) use a heterogeneous beliefs model to show that purchases of houses more exposed to SLR are more likely to be leveraged despite lower property prices. Moreover, Bakkensen et al. (2022) find that the underlying mortgage contracts feature a longer maturity and that climate change pessimists are more likely to trade their climate risk exposure with banks via long-term debt contracts. Issler et al. (2019) focus on the implications of wildfires on mortgage delinquency and foreclosure rates and find that both increase significantly after a wildfire. The authors show that the size of this effect declines in the size of the disaster. This is because coordination externalities afforded by large fires positively affect the quality of rebuilding projects and thus makes the neighbourhood more valuable than it was before the wildfire occurred. Finally, Ouazad and Kahn (2019) research whether mortgage originators are more likely to transfer default risk after natural disasters. The authors find confirm lenders are more likely to increase the share of mortgages originated below the conforming loan limit and that securitization rates increase after the occurrence of a natural disaster.

Roadmap The remainder of this paper is structured as follows: Section 2 describes the theoretical framework. The conditions relevant for the definition of an equilibrium are derived in Section 3. In Section 4, adaptation is introduced and credit constraints are included in Section 5. Section 6 discusses the main quantitative results and Section 7 concludes.

2 Theoretical Framework

Time is discrete and denoted by $t \in \{0, 1, ..., \infty\}$. The economy is characterized by two overlapping generations (young and old), each consisting of a unit mass of households. There is a mass of firms, which live for one period and produce a non-durable consumption good.

2.1 Households

Households live for two periods and derive utility from consuming housing and a non-durable consumption good. When young, households purchase housing capital, denoted by L, from the old generation at a relative price p. In addition, young households can invest in the corporate debt and equity of a firm, f. Corporate debt and equity are denoted by D_f respectively

e_f. Once old, households sell their housing capital and use the return on their housing and financial wealth to purchase a non-durable consumption good, denoted by c. There is an initial generation at $t = 0$, which is endowed with the supply of houses.

2.1.1 Preferences

Households have preferences over housing and non-durable consumption according to

$$U(c_{i,t+1}, L_{i,t}) = c_{i,t+1} + v\left(L_{i,t}\right)$$

where $v\left(L_{i,t}\right)$ captures the utility that household i obtain in period t from owning $L_{i,t}$ housing capital and $v'(\cdot) > 0$, $v''(\cdot) < 0$. Households maximize expected lifetime utility and do not discount the future.

2.1.2 Labour Endowments

Households are heterogeneous in terms of skills level, which are exogenously given. A fraction ϕ of households is high-skilled, h, and is endowed with $\bar{h}$ high-skilled labour. The remaining households are low-skilled, l, and are endowed with $\bar{l}$ manual labour. Workers supply their labour inelastically in a perfectly competitive labour market when young (Döttling and Perotti (2017)). Income is given by $y_{i,t} = \{q_t\tilde{h},\ w_t\tilde{l}\}$, where q_t denotes the high-skilled workers' wage and w_t the low-skilled workers' wage.

2.1.3 Innovators

A fraction ε of high-skilled workers has some entrepreneurial talent. When young, these 'innovators' set up a firm, f and create create intangible capital, H, by investing $I_{H,t}$, where $I_{H,t} = H_{t+1}$. This is a costly investment, as innovators incur an effort cost of $C(I_{H,t}) = \frac{1}{2}I^2_{H,t}$.

The firm operates with the intangible capital once the innovator turns old. Innovators issue equity shares, which are granted to its high-skilled colleagues. When the firm starts operating, however, shareholders try to capture part of the value of intangible capital. Due to the inalienability of human capital (Hart and Moore (1994)), innovators can threaten to leave the firm. Denote ω as the share of intangible capital that innovators appropriate if they leave. Then, shareholders capture a fraction $(1-\omega)$ of the value of intangible capital.

2.1.4 Climate Risk and Housing Capital

The economy is exposed to climate risk and experiences extreme weather events in each period. These extreme weather events occur at the start of the period and destroys housing capital. Let γ_t denote the probability that the housing capital of a given household becomes damaged due to the occurrence of an extreme weather event in period, t. By the law of large numbers, γ corresponds to the fraction of households that suffer climate-related damages in any period t (Fried (2021)). This probability increases over time according to the following law of motion:[1]

[1] Under the assumption that climate risk rises linearly in temperatures, the linear approximation is consistent with the predicted temperature paths under RCP 2.6, RCP 4.5 and RCP 8.5 (IPCC (2013)). Note that the slope coefficient is differs under each temperature path and is the highest under RCP 8.5.

$$\gamma_{t+1} = a + b\gamma_t$$

where $a \in [0,1)$ and $b \geq 0$, which reflects that higher climate risk today also indicates that climate risk is higher in the future. Households have common knowledge on the level of γ_t and its evolution, which implies that there is no aggregate uncertainty. Denote μ_L as the expected loss of housing capital conditional on being hit by an extreme weather event and denote by $\xi_{i,t}$ the losses suffered by a given household, i, in period, t. Since extreme weather events hit certain households harder than others, $\xi_{i,t}$ is stochastic and follows some distribution, $F\left(\xi_{i,t}\right)$, which is i.i.d. across households and has a mean of

$$\begin{aligned}\mathbb{E}\left(\xi_{i,t}\right) &= \mathbb{E}\left(\xi_{i,t}\middle|\text{Extreme weather event}\right)\cdot\mathbb{P}\left(\text{Extreme weather event}\right)\\ &= \mu_L\gamma_t\end{aligned}$$

As the probability with which extreme weather events occur rises, the expected losses to housing capital rise. I assume that repair costs are too high for households to repair the damages in their lifetime. This means that the damaged part becomes 'uninhabitable' and this permanently reduces the amount of housing capital which household i own:

$$L_{i,t+1} = \left(1-\xi_{i,t}\right)L_{i,t}$$

Denote by the total stock of houses in a given period $\phi\epsilon\bar{L}_t$ (where $\phi\epsilon$ is a normalization). Then, the stock of houses declines according to the following law of motion

$$\begin{aligned}\bar{L}_{t+1} &= \int_0^1\left(1-\xi_{i,t}\right)di\cdot\bar{L}_t\\ &= \left(1-\mu_L\gamma_{t+1}\right)\cdot\bar{L}_t\end{aligned}$$

where the second equality is results from the law of large numbers. This law of motion reflects that a rise in physical climate risk causes a gradual loss of habitat as the rise in the exposure to physical climate risk renders certain 'areas' uninhabitable (Burzyński et al. (2019)).

2.2 Firms

Innovators set up identical firms, which operate one period and maximize profits. Firms employ both high- and low-skilled workers as well as tangible and intangible capital in the production process.

2.2.1 Production Technology

Each firm produces a non-durable consumption good and use tangible and intangible capital in the production process. Intangible capital (H) is complementary to high-skilled labour (h) and tangible capital (K) is complementary to manual labour (l). Output, Y_t, is produced according to the following constant elasticity of substitution production technology:

$$Y_t = A\left[\eta\left(H_t^{\alpha}h_t^{1-\alpha}\right)^{\rho} + (1-\eta)\left(K_t^{\alpha}l_t^{1-\alpha}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

where A is a technology parameter, $\rho \geq 0$ is the substitution parameter, and η is a distribution parameter reflecting the relative productivity of intangible capital and high-skilled labour. As a result of skill-biased technological change, η rises exogenously over time (Corrado et al. (2009); Döttling and Perotti (2017)), and this generates an increase in the economy's reliance on intangible inputs.

2.2.2 Scarcity of High-Skilled Labour

To ensure that wages of high-skilled workers are higher than those of low-skilled workers, high-skilled labour is relatively scarce, i.e. (Döttling and Perotti (2017))

Assumption 1.

$$\frac{\phi}{1-\phi} \leq \frac{\eta}{1-\eta}$$

2.2.3 Capital Accumulation

Intangible capital is created by innovators, who capture a fraction of its return (Döttling and Perotti (2017)). Tangible capital follows from upfront investment, where $I_{f,K,t} = K_{f,t+1}$. Both types of capital do not have any productive use after the production period and depreciate fully (i.e. $\delta_K = \delta_H = 1$). Firms operating in $t = 0$ are endowed with an initial stock of tangible capital, K_0, and old innovators are endowed with an initial stock of intangible capital, H_0.

2.2.4 Firm Financing

To finance the set-up of the firm, innovators issues shares. The price of a share of firm f is denoted by $e_{f,t}$ and the quantity of shares of each firm is normalized to 1. Firm equity is backed by the share of intangible capital that is appropriated by equity holders, who receive its value in the form of a dividend payment, $d_{f,t}$, at the end of the production period. To finance the investment in tangible capital, each firm f issues one-period corporate debt, $D_{f,t}$. Corporate debt is repaid each period.

2.2.5 Climate Risk and Firm Capital

Firms are also exposed to climate change, as extreme weather events destroy its capital. As production relies on the use of two distinct types of capital, which may face different exposures to climate change risk, I introduce direct damages to the firm's capital rather than a decline in the TFP factor.[2] For simplicity, only damages to tangible capital are considered and these damages are therefore to be interpreted as the damages to tangible capital *relative* to the damages to intangible capital.[3]

[2]The literature following Nordhaus (1992) models climate-related damages to production as a TFP factor that declines in temperatures. When only one type of capital is used in production and the production function is Cobb-Douglas, damages to capital of size $\xi_{f,t}$ are equivalent to a decline in the TFP factor by $\left(1-\xi_{f,t}\right)^{\alpha}$. Since production relies on both tangible and intangible capital in the present model and only damages to tangible capital are considered, this is equivalent to a decline in the productivity of *physical* inputs by $\left(1-\xi_{f,t}\right)^{\alpha}$. Therefore, the modeling approach also captures a potential decline in the productivity of manual labor (Acharya et al. (2022)).

[3]This is consistent with the classification of (mainly) tangible industries as industries with a high exposure to outdoor heat Acharya et al. (2022).

Let γ_t also capture the probability that a given firm looses a fraction of its tangible capital due to the occurrence of an extreme weather event in period, t. I assume that firms all locate on some production site and therefore are hit equally hard when an extreme weather event occurs. Denote by μ_K the expected losses to tangible capital conditional on being hit by an extreme weather event and by $\xi_{f,t}$ the losses of tangible capital suffered by a given firm, f, in period, t. As firms do not suffer idiosyncratic risk, $\xi_{f,t}$ is deterministic and simply equals

$$\xi_{f,t} = \mu_K \gamma_t$$

Production takes place after an extreme weather event occurred and damaged tangible capital does not have any productive value. Denote by K_t^{net} the tangible capital which remains of use to the firm in the production process, i.e.

$$\begin{aligned} K^{net} &= \left(1 - \xi_{f,t}\right) K_t \\ &= \left(1 - \mu_K \gamma_t\right) K_t \end{aligned}$$

Then, the firm's production is given by

$$Y_t^{net} = A\left[\eta \left(H_t^{\alpha} h_t^{1-\alpha}\right)^{\rho} + (1-\eta)\left(K_t^{net\,\alpha} l_t^{1-\alpha}\right)^{\rho}\right]^{\frac{1}{\rho}}$$

I assume that net income falls in climate-related damages.[4]

Assumption 2.

$$\partial Y_t^{net} / \partial \gamma_t \leq 0$$

and there is no firm default in the baseline model.[5]

Note, finally, that there is a real difference between the climate risk exposure of households and of firms. Since firms only operate one period and its capital stock depreciates fully, climate-related damages only affect firm productivity in a given period (i.e. climate risk has a *flow effect*). In contrast, the supply of housing shrinks over time as a result of climate-related damages to housing capital (i.e. climate risk has a *stock effect*).

2.3 Housing Market Dynamics

There is one housing market on which all home purchases and sales take place. Sales transactions take place after the occurrence of an extreme weather. Since destroyed housing capital is lost and once losses become sufficiently large, households with a mortgage risk default. Define $S_{i,t}$ as the net savings of a household, i in period t, i.e.

[4]While it is unlike that climate-related damages lead to an increase in income, this can occur in general equilibrium happen when households' savings are withdrawn from the housing market and are reallocated to the market for corporate debt. This would not occur if there were a government, however, which attracts funding on capital markets to provide some relief to those suffering climate-related damages. In this latter case, households' savings would flow from the real estate market to the market for sovereign bonds. As this would prevent capital from flowing to the firm and therefore prevent income from rising, the case in which $\partial Y_t / \partial \gamma_t \geq 0$ is ruled out by assumption.

[5]As all financial claims are short-term, this holds for medium values of γ_t. In the limit, where losses become extreme, firm default may, however, occur.

$$S_{i,t} = y_{i,t} - p_t L_{i,t} - s_{i,t} e_t$$

where

$$S_{i,t} \begin{cases} \geq 0 & \text{net lender} \\ < 0 & \text{net borrower} \end{cases}$$

Default occurs when

$$p_{t+1} L_{i,t+1} \leq (1 + r_{t+1})(-S_{i,t})$$

Define the loan-to-value ratio in period $t+1$ as

$$LTV_{i,t+1} = \frac{(1 + r_{t+1})(-S_{i,t})}{p_{t+1} L_{i,t}}$$

then, the level of losses above which a homeowner defaults is

$$\hat{\xi}_{i,t+1} = 1 - LTV_{i,t+1}$$

and the probability that a homeowner defaults in a given period, $t+1$ is given by $\left(1 - F(\hat{\xi}_{i,t+1})\right)$.

2.4 Financial Markets and Credit Risk

Households purchase shares, $s_{i,f,t}$ and lend to the firm, $D_{i,f,t}$ as well as to each other, on the financial market. Lending only occurs against collateral and takes the form of mortgage debt backed by housing capital or corporate debt backed by tangible capital. As firms do not face default risk, equity and corporate debt are in essence equivalent and corporate debt earns the risk-free rate of return, r_t. Household debt is subject to default risk as households face idiosyncratic losses. Therefore, mortgagors pay the risky rate of return, $\hat{r}_t > r_t$.

3 Equilibrium

3.1 Household Optimization Problem

Households maximize utility subject to the budget constraint and limited liability constraint:

$$\begin{aligned} \max_{c_{i,t+1}, L_{i,t}, s_{i,t}, S_{i,t}} \quad & \mathbb{E}\left(U(c_{i,t+1}, L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right) \\ s.t. \quad & y_{i,t} \leq p_t L_{i,t} + s_{i,t} e_t + S_{i,t} \\ & c_{i,t+1} \leq \max\{y_{i,t+1} + p_{t+1}(1 - \xi_{i,t+1}) L_{i,t} + d_{t+1} s_{i,t} + (1 + \hat{r}_{t+1}) S_{i,t}, 0\} \\ & c_{i,t+1}, L_{i,t}, s_{i,t} \geq 0, \end{aligned}$$

where $c_{i,t+1}$ is the consumption of agent i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

3.1.1 Optimal Demand for Housing Capital

The optimal demand for housing capital in a given period t determines its price :

Lemma 1. *The demand for housing capital of each household i in period t is given by*

$$L_t^* = v'^{-1}\left((1+r_{t+1})p_t - \left(1-\mu_L\gamma_{t+1}\right)p_{t+1}\right)$$

Then, the price of housing capital in a given period, t, becomes

$$p_t = \frac{p_{t+1}(1-\mu_L\gamma_{t+1}) + v'\left(L_t^*\right)}{(1+r_{t+1})}$$

The house price today is equal to the discounted benefit of owning a house, where the benefit consists of the marginal benefits of owning house, $v'\left(L_t^*\right)$, as well as the revenue from selling the undamaged housing capital in the next period. The revenue from selling the undamaged housing capital falls in *future* climate risk, γ_{t+1}, which is consistent with the empirically documented existence of a discount against which houses, that are exposed to climate risk, are traded in the market (Bernstein et al. (2019); Bosker et al. (2019); Baldauf et al. (2020)).

3.1.2 Optimal Demand for Shares and Corporate Debt

The price of shares follows from the FOC for share holdings, $s_{i,t}$ and is equal to the discounted value of the dividend payment, d_{t+1}

$$e_t = \frac{d_{t+1}}{(1+r_{t+1})}$$

The investment in corporate and household debt follows as residual. Households with net savings lend to households and firms. Other households take out a mortgage (Döttling and Perotti (2017)).

3.2 Firm Optimization problem

Firms maximize the value to its equity holders. As firms only operate for one period and pay out all profits, the firm maximization is given by

$$\max_{H_t,h_t,K_t,l_t} \pi_{f,t} = Y_t^{net}(A,H_t,h_t,K_t^{net},l_t) - \omega R_t H_t - q_t h_t - [I_{K,t-1} - D_{t-1}] - (1+r_t)K_t - w_t l_t$$

3.2.1 High- and Low-Skilled Workers' Wage

Labour markets are perfectly competitive so high-skilled and low-skilled workers earn their marginal productivity

Lemma 2. *Wages of high-skilled workers, q_t, and wages of low-skilled workers, w_t are given by:*

$$q_t^* = A^\rho(1-\alpha)\eta\frac{{Y_t^{net}}^{1-\rho}}{h_t^{1-(1-\alpha)\rho}}H_t^{\alpha\rho}$$

$$w_t^* = A^\rho(1-\alpha)(1-\eta)\frac{{Y_t^{net}}^{1-\rho}}{l_t^{1-(1-\alpha)\rho}}(1-\mu_K\gamma_t)^{\alpha\rho}K_t^{\alpha\rho}$$

and the wage ratio, defined as $\frac{q_t^*}{w_t^*}$, becomes:

$$\frac{q_t^*}{w_t^*} = \frac{\eta}{1-\eta} \cdot \left(\frac{H_t}{(1-\mu_K\gamma_t)K_t}\right)^{\alpha\rho} \cdot \left(\frac{l_t}{h_t}\right)^{1-(1-\alpha)\rho}$$

Income falls with climate-related damages, and this reduces wages of high- and low-skilled workers. Investments in tangible as well as intangible capital also drop as income falls, with firms scaling back investments in tangible capital to a larger extent since climate-related damages render tangible capital less productive. This offers an additional channel via which climate risk suppresses low-skilled workers wages, and climate risk thereby exacerbates wage inequality.

Proposition 1. *Wage inequality rises in climate-related damages.*

Proof: See Appendix A.1

3.2.2 Return on Tangible Capital

Firms are financially unconstrained and borrow up to the point where the costs of tangible capital are equal to its marginal productivity

Lemma 3. *The return to tangible capital is given by*

$$(1+r_t^*) = A^\rho \alpha(1-\eta) \frac{{Y_t^{net}}^{1-\rho}}{K_t^{1-\alpha\rho}} l_t^{(1-\alpha)\rho} \cdot (1-\mu_K\gamma_t)^{\alpha\rho}$$

and firms fully finance the investment in tangible capital by debt in each period, $I_{K,t}^* = D_t$.

Proposition 2. *Climate-related damages increase the firm's financing costs for $\rho \in [0, \hat{\rho}_t]$.*

Proof: See Appendix A.2

Climate-related damages render tangible capital less productive and reduce income growth, both suppressing the return on tangible capital. On the other hand, extreme weather events make capital relatively more scarce and this pushes rates up. Whether this latter effect dominates depends on the value of ρ, which reflects the degree of substitutability between tangible inputs - which face climate-related damages - and intangible inputs - which are not directly exposed to extreme weather events. For $\rho = 0$, the production function becomes Cobb-Douglas, and as the elasticity of tangible capital is larger than the elasticity of income with respect to climate related-damages, the rise in the scarcity of capital dominates and firm's financing costs *rise*.

As ρ increases, substitutability becomes higher and this improves the ability of firms to reduce their exposure to climate-related damages. While this weakens the effect driven by income growth, and reduces firm's reliance on capital, this does not compensate for the fall in productivity. Therefore, once substitutability become sufficiently large (i.e. $\rho > \hat{\rho}$), the productivity effect dominates and firm's financing costs *fall* in climate-related damages.

3.2.3 Return on Intangible Capital

Competitive firms pay a return on intangible capital equal to its marginal productivity

Lemma 4. *The return on intangible capital is given by:*

$$R_t^* = A^\rho \alpha\eta \frac{Y_t^{net\,1-\rho}}{H_t^{1-\alpha\rho}} h_t^{(1-\alpha)\rho}$$

and the amount of intangible capital created by innovators is

$$I_t^* = \frac{\omega R_{t+1}^*}{\beta}$$

where $I_t^* = H_{t+1}^*$ (Döttling and Perotti (2017)).

3.2.4 Dividends and Share Prices

Since innovators only capture a share ω of the return to intangibles capital, equilibrium dividends are equal to the value captured by the shareholders:

$$\begin{aligned} d_t^* &= Y_t^{net}(H_t, K_t^{net}, h_t, l_t) - \left(\omega R_t^* H_t + q_t^* h_t + (1+r_t^*)K_{t-1} + w_t^* l_t\right) \\ &= (1-\omega)R_t^* H_t \end{aligned}$$

Lemma 5. *Dividends decline in climate-related damages.*

The downwards revision of future dividends is a result of the reduction in firm profitability, as caused by climate-related damages. Such revisions trigger a revaluation of firm's equity, suppressing equity prices (Campiglio and der Ploeg (2021), which are given by

$$e_t^* = \frac{(1-\omega)R_{t+1}H_{t+1}}{1+r_{t+1}}$$

Proposition 3. *Equity prices fall in climate risk for $\rho \in [0, \bar{\rho}_t]$, where $\bar{\rho}_t \geq \hat{\rho}_t$.*[6]

Proof: See Appendix A.3

Proposition 2 and 3 have important implications for (financial) wealth inequality. As equity values decline, this reduces the financial wealth of shareholders and (financial) wealth inequality falls. This implies that asset price changes form an equalizing force for wealth inequality. On the other hand, however, the rate at which households with positive savings accumulate wealth rises. The latter increases wealth inequality, and savings, therefore, form an unequalizing force (Bauluz et al. (2022)).

[6] As dividends fall in climate risk, incentives for substitution must be sufficiently strong to ensure that the fall in dividends - which reduces share prices - is offset by the fall in firm's financing costs - which puts upwards pressure on share prices. In the model simulations (Section 6.1.3), I show that Y rises when $\rho \geq \hat{\rho}$. This leads to a fall in firm's financing costs. Additionally, dividends rise once this threshold is passed. This implies that the condition holds with inequality, i.e. $\bar{\rho}_t = \hat{\rho}_t$.

3.3 Equilibrium and Market Clearing

A competitive equilibrium is defined as an allocation $\{c_t^l, c_t^h, L_t^l, L_t^h, s_t^l, s_t^h, D_t^l, D_t^h, K_t, H_t, l_t, h_t\}_{t=0}^T$ and prices $\{p_t, e_t, r_t, R_t, w_t, h_t\}_{t=0}^T$ such that in each period, t, given prices

1. Households maximize lifetime utility
2. Firms maximize profits
3. Innovators choose intangible investment

and markets clear, i.e.

1. Total labour demand equals total labour supply:

$$\int_0^{\phi\epsilon} \left[h_{f,t}^d, l_{f,t}^d\right] df = [h^s, l^s]$$

2. Total housing demand equals total housing supply:

$$\int_0^1 L_{i,t}^* di = \phi\epsilon \bar{L}_t$$

3. Total demand for shares equals total supply of shares:

$$\int_0^1 s_{i,t}^* di = \phi\epsilon$$

4. Total net savings are equal to the value of the firm's market capitalization and its corporate debt:

$$(1-\alpha) Y_t^{net} - p_t \bar{L}_t = e_t + D_t$$

where $(1-\alpha)\phi\epsilon Y_t^{net} = q_t \phi \tilde{h} + w_t (1-\phi)\tilde{l}$.

3.3.1 Labour Market Clearing

Households supply their entire labour endowment since its marginal product is strictly positive, i.e. $[h^s, l^s] = \{\phi\bar{h}, (1-\phi)\bar{l}\}$ and

$$\int_0^{\phi\epsilon} \left[h_{f,t}^d, l_{f,t}^d\right] df = \{\phi\bar{h}, (1-\phi)\bar{l}\}$$

3.3.2 Housing Market Clearing

The housing market clearing condition pins down the equilibrium price of housing capital in a given period, t

$$\begin{aligned} p_t^* &= \frac{p_{t+1}(1-\mu_L \gamma_{t+1}) + v'\left(\phi\epsilon\bar{L}_t\right)}{1+r_{t+1}} \\ &= \sum_{j=t}^{\infty} \left(\prod_{\tau=t}^{j} \frac{1}{1+r_{\tau+1}}\right) \left[v'\left(\phi\epsilon\bar{L}_j\right)\right] \prod_{i=t}^{j-1} (1-\mu_L\gamma_{i+1}) \end{aligned}$$

Proposition 4. *In a partial equilibrium, the price of housing capital rises in future climate risk when Currently being updated*

Proof: See Appendix A.4

The equilibrium price of housing capital reveals a *novel* channel via which climate risk affects the price of housing capital As climate-related damages materialize, this reduces the supply of housing capital and increases the marginal benefit of owning a house. The latter effect puts upwards pressure on the contemporaneous prices of housing capital. This general equilibrium effect propagates as the supply of housing capital is permanently reduced, but is also amplified due to the further rise in climate risk. When this general equilibrium effect dominates, house prices *rise* faster as climate risk becomes higher.

3.3.3 Financial Market Clearing

The financial market clearing condition requires the value of assets that carry savings over time (RHS), i.e. housing, shares and corporate debt, to equal total savings from labour income of households (LHS). Recall that $D_t = K_t$. Then, the supply of tangible capital is given by

$$K_t^S = (1-\alpha)Y_t^{net} - p_t\bar{L}_t - e_t$$

which falls in climate-related damages.

Additionally, the financial market clearing condition requires total savings of labour income to be large enough to cover the purchase of the stock of houses and the firm's equity as well as its corporate debt. This implies that at least one type of workers must have positive savings (Döttling and Perotti (2017)). Assumption 1 ensures that the wage high-skilled workers is higher than the wage of low-skilled workers and therefore high-skilled workers are net lenders. Low-skilled, then, are either net lenders or net borrowers in equilibrium and the volume of mortgage credit in the economy, m, becomes

$$m_t = \max\left\{0, (1-\phi)\left(p_t\phi\epsilon\bar{L}_t - w_t\tilde{l}\right)\right\}$$

Corollary 1. *When the price of housing capital in a given period, t, rises in future climate risk (i.e. when the condition in Proposition 4 is satisfied), mortgage credit volumes rise in future climate risk.*

Note that climate-related damages affects mortgage credit volumes in two opposing directions in the current period, t. Since climate-related damages reduces wages of low-skilled workers, this increases mortgage credit demand. On the other hand, as climate related-damages reduce the supply of housing capital and households are confined to purchase a lower amount of housing capital, this reduces funding needs and therefore reduces their mortgage credit demand.

4 Adaptation to Climate Change

Households cannot take measures to mitigate climate change, as climate change is exogenous for the economy under consideration. However, households can adapt to climate change by investing in measures that reduce their vulnerability to the effects of extreme weather events. Importantly, climate change adaptation does not affect the level of the economy's climate risk exposure in a given period (γ_t) nor it's evolution ($\gamma_{t+1},...,\gamma_\infty$), but rather shifts the distribution of the *idiosyncratic* losses to the left. This reduces damages to housing capital once an extreme weather event occurs. It is assumed that adaptation cannot perfectly insure households against the consequences of climate change. Therefore, losses cannot be fully eliminated. Rather, adaptation reduces the speed at which the housing stock falls.

4.1 Adaptation

Households invest in climate change adaptation after purchasing housing capital. Denote by $x_{i,t} \in [0,1)$ the degree to which household i adapt to climate change in period t. To reach a certain degree of adaptation, $x_{i,t}$, households must make a costly investment, of $\psi(x_{i,t}) = \frac{\theta}{2}L_{i,t}(x_{i,t})^2$, where θ captures the ease of adapting. The cost of investment rises with the housing capital owned by household i, as larger housing capital means more adaptation measures are required to achieve a similar level of protection (Fried (2021)). For example,a bigger house requires more storm-proof windows or longer seawalls for similar protection as a smaller house. The investment cost increases convexly with the level of adaptation. This is because low-cost measures may reduce damage, but full prevention of climate impacts requires advanced measures.

Adaptation reduces the idiosyncratic losses suffered when hit by an extreme weather event. Let the degree of adaptation, $x_{i,t}$, represent the fraction of expected losses (i.e. $\mu\gamma_{t+1}$) by which the idiosyncratic losses are reduced, i.e. for a degree of adaptation of $x_{i,t}$, the distribution of idiosyncratic losses is shifted to the left by $x_{i,t}\mu\gamma_{t+1}$. The degree of adaptation therefore reflects the extent to which households have adapted to climate change, where $x_{i,t} = 0$ indicates that a given household did not undertake any measures to reduce climate-related losses, whereas $x_{i,t} \to 1$ indicates that the household has perfectly adapted to climate change and has reduced nearly all expected losses. Since adaptation shifts the distribution of idiosyncratic losses to the left, the mean of $F(\xi_{i,t})$ becomes

$$\mathbb{E}\left(\xi_{i,t+1}\right) = \left(1 - x_{i,t}\right)\mu_L\gamma_{t+1}$$

In expectation, a fraction $x_{i,t}\mu_L\gamma_{t+1}$ of the housing capital of household i remains preserved when hit by an extreme weather event. This means that adaptation reduces the speed by which the stock of houses declines

$$\bar{L}_{t+1} = \int_0^1 \left(1 - \xi_{i,t}\right) di L_t$$

which, by the law of large numbers, becomes

$$\bar{L}_{t+1} = \int_0^1 \left(\left(1 - x_{i,t}\right)\mu\gamma_{t+1}\right) di L_t$$

Define X_t as the aggregate degree of adaptation, i.e.

$$X_t = \int_0^1 x_{i,t} di$$

Then, the stock of housing capital declines according to the following law of motion

$$\bar{L}_{t+1} = \left(1 - (1 - X_t)\mu_L \gamma_{t+1}\right) \bar{L}_t$$

4.2 Equilibrium with Adaptation

4.2.1 Household Optimization Problem

With adaptation, households' optimization problem becomes

$$\begin{aligned}
\max_{c_{i,t+1}, x_{i,t}, L_{i,t}, S_{i,t}, s_{i,t}} \quad & \mathbb{E}\left(U(c_{i,t+1}, L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right) \\
s.t. \quad & y_{i,t} \leq \left(p_t + \frac{\theta}{2} x_{i,t}^2\right) L_{i,t} + s_{i,t} e_t + S_{i,t} \\
& c_{i,t+1} \leq \max\left\{y_{i,t+1} + p_{t+1}\left(1 - \xi_{i,t+1}\right) L_{i,t} + (e_{t+1} + d_{t+1}) s_{i,t} + (1 + \hat{r}_{t+1}) S_{i,t}, 0\right\} \\
& c_{i,t+1}, x_{i,t}, L_{i,t}, s_{i,t} \geq 0
\end{aligned}$$

4.2.2 Optimal Demand for Housing and Adaptation

The introduction of adaptation affects housing demand in two opposing directions. As the investment in adaptation absorbs part of households' savings, this reduces demand for housing capital. On the other hand, adaptation reduces the expected losses of housing capital, and therefore increases the revenue from selling housing capital in the next period. This latter effect increases demand for housing capital. As the optimal demand for housing capital in a given period, t, determines its price, the trade-off between the present costs and future benefits of adaptation is also reflected in the price of housing capital:

Lemma 6. *With adaptation, the demand for housing capital of each household i in a given period t is given by*

$$L_t^* = v'^{-1}\left((1 + r_{t+1})\left(p_t + \frac{\theta}{2} x_{i,t}^{*2}\right) - \left(1 - (1 - x_{i,t}^*)\mu_L \gamma_{t+1}\right) p_{t+1}\right)$$

and the price of housing capital in a given period, t, is given by

$$p_t = \frac{\left(1 - (1 - x_{i,t}^*)\mu_L \gamma_{t+1}\right) p_{t+1} + v'(L_t^*)}{(1 + r_{t+1})} - \frac{\theta}{2} x_{i,t}^{*2}$$

Moreover, as households only invest in adaptation as long as its marginal benefits outweigh its marginal costs, the optimal level of adaptation becomes

Lemma 7. *The privately optimal degree of adaptation for each household i is given by*

$$x_t^* = \frac{\mu_L \gamma_{t+1} \cdot p_{t+1}}{\theta \cdot (1 + r_{t+1})}$$

As climate risk is priced in the market, adaptation increases the revenue from selling housing capital to the next generation, and households internalize the benefits of their adaptive efforts for future generations. This prevents "tragedy of the horizon" effects (Carney (2015)), and the privately optimal investment in adaptation is therefore efficient.

Proposition 5. *The privately optimal investment of climate change adaptation is efficient.*

Proof: See Appendix A.5

Note that this finding relies on the underlying assumption that climate risk is (fairly) priced in the market. When climate risk is imperfectly priced, prices fail to signal the risk to which households are exposed, and households underinvest in adaptation. Proposition 5 therefore highlights the importance of policies that contribute to the accurate pricing of climate risk in housing markets, as such policies encourage households to adapt optimally. Additionally, this finding is based on the assumption that the social planner maximizes utilitarian welfare and weights generations based on market discount rates. When the welfare of future generations is evaluated at a rate smaller than market discount rates, underinvestment in adaptation again follows.

Corollary 2. *When the social planner discounts the welfare of future generations at rate* $r^{SP} \in [0,1]$ *and* $\frac{1}{1+r^{SP}} > \frac{1}{1+r_{t+1}}$, *households* underinvest *in adaptation. The size of the underinvestment is given by*

$$\sum_{j=t+1}^{\infty}\left(\frac{1}{1+r^{SP}}\right)^{j-t} - \prod_{\tau=t+1}^{j}\left(\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right]\prod_{i=t+1}^{j-1}(1-(1-x_i)\mu_L\gamma_{i+1})$$

Proof: See Appendix D.1

Corollary 2 highlights that the choice of discount rates is essential to ensure that the privately optimal outcome is efficient, indicating that the discounting debate in climate change (Stern (2007); Nordhaus (2008)), is also relevant for the determination of optimal climate change adaptation policies.

4.2.3 Optimal Demand for Shares and Corporate Debt

The price of shares remains the same as in Section 3.1.2. and investments in corporate and household debt continue to follow as residual. Since adaptation increases the total costs associated with the purchase of housing capital, it reduces households' savings.

4.2.4 Equilibrium and Market Clearing

A competitive equilibrium is defined as an allocation $\{c_t^l, c_t^h, L_t^l, L_t^h, x_t^l, x_t^h, s_t^l, s_t^h, D_t^l, D_t^h, K_t, H_t, l_t, h_t\}_{t=0}^T$ and prices $\{p_t, e_t, r_t, R_t, w_t, q_t\}_{t=0}^T$ such that given prices

1. Households maximize lifetime utility

2. Firms maximize profits

3. Innovators choose intangible investment

and markets clear. With adaptation, the labour market clearing condition and stock market clearing condition remain the same as in Section 3.3. The housing market clearing becomes

$$\int_0^1 L_{i^*,t} di = \phi\epsilon\bar{L}_t$$

where $\bar{L}_t = (1-(1-X_t)\mu\gamma_t)\bar{L}_{t-1}$ (see Section 4.1).

This clearing condition again determines the equilibrium price of a house in period t, which becomes

$$\begin{aligned} p_t^* &= \frac{\left(1-(1-X_t^*)\mu_L\gamma_{t+1}\right)p_{t+1} + v'(\phi\epsilon\bar{L}_t)}{(1+r_{t+1})} - \frac{\theta}{2}X_t^{*,2} \\ &= \sum_{j=t}^{\infty}\left(\prod_{\tau=t}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}X_j^{*,2} + v'(\phi\epsilon\bar{L}_j)\right]\prod_{i=t}^{j-1}\left(1-(1-X_i)\mu_L\gamma_{i+1}\right) \end{aligned}$$

The financial market clearing condition accounts for the investments in adaptation since this absorb part of workers' savings:

$$(1-\alpha)Y_t - \left(p_t(X_t) + \frac{\theta}{2}X_t^{2*}\right)\bar{L}_t = e_t + D_t$$

Moreover, investments in adaptation increase the funding needs of those who borrow. Therefore, the volume of mortgage credit demand becomes:

$$m_t = \max\left\{0, (1-\phi)\left(p_t(X_t) + \frac{\theta}{2}X_t^2\right)\phi\epsilon\bar{L}_t - w_t\tilde{l}\right\}$$

5 Endogenous Credit Constraints

To prevent low-skilled households with large amounts of debt from repudiating on their debt contract, creditors collateralize their housing capital. Thus far, creditors did not take into consideration that the colleteralized housing capital is at risk of becoming destroyed, and destroyed housing capital has zero liquidation value. However, as creditors have perfect foresight on the evolution of climate risk,and losses of housing capital are therefore foreseeable, creditors would never allow the size of the debt to exceed the liquidation value of the collateral (Kiyotaki and Moore (1997)). Specifically, when a low-skilled households purchases $L_{l,t}$ housing capital in period t and pledges this housing capital as collateral, this household cannot borrow more than the period $t+1$ market value of the, in expectation, undamaged fraction of the underlying housing capital (Kiyotaki and Moore (1997)):

$$-S_{l,t} \leq \left(1-\left(1-\mathbb{E}\left(\bar{x}_t^l\right)\right)\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}$$

At a constant prices of housing capital, credit constraints tighten over time as the economy's climate risk exposure rises. Given rational expectations and as there is no aggregate uncertainty about γ_t, creditors also have perfect foresight of future prices of housing capital (Kiyotaki and Moore (1997)).

Low-skilled households adapt to rising climate risk, but cannot commit to a certain level of investment. This means that creditors must form expectations about the level of adaptation by low-skilled households, denoted by $\bar{x}_{l,t}$, and set the credit constraint accordingly. As climate change adaptation reduces the expected losses to housing capital, the liquidation value of the pledged housing capital becomes higher. Adaptation, therefore provides a countervailing force against the tightening of the credit constraint due to the rise in the economy's climate risk exposure.

5.1 Equilibrium with Credit Constraints

Low-skilled households maximize expected utility subject to their budget constraint and the credit constraint

$$
\begin{aligned}
\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},L_{l,t})\right) &= \mathbb{E}_t\left(c_{l,t+1}\right) + v\left(L_{l,t}\right) \\
s.t. \quad w_t &\leq \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t} + S_{l,t} \\
c_{l,t+1} &\leq \max\left\{p_{t+1}\left(1-\xi_{l,t+1}\right)L_{l,t} + (1+\hat{r}_{t+1})\,S_{l,t}, 0\right\} \\
-S_{l,t} &\leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t} \\
c_{l,t+1}, x_{l,t}, L_{l,t} &\geq 0
\end{aligned}
$$

5.1.1 Optimal Demand for Housing and Adaptation of Low-Skilled Workers

Credit constraint low-skilled workers have limited resources to purchase housing capital and invest in adaptation. In equilibrium, low-skilled workers borrow up to the point where the constraint bind and their demand for housing capital is defined in terms of $x_{l,t}^*$:

Lemma 8. *The demand for housing capital of low-skilled workers in terms of $x_{l,t}^*$ is given by:*

$$
L_{l,t}^* = \frac{w_t}{\left(1-\left(1-x_{l,t}^*\right)\mu_L\gamma_{t+1}\right)p_{t+1} + \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)}
$$

where $\mathbb{E}(x_{l,t}) = x_{l,t}^*$ in a symmetric equilibrium.

The degree to which low-skilled households adapt, $x_{l,t}^*$, is given by

$$
x_{l,t}^* = \frac{\mu_L\gamma_{t+1}\cdot p_{t+1}}{\theta\cdot(1+r_{t+1}+\lambda)}
$$

where $\lambda \geq 0$ denotes the shadow price.

Proposition 5. *Low-skilled workers adapt less to climate change than high-skilled workers when the credit constraint binds.*

Proof: See Appendix A.5

When credit constraints bind, low-skilled households do not have sufficient resources to adapt efficiently. As a result, these households lose a relatively larger part of their housing wealth once an extreme weather event occurs. Importantly, as credit-constraint low-skilled households possess less housing wealth than other households, the failure to adapt efficiently to climate change exacerbates wealth inequality.

6 Model Simulation and Counterfactual Analysis

To evaluate the equilibrium effects of rising climate risk on house prices, mortgage credit volumes and default rates, I run model simulations. I focus on flood risk in particular and restrict my attention to the Netherlands, a country with a long history in flood risk management. I set one time period equal to 30 years and run the model forward from 2010 to 2100. Additonally, I conduct counterfactual analysis, in which I consider the low (RCP 2.6), medium (RCP 4.5), and high (RCP 8.5) greenhouse gas concentration trajectories from the IPCC (IPCC (2013)).

6.1 Parameterization, Specification of Functional Forms and Distributions

6.1.1 Parameters

Parameter values are based on the existing literature as well as data. The values are reported as off 2010 and can be found in Table 1. I also use a number of normalizations.

Parameter	Description	Value	Target/Source
A	TFP in final-good production	1	Standard Value/Normalization
$\tilde{h}$	Inelastic Supply of High-Skilled Labour	24.438	Döttling and Perotti (2017)
$\tilde{l}$	Inelastic Supply of Low-Skilled Labour	15.945	Döttling and Perotti (2017)
$\bar{L}$	Initial Stock of Houses	1	Normalization
α	Capital share in final-good production	0.36	Bengtsson and Waldenström (2018)
β	Ease of innovating	1	Normalization
ϵ	Fraction of High-Skilled Entrepreneurs	0.05	
η	Relative productivity of intangible inputs	0.60	Döttling and Perotti (2017)
θ	Ease of adapting	1	Normalization
μ_L	Fraction of damages to housing capital	1	Normalization
μ_K	Fraction of damages to tangible capital	0.7	Target $\mu_L/\mu_K = 0.7$ (Fried (2021))
ρ	Substitution parameter	0	Cobb-Douglas Production
ϕ	Fraction of high skilled labour	0.28	Van der Mooren and De Vries (2022)
ω	Bargaining power of entrepreneurs	0.1	

Table 1: *Values of Parameter Used in the Model Simulations*

Note that as the production function becomes Cobb-Douglas (i.e. $\rho = 0$), climate risk no longer affects wages via changes in γ_t, H_t, K_t but only does so via changes in Y_t. For the purpose of these simulations, I relax Assumption 2 such that there are no restrictions on the change in Y_t in response to rising climate risk. Additionally, I assume that $v(L) = ln(L)$ in all model simulations.

6.1.2 Determination of γ_t

To approximate the evolution of γ_t, which pins down a number of key variables of interest, I determine the fraction of houses at risk of flooding for a given level of sea level rise. I use the Climate System Scenario Tables of the IPCC to obtain projection on global mean sea level rise for each decade, from 2010 to 2100 onwards (the period 1986-2005 serves as reference period) (IPCC (2013)). The projections are reported under a low (RCP 2.6), medium (RCP 4.5, RCP 6.0) and high (RCP 8.5) greenhouse gas concentration trajectory.[7]

To establish the relation between the rise in sea levels and the additional houses at risk of flooding in the Netherlands and the rise in sea levels, a back of the envelope calculation is made based on estimates of Bosker et al. (2019), who use Dutch elevation data to obtain the number of houses at risk of flooding under a best- (24cm), medium- (100cm). The estimates are used in combination with the 50^{th} percentile of the projected gloabl mean sea level rise of the IPCC to generate values for γ_t. The result is depicted in Figure 1 for different RCP trajectories.

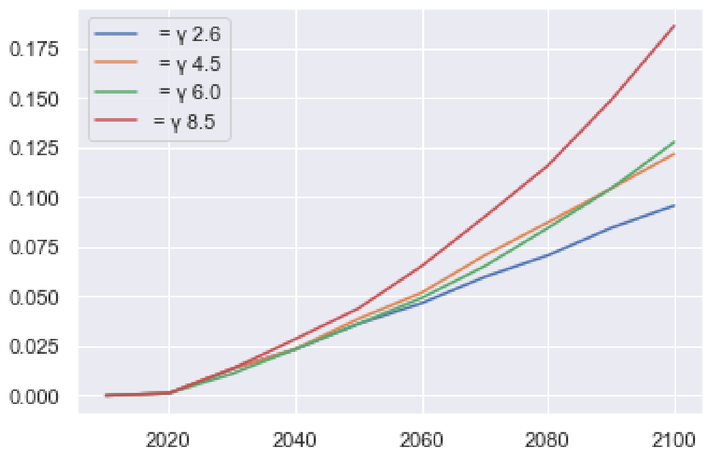

Figure 1: *The evolution of γ_t under the RCP 2.6, RCP 4.5, RCP 6.0 and RCP 8.5 trajectory.*

6.1.3 Distribution of $\xi_{i,t}$

I assume that $\xi_{i,t}$ follows a Beta distribution with shape parameters $\upsilon \geq 0, \nu \geq 0$, i.e. $\xi_{i,t} \sim Beta(\upsilon, \nu)$. The Beta distribution is defined on the interval $[0,1]$, and its shape depends on the values of its shape parameters. To reflect the fact that it is relatively rare for a household to lose its entire property due to a flood, but relatively common for a household to suffer some (small) amount of damage, I set the values of the shape parameters to ensure that the proba-

[7] The Representative Concentration Pathways trajectories describe different climate futures depending on the volume of future greenhouse gas emissions (IPCC (2014)). Under the RCP 2.6 (RCP 4.5 respectively RCP 6.0) trajectory, emissions peak in 2020 (2040 respectively 2080) and the rise in global mean temperatures is likely to stay between 0.3 to 1.7 (1.1 to 2.6 respectively 1.4 to 3.1) degrees Celsius, relative to the reference period. This translates into a rise in global mean sea levels as of 2100 of 0.26 to 0.55 (0.32 to 0.63 respectively 0.33 to 0.63) meters relative to the reference period (IPCC (2014)). Under RCP 8.5, emissions continue to rise throughout the 21^{st} century and global mean temperatures are likely to rise by approximately 2.6 to 4.8 degrees Celsius. This translates into a rise in global mean sea levels of 0.45 to 0.82 meters (IPCC (2014)).

bility is concentrated among low values of $\xi_{i,t}$, with large values in the tail of the distribution. Specifically, I set $\upsilon = \mu_L$, which is equal to one in the model simulation. To determine the value of ν, note that the expectation of $\xi_{i,t}$ was given by

$$\mathbb{E}\left(\xi_{i,t}\right) = \mu_L \gamma_t$$

I set this value equal to the expectation of $\xi_{i,t}$ as implied by the $Beta(\upsilon, \nu)$ distribution, i.e.

$$\mathbb{E}\left(\xi_{i,t}\right) = \frac{\upsilon}{\upsilon + \nu}$$

Then, ν is given by

$$\nu_t = \frac{1}{\mu_L \gamma_t} - 1$$

and becomes time-varying as it depends direct on γ_t. As γ_t remains relatively small under each RCP trajectory for a large part of the time period covered, ν_t becomes very large. This creates computational difficulties for estimating the probability density function as its determination involves the gamma function $\Gamma(\nu_t)$, which is computationally not defined for most of the values ν_t. To overcome this, I calculate the probability density function without applying normalization. Then, to ensure that the area under the curve is equal to 1, I divide each value along the curve by the integral of the non-normalized probability density function. The results are displayed in Figure 2.

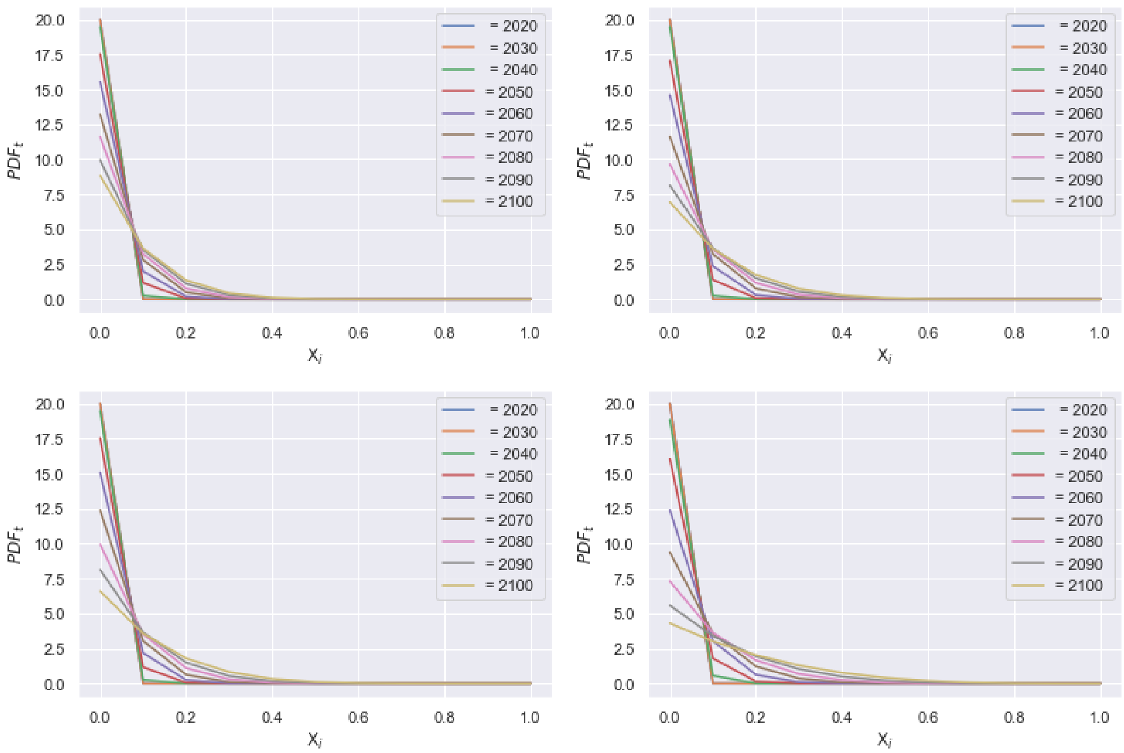

Figure 2: *The probability density function of $\xi_{i,t}$ for different periods in time. The left upper panel plots the probability density functions over time under the RCP 2.6 scenario, and the right upper panel plots it under the RCP 4.5 scenario. The left lower panel plots the probability density functions over time under the RCP 6.0 scenario, and the right lower panel plots it under the RCP 8.5 scenario.*

The figure shows that small levels of damages (i.e., ξ_t) are relatively more common under each scenario. As the probability of a flood increases (i.e., γ_t), small values of damages become relatively less common while larger values of damages are suffered more frequently. This is visible when comparing the probability density functions over different time periods within each plot as well as when comparing different RCP trajectories across plots.

The cumulative density function is found subsequently by integrating the area under each normalized probability density function, for different levels of damages. The cumulative density functions are shown graphically in Figure 3.

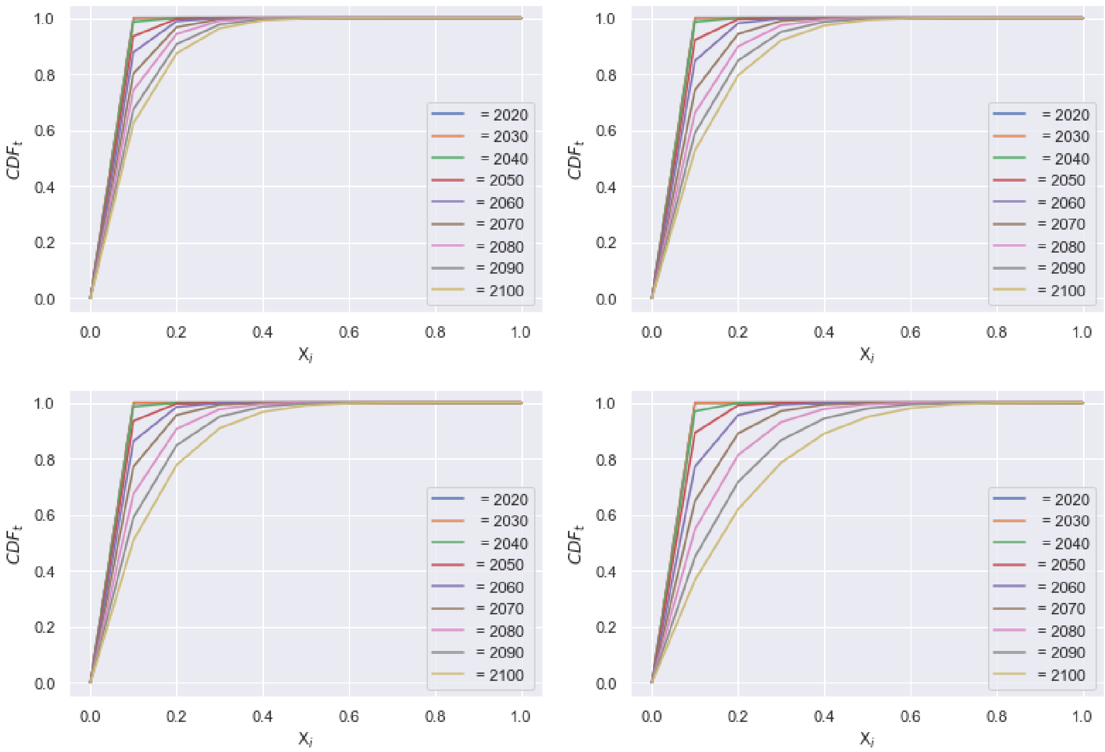

Figure 3: *The cumulative density function of $\xi_{i,t}$ for different periods in time. The left upper panel plots the cumulative density functions over time under the RCP 2.6 scenario, and the right upper panel plots it under the RCP 4.5 scenario. The left lower panel plots the cumulative density functions over time under the RCP 6.0 scenario, and the right lower panel plots it under the RCP 8.5 scenario.*

6.2 Simulation Results

The simulation of the model is carried out under the parameterization described in Section 6.1.1. Climate risk is absent in the baseline model, and only η grows over time, following the growth path provided in Döttling and Perotti (2017). The steady-state equations are determined and used to solve the model backwards. The simulation assumes that the economy moves instantaneously to a new steady-state, and therefore shows the balanced growth path of the economy. Counterfactual analysis is performed to compare the outcomes of the baseline model to the results found under different projections of the Representative Concentration Pathways (RCP) provided by the Intergovernmental Panel on Climate Change (IPCC). The results are to be interpreted as deviations from the balanced growth path.

For each RCP trajectory, γ follows the path described in Section 6.1.2. Since estimates of projected global mean sea level rise are only provided up to 2100, γ is assumed to reach a steady-state by 2100 and remains constant thereafter. Likewise, η is assumed to stop growing after 2100. The simulation is extended to include the case *(i)* when households adapt to climate change, and *(ii)* when households face financial constraints. A detailed explanation of the simulation method can be found in Appendix C.

6.2.1 Baseline Model

The results of the baseline model are depicted in below. In each figure, the balanced growth path is shown absent climate risk and deviations of the balanced growth path are provided under each RCP trajectory. Figure 4 illustrates the growth path of the economy's income. This figure reveals that income only falls in climate risk relative to the balanced growth path until the year 2080, and rises afterwards. This reversal is driven by firm's substitution from tangible to intangible capital. As climate-related damages become larger, tangible capital becomes less attractive for firms due to the decline in its productivity and the simultaneous increase in its price. This occurs against a background of skill-biased technological change, which increases the relative productivity of intangible capital. Climate risk, therefore, provides an additional incentive for firms to shift to the use of intangible inputs in its production process. The larger climate risk, the stronger the incentive for substitution becomes, and the more the firm benefits from the rise in the relative productivity of intangibles.

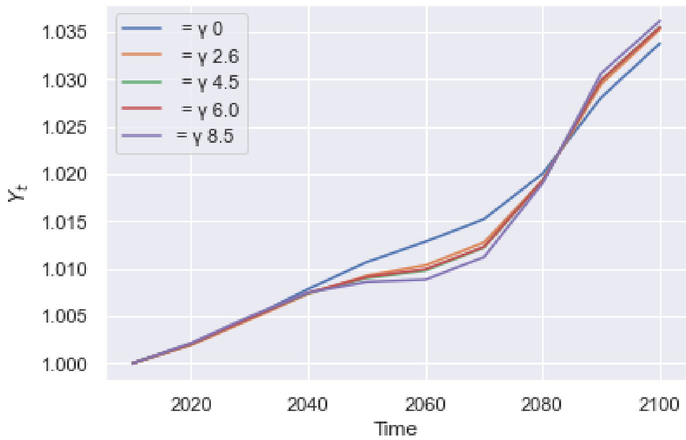

Figure 4: *The balanced growth path of income in the absence of climate change and deviations of the balanced growth path under the different RCP trajectories.*

Figure 5 shows that house prices to income grow faster when climate risk is higher. This trend is observed over a number of decades and becomes even more pronounced as income rises in climate risk. This confirms that under the present parameterization the condition established in Proposition 4 is satisfied, and the general equilibrium effect therefore dominates the direct damage-effect. As the supply of inhabitable houses decreases, the upwards pressure on house prices becomes lager than the price discount due to climate-related damages, leading to a stronger growth of house prices to income over time as compared to growth path in the absence of climate risk.

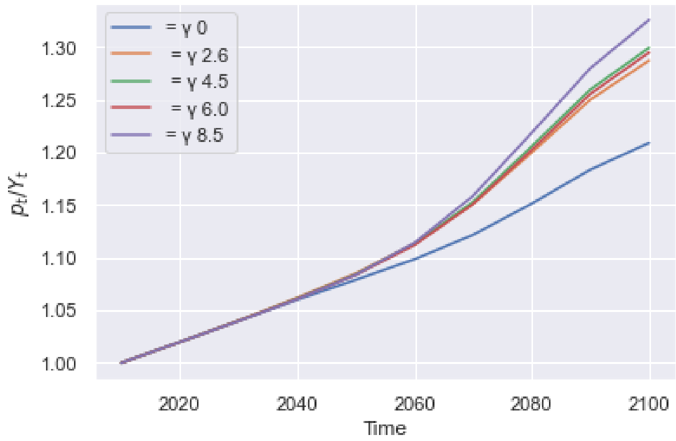

Figure 5: *The balanced growth path of house prices to income in the absence of climate change and the deviations from the balanced growth path under the different RCP trajectories.*

While the effect is evident for house prices to income, this is not the case for mortgage credit to income. Figure 6) highlights that the effect of climate risk on mortgage credit volumes is negligible, suggesting that the rise in house prices to income and the fall in the stock of houses balance out.

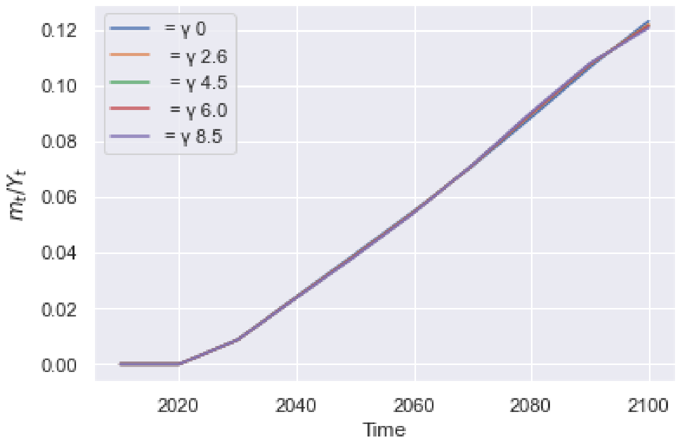

Figure 6: *The balanced growth path of mortgage credit to income in the absence of climate change and the deviations from the balanced growth path under the different RCP trajectories.*

Figure 7 presents the balanced growth path of firm's financing costs, which displays a downwards trend. Consistent with Proposition 2, firm's financing costs rise in climate risk relative to the balanced growth path for all periods when income falls in climate risk. Once income rises, however, firm's financing costs fall in climate risk. As established in Proposition 2, firm's financing costs fall up to the point where $\rho < \hat{\rho}_t$. As ρ is equal to zero in the model simulations, the above result suggests that $\hat{\rho}_t$ falls below 0 around 2080. Furthermore, combining the findings from Figure 4 with the conclusion mentioned above, reveals that the relevant threshold for ρ is the point at which income begins to increase with climate risk. In essence, as the incentives for substitution become sufficiently strong (i.e. when $\hat{\rho}_t$ falls sufficiently), income grows in climate risk relative to its balanced growth path. This reverses the effect of climate risk on the scarcity of capital as well as on income growth, and thus leads to an increase in firm's financing costs as well.

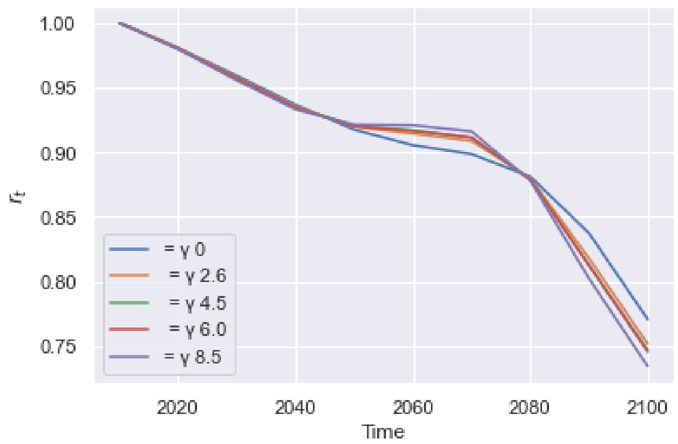

Figure 7: *The balanced growth path of firm's financing costs in the absence of climate change and deviations from the balanced growth path under the different RCP trajectories.*

The above findings also have implications for share prices, which are depicted in Figure 8. Consistent with Proposition 3, share prices fall in climate risk relative to the balanced growth path for all periods when income falls. Theoretical results pointed out share prices only rise in climate risk when the incentives for substitution are at least as large as needed for the reversal in firm's financing costs (i.e. $\bar{\rho} \geq \hat{\rho}$), as the fall in dividends must be offset in addition. However, as income rises in climate risk once $\hat{\rho}_t$ falls below 0, this directly translates into a rise in share prices. As income rises in climate risk, this increase both firm's financing costs (as displayed in Figure 7) as well as dividends. Therefore, reversal in share prices coincides with the reversal in income and firm's financing costs as well as the reversal in share prices, implying that $\bar{\rho}_t = \hat{\rho}_t$.

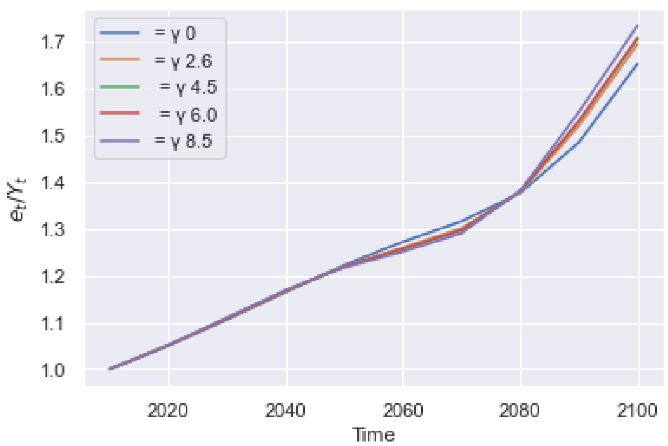

Figure 8: *The balanced growth path of share prices to income in the absence of climate change and deviations from the balanced growth path under the different RCP trajectories.*

6.2.2 Simulations with Adaptation

Currently being updated

6.3 Simulations with Credit Constraints

Currently being updated

7 Conclusion

This paper explores the broader implications of climate change and adaptation through a general equilibrium model. The analysis shows that climate change has a redistributive effect that exacerbates wage inequality, as low-income households face larger declines in their wages as a result of climate-related damages. While several existing papers have emerged that study the effects of climate change mitigation policies on wealth inequality, this paper is - to my knowledge - the first that focuses on the relation of climate adaptation and wealth inequality. I show that there are feedback effects of adaptation on wealth inequality in the presence of financial constraints. As credit-constrained low-income households do not have sufficient resources to adapt efficiently, these households lose a relatively larger part of their housing wealth once an extreme weather event occurs. Importantly, as credit-constraint low-skilled households possess less housing wealth than other households, the failure to adapt to climate change exacerbates wealth inequality. Recent analyses (e.g. Havlinova et al. (2022)) show that a significant number of homeowners are unable to finance investments that improve the sustainability of their homes due to credit constraints, and this is in particular the case for homeowners in the bottom deciles of the income distribution This finding, therefore, argues for targeted policies, which support investments in adaptation by low-skilled households to prevent such underinvestment to translate into a further rise in wealth inequality. This finding has further implications due tot the interaction of climate change and skill-biased technological change. While both climate-change and skill-biased technological change reduce wages of low-skilled households over time, technological change drives house prices up. While houses exposed to climate risk trade at a discount in the market, the decline in the supply of houses reinforces this effect. Unless households continuously opt for smaller houses, this implies that households become more indebted over time. As financing costs rise, this worsens the financial position of households over time even in the absence of the wealth effects driven by a failure to adapt efficiently to climate change.

Finally, this paper has shown that climate risk has implications for firm's financing costs. Due to climate-related damages, capital becomes more scarce, and its effect on firm's financing costs outweighs the effects on the productivity of capital as well as on income growth. As a result, firm's financing costs fall slower as compared to its growth path in the absence of climate risk. On the other hand, equity prices rise at a slower pace as compared to its growth path in the absence of climate risk. These finding follow from an analysis in which only *physical* climate risk is taken into consideration. As the transition to the green economy requires large investments to be made, this puts additional pressure on financing costs (Bolton et al. (2021); Mongelli et al. (2022)). Additionally, there is empirical evidence that investors already demand compensation for exposure to carbon emission risk (Bolton and Kacperczyk (2021)), or more specifically, to environmental regulatory risk (**?**. Such a carbon premium would depress share prices further. While the implications of climate *transition* risk, therefore, seem to reinforce those driven by physical climate risk, a proper investigation of the interaction of both types of climate risk is left for further research.

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Appendix A: Proof of Propositions

A.1 Proof of Proposition 1

Wage inequality increases with climate-related damages when

$$\frac{\partial q_t/w_t}{\partial \gamma_t} = \frac{\eta}{(1-\eta)} \cdot \left(\frac{(1-\phi)\tilde{l})}{\phi \tilde{h}}\right)^{1-(1-\alpha)\rho} \cdot \frac{\partial}{\partial \gamma_t}\left(\frac{H_t}{(1-\mu_L\gamma_t)K_t}\right)^{\alpha\rho} \geq 0$$

For this to hold, it must be that

$$\frac{\mu_L}{(1-\mu_L\gamma_t)} + \frac{\partial H_t/\partial \gamma_t}{H_t} - \frac{\partial K_t/\partial \gamma_t}{K_t} \geq 0$$

or equivalently

$$\frac{\mu_L}{(1-\mu_L\gamma_t)} \geq \frac{\partial}{\partial \gamma_t} ln\left(\frac{K_t}{H_t}\right)$$

which implies that the losses of tangible capital (*i.e. the direct effect*) should be larger than the change in investment in tangible capital relative to that of intangible capital *(i.e. the indirect effect)*.To proof this, it suffices to show that

$$\frac{\partial}{\partial \gamma_t} ln\left(\frac{K_t}{H_t}\right) \leq 0$$

Note first that

$$H_t = I^*_{t-1} = \frac{\omega}{\beta} \cdot A^\rho \alpha\eta \frac{Y_t^{net(1-\rho)}}{H_t^{1-\alpha\rho}} h_t^{(1-\alpha)\rho}$$

Using logarithmic differentiation, the derivative of H_t to γ_t becomes

$$(2-\alpha\rho)\frac{\partial}{\partial \gamma_t} ln(H_t) = (1-\rho)\frac{\partial}{\partial \gamma_t} ln(Y_t^{net})$$

Note that the derivative of Y_t^{net} to γ_t is given by

$$\begin{aligned}\frac{\partial}{\partial \gamma_t} Y_t^{net} &= \frac{\partial Y_t^{net}}{\partial H_t} \cdot \frac{\partial H_t}{\partial \gamma_t} + \frac{\partial Y_t^{net}}{\partial K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} \\ &= R_t \cdot \frac{\partial H_t}{\partial \gamma_t} + (1+r_t) \cdot \frac{\partial K_t}{\partial \gamma_t}\end{aligned}$$

This equation is used to find a relation between the partial derivatives of capital to climate-related damages

$$\frac{\partial K_t}{\partial \gamma_t} = \frac{\frac{(2-\alpha\rho)\cdot Y_t^{net}}{H_t \cdot (1-\rho)} - R_t}{(1+r_t)} \cdot \frac{\partial H_t}{\partial \gamma_t}$$

Now, it must be shown that

$$\frac{\partial}{\partial \gamma_t} ln\left(\frac{K_t}{H_t}\right) \leq 0 \Leftrightarrow \frac{1}{H_t} \cdot \frac{\partial H_t}{\partial \gamma_t} \geq \frac{1}{K_t} \cdot \frac{\partial K_t}{\partial \gamma_t}$$

There are two cases;

1. $\partial H_t/\partial\gamma_t \leq 0$. In this case, the following condition is obtained by combining equation (81) and inequality (82)

$$R_t H_t + (1+r_t)K_t \leq \frac{(2-\alpha\rho)Y_t^{net}}{(1-\rho)}$$

   Using that $R_t H_t + (1+r_t)K_t = \alpha Y_t^{net}$ gives

$$\alpha \leq 2$$

   which is always satisfied. Therefore,

   Lemma 9. *The elasticity of tangible capital to climate-related damages is higher than the elasticity of intangible capital, i.e.*

$$\frac{1}{K_t}\cdot\left|\frac{\partial K_t}{\partial\gamma_t}\right| \geq \frac{1}{H_t}\cdot\left|\frac{\partial H_t}{\partial\gamma_t}\right|$$

   which proves that wage inequality increases in climate-related damages when $\partial H_t/\partial\gamma_t \leq 0$.

2. $\partial H_t/\partial\gamma_t \geq 0$. Lemma 6 implies that

   Corollary 3. *The partial derivatives of H_t, K_t and Y_t to γ_t have the same sign, i.e.*

$$\frac{\partial H_t}{\partial\gamma_t} \geq 0 \implies \frac{\partial K_t}{\partial\gamma_t} \geq 0 \implies \frac{\partial Y_t}{\partial\gamma_t} \geq 0$$

   Therefore, $\partial H_t/\partial\gamma_t \geq 0 \implies \partial Y_t/\partial\gamma_t \geq 0$. This case is ruled out by Assumption 2.

   Concluding, wage inequality increases in climate-related damages.

A.2 Proof of Proposition 2

The return to tangible capital is given by

$$(1+r_t^*) = A^\rho \alpha(1-\eta) \frac{{Y_t^{net}}^{1-\rho}}{K_t^{1-\alpha\rho}} l_t^{(1-\alpha)\rho} \cdot (1-\mu_K\gamma_t)^{\alpha\rho}$$

Using logarithmic differentiation, the derivative of r_t^* to γ_t becomes

$$\frac{\partial r_t^*}{\partial \gamma_t} = \frac{(1-\rho)}{Y_t} \cdot \frac{\partial Y_t^{net}}{\partial \gamma_t} - \frac{(1-\alpha\rho)}{K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} - \frac{\alpha\rho \cdot \mu_K}{(1-\mu_K\gamma_{t+1})}$$

For $\rho = 0$, this derivative becomes

$$\left.\frac{\partial r_t^*}{\partial \gamma_t}\right|_{\rho=0} = \frac{1}{Y_t^{net}} \cdot \frac{\partial Y_t}{\partial \gamma_t} - \frac{1}{K_t} \cdot \frac{\partial K_t}{\partial \gamma_t}$$

Recall that

$$\begin{aligned}\frac{\partial}{\partial \gamma_t} Y_t^{net} &= \frac{\partial Y_t^{net}}{\partial H_t} \cdot \frac{\partial H_t}{\partial \gamma_t} + \frac{\partial Y_t^{net}}{\partial K_t} \cdot \frac{\partial K_t}{\partial \gamma_t} \\ &= R_t \cdot \frac{\partial H_t}{\partial \gamma_t} + (1+r_t) \cdot \frac{\partial K_t}{\partial \gamma_t}\end{aligned}$$

and that

$$\left.\frac{\partial K_t}{\partial \gamma_t}\right|_{\rho=0} = \frac{\frac{2\cdot Y_t^{net}}{H_t} - R_t}{(1+r_t)} \cdot \frac{\partial H_t}{\partial \gamma_t} \Leftrightarrow \left.\frac{\partial H_t}{\partial \gamma_t}\right|_{\rho=0} = (1+r_t) \cdot \frac{\partial K_t}{\partial \gamma_t} \cdot \frac{H_t}{2Y_t^{net} - R_t H_t}$$

then

$$\begin{aligned}\left.\frac{\partial r_t^*}{\partial \gamma_t}\right|_{\rho=0} &= \frac{\partial K_t}{\partial \gamma_t} \cdot \left[(1+r_t) \cdot \left(\frac{2}{2Y_t^{net} - R_t H_t}\right) - \frac{1}{K_t}\right] \\ &= \frac{\partial K_t}{\partial \gamma_t} \cdot \frac{2}{K_t \cdot (2Y_t^{net} - R_t H_t)} \cdot \left((1+r_t)K_t + R_t H_t - Y_t^{net} - 1/2 R_t H_t\right)\end{aligned}$$

Recall that $R_t H_t + (1+r_t)K_t = \alpha Y_t^{net}$.

Then

$$\left.\frac{\partial r_t^*}{\partial \gamma_t}\right|_{\rho=0} = \frac{\partial K_t}{\partial \gamma_t} \cdot \frac{2}{K_t \cdot (2Y_t^{net} - R_t H_t)} \cdot \left((\alpha-1) Y_t^{net} - 1/2 R_t H_t\right)$$

which is larger than 0.

(...)

Since the derivative increases in ρ, there exists a threshold, $\hat{\rho}$ for which $\partial r_t / \partial \gamma_t$ becomes *negative*.

A.3. Proof of Proposition 3

Share prices are given by

$$e_t^* = \frac{(1-\omega)R_{t+1}H_{t+1}}{1+r_{t+1}}$$

i.e.

$$e_t^* = (1-\omega)\cdot\frac{\eta}{(1-\eta)}\cdot\left(\frac{\tilde{h}}{\tilde{l}}\right)^{(1-\alpha)\rho}\cdot\frac{H_{t+1}^{\alpha\rho}\cdot K_{t+1}^{(1-\alpha\rho)}}{(1-\mu_K\gamma_{t+1})^{\alpha\rho}}$$

Using logarithmic differentiation, the derivative of e_t^* to γ_{t+1} becomes

$$\frac{\partial e_t^*}{\partial\gamma_{t+1}} = \frac{(1-\alpha\rho)}{K_{t+1}}\cdot\frac{\partial K_{t+1}}{\partial\gamma_{t+1}} + \frac{\alpha\rho}{H_{t+1}}\cdot\frac{\partial H_{t+1}}{\partial\gamma_{t+1}} + \frac{\alpha\rho\cdot\mu_K}{(1-\mu_K\gamma_{t+1})}$$

For $\rho = 0$, this derivative becomes

$$\left.\frac{\partial e_t^*}{\partial\gamma_{t+1}}\right|_{\rho=0} = \frac{1}{K_{t+1}}\cdot\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}$$

which, trivially, is negative.

However, $\frac{\partial e_t^*}{\partial\gamma_{t+1}}$ increases in ρ. To see this, note that

$$\frac{\partial e_t^*/\partial\gamma_{t+1}}{\partial\rho} = \alpha\left[\frac{\mu_K}{(1-\mu_K\gamma_{t+1})} + \frac{1}{H_{t+1}}\cdot\frac{\partial H_{t+1}}{\partial\gamma_{t+1}} - \frac{1}{K_{t+1}}\cdot\frac{\partial K_{t+1}}{\partial\gamma_{t+1}}\right]$$

Given that $\mu_K, \mu_L \geq 0$, it follows from the proof in Appendix A.1 that the expression in the brackets is positive.

Since the derivative increases in ρ, there exists a threshold, $\bar{\rho}$ for which $\partial e_t^*/\partial\gamma_{t+1}$ becomes *positive.*

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A.4. Proof of Proposition 4

Currently being updated

A.5 Proof of Proposition 5

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The unconstrained social planner maximizes utilitarian welfare, i.e.

$$\sum_{t=0}^{\infty}\left(\prod_{\tau=1}^{t}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{t+1})\frac{\theta}{2}x_t^2L_t+v(L_t)\right]$$

subject to

$$L_j=L_t\prod_{i=t}^{j-1}(1-(1-x_i)\mu_L\gamma_{i+1})$$

The first order condition for x_t^S is

$$\left(\prod_{\tau=1}^{t}\frac{1}{1+r_{\tau+1}}\right)(1+r_{t+1})\theta x_t^S L_t=\sum_{j=t+1}^{\infty}\left(\prod_{\tau=1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}{x_j}^{S},2+v'(L_j)\right]\frac{\partial L_j}{\partial x_t}$$

Using

$$\frac{\partial L_j}{\partial x_t}=\mu_L\gamma_{t+1}L_t\prod_{i=t+1}^{j-1}(1-(1-x_i)\mu_L\gamma_{i+1})$$

this becomes

$$(1+r_{t+1})\theta x_t=\mu_L\gamma_{t+1}\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(L_j)\right]\prod_{i=t+1}^{j-1}(1-(1-x_i)\mu_L\gamma_{i+1})$$

The first-order condition of the unconstrained household is

$$(1+r_{t+1})\theta x_t=\mu_L\gamma_{t+1}\cdot p_{t+1}$$

and the first-order condition of the unconstrained social planner is

$$(1+r_{t+1})\theta x_t=\mu_L\gamma_{t+1}\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(L_j)\right]\prod_{i=t+1}^{j-1}(1-(1-x_i)\mu_L\gamma_{i+1})$$

A necessary and sufficient condition for the privately optimal level of investment to be efficient is

$$p_{t+1}=\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(L_j)\right]\prod_{i=t+1}^{j-1}(1-(1-x_i)\mu_L\gamma_{i+1})$$

Note that the demand for housing capital of unconstrained households is

$$L_{t+1}^*=v'^{-1}\left((1+r_{t+2})\left(p_{t+1}+\frac{\theta}{2}x_{t+1}^2\right)-\left(1-(1-x_{t+1})\mu_L\gamma_{t+2}\right)p_{t+2}\right)$$

Therefore

$$p_{t+1}=\left(\frac{1}{1+r_{t+2}}\right)\left[-(1+r_{t+2})\frac{\theta}{2}x_{t+1}^2+v'(L_{t+1})+\left(1-(1-x_{t+1})\mu_L\gamma_{t+2}\right)\cdot p_{t+2}\right]$$

Forward substitution of this expression for p_{t+1} gives

$$p_{t+1}=\sum_{j=t+1}^{\infty}\left(\prod_{\tau=t+1}^{j}\frac{1}{1+r_{\tau+1}}\right)\left[-(1+r_{j+1})\frac{\theta}{2}x_j^2+v'(L_j)\right]\prod_{i=t+1}^{j-1}(1-(1-x_i)\mu_L\gamma_{i+1})$$

Therefore, the first-order condition of the unconstrained social planner is equivalent to the first-order condition of the unconstrained household, which implies that the market outcome is efficient.

A.6 Proof of Proposition 6

The first-order condition for L^{l*} is derived from the constrained household problem as

$$-(1+r_{t+1}+\lambda)\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda)+p_t\right)+\left(1-\left(1-x_{l,t}^*\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)+\lambda p_{t+1}\left(1-\left(1-x_{l,t}^*\right)\mu\gamma_{t+1}\right)=0$$

This condition defines an implicit expression for λ, i.e.

$$\lambda=\frac{\left(1-\left(1-x_{l,t}^*(\lambda)\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_t^*\right)-(1+r_{t+1})\left(\frac{\theta}{2}x_{l,t}^{*,2}(\lambda)+p_t\right)}{\left(\frac{\theta}{2}x_{l,t}^*(\lambda)+p_t\right)-p_{t+1}\left(1-\left(1-x_{l,t}^*\right)\mu\gamma_{t+1}\right)}$$

Since λ denotes the change in the optimal level of utility for loosening the constraint, it is by construction that $\lambda\geq 0$. What remains to be determined, is under which condition $\lambda=0$.

For λ to be zero, it must be that

$$p_t=\frac{\left(1-\left(1-x_{l,t}^*(\lambda)\right)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{l,t}^*\right)}{1+r_{t+1}}-\frac{\theta}{2}x_{l,t}^{*,2}(\lambda)$$

Since the demand for housing and the degree of adaptation of high-skilled households determine the price, i.e.

$$p_t=\frac{\left(1-(1-x_{h,t}^*)\mu_L\gamma_{t+1}\right)p_{t+1}+v'\left(L_{h,t}^*\right)}{1+r_{t+1}}-\frac{\theta}{2}x_{h,t}^{*,2}$$

Then, the following condition must hold

$$v'\left(L_{h,t}^*\right)-v'\left(L_{l,t}^*\right)=\left(x_{l,t}^*-x_{h,t}^*\right)\mu_L\gamma_{t+1}\cdot p_{t+1}-(1+r_{t+1})\frac{\theta}{2}\left(x_{l,t}^{*,2}-x_{h,t}^{*,2}\right)$$

Since $\lambda=0 \implies x_{h,t}^*=x_{l,t}^*$, the RHS of the above condition becomes zero and a necessary and sufficient condition for $\lambda=0$ becomes $v'\left(L_{h,t}^*\right)=v'\left(L_{l,t}^*\right)$. This condition cannot not hold when the credit constraint binds, as $v'\left(L_{h,t}^*\right)<v'\left(L_{l,t}^*\right)$. Therefore, when the credit constraint binds, λ cannot be zero and it must be that $\lambda>0$.

Appendix B: Proof of Lemmas

B.1 Proof of Lemma 1

Households maximize utility subject to the budget constraint and limited liability constraint:

$$\max_{c_{i,t+1},L_{i,t},s_{i,t},S_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right)$$
$$s.t. \quad y_{i,t} \le p_t L_{i,t} + s_{i,t}e_t + S_{i,t}$$
$$c_{i,t+1} \le \max\left\{y_{i,t+1} + p_{t+1}\left(1-\xi_{i,t+1}\right)L_{i,t} + (e_{t+1}+d_{t+1})s_{i,t} + (1+\hat{r}_{t+1})S_{i,t}, 0\right\}$$
$$c_{i,t+1}, L_{i,t}, s_{i,t} \ge 0,$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

Using the probability of default, $\left(1-G\left(\hat{\xi}^i_{t+1}\right)\right)$, the expectation of household i's consumption in period $t+1$, $c_{i,t+1}$, as formed at date t, becomes:

No arbitrage requires that the expected payoff of investing in corporate debt is equal to the expected payoff of holding household debt:

$$(1+r_{t+1})(-S_{i,t}) = G\left(\hat{\xi}_{i,t+1}\right)(1+\hat{r}_{t+1})(-S_{i,t}) + \left(1-G\left(\hat{\xi}_{i,t+1}\right)\right)p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{i,t+1} > \hat{\xi}_{i,t+1}\right)\right)L_{i,t}$$

where the expected payoff of holding household debt is equal to the repayment of the loan with interest in case the household does not default and the revenue from selling the (undamaged) collateral in case of default.

The no-arbitrage condition can be rewritten as

$$G(\hat{\xi}_{i,t+1})\left(p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}|\xi_{i,t+1} \le \hat{\xi}_{i,t+1}\right)\right)L_{i,t} + (1+\hat{r}_{t+1})(S_{i,t})\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right)L_{i,t+1}$$

and the expectation of household i's consumption in period $t+1$, $c_{i,t+1}$, as formed at date t, becomes

$$\mathbb{E}_t\left(c_{i,t+1}\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{i,t+1}\right)\right)L_{i,t} + d_{t+1}s_{i,t}$$

Using that $\mathbb{E}\left(\xi_{i,t+1}\right) = \mu\gamma_{t+1}$, the household optimization problem can be written as

$$\max_{c_{i,t+1},L_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = (1+r_{t+1})(S_{i,t}) + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v\left(L_{i,t}\right)$$
$$s.t. \quad y_{i,t} \le p_t L_{i,t} + s_{i,t}e_t + S_{i,t}$$
$$c_{i,t+1}, L_{i,t}, s_{i,t} \ge 0,$$

and the budget constraint is substituted to obtain

$$\max_{c_{i,t+1},L_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},L_{i,t})\right) = (1+r_{t+1})(y_{i,t} - p_t L_{i,t} - s_{i,t}e_t) + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v\left(L_{i,t}\right)$$
$$s.t. c_{i,t+1}, L_{i,t}, s_{i,t} \ge 0,$$

The FOC for $L_{i,t}$ is given by

$$-(1+r_{t+1})p_t + p_{t+1}\left(1-\mu_L\gamma_{t+1}\right) + v'\left(L_{i,t}\right) = 0$$

and the demand for housing capital of each household i in period t is given by

$$L_t^* = v'^{-1}\left((1+r_{t+1})p_t - p_{t+1}\left(1-\mu_L\gamma_{t+1}\right)\right)$$

and the price of housing capital in a given period, t, becomes

$$p_t = \frac{\left(1-\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(L_t^*\right)}{(1+r_{t+1})}$$

B.2 Proof of Lemma 2

The wages of high-skilled workers, q_t, follows from the FOC for h_t from the firm optimization problem:

$$\begin{aligned} q_t^* &= \frac{\partial Y_t^{net}}{\partial h_t} \\ &= A^\rho(1-\alpha)\eta\frac{{Y_t^{net}}^{1-\rho}}{h_t^{1-(1-\alpha)\rho}}H^{\alpha\rho} \end{aligned}$$

and the wages low-skilled workers, l_t, follow from the FOC for l_t from the firm optimization problem:

$$\begin{aligned} w_t^* &= \frac{\partial Y_t^{net}}{\partial l_t} \\ &= A^\rho(1-\alpha)(1-\eta)\frac{{Y_t^{net}}^{1-\rho}}{l_t^{1-(1-\alpha)\rho}}K^{\alpha\rho} \end{aligned}$$

Then, the wage ratio becomes

$$\frac{q_t^*}{w_t^*} = \frac{\eta}{(1-\eta)}\cdot\left(\frac{H_t}{(1-\mu_K\gamma_t)K_t}\right)^{\alpha\rho}\cdot\left(\frac{l_t}{h_t}\right)^{1-(1-\alpha)\rho}$$

B.3 Proof of Lemma 3

The return to tangible capital follow from the FOC for K_t from the firm optimization problem:

$$\begin{aligned} (1+r_t^*) &= \frac{\partial Y_t^{net}}{\partial K_t} \\ &= A^\rho\alpha(1-\eta)\frac{{Y_t^{net}}^{1-\rho}}{K_t^{1-\alpha\rho}}l^{(1-\alpha)\rho}\cdot\left(1-\mu_K\gamma_t\right)^{\alpha\rho} \end{aligned}$$

B.4 Proof of Lemma 4

The return to intangible capital follow from the FOC for H_t from the firm optimization problem:

$$\begin{aligned} R_t^* &= \frac{\partial Y_t^{net}}{\partial H_t} \\ &= A^\rho\alpha\eta\frac{{Y_t^{net}}^{1-\rho}}{H_t^{1-\alpha\rho}}h^{(1-\alpha)\rho} \end{aligned}$$

The productive use of intangible capital requires the commitment of innovators, who capture a fraction ω of its value. This means that the return earned by innovators on the intangible capital they create, H_{t+1}, is $\omega R_{t+1} H_{t+1}$. The effort cost associated with creating H_{t+1} units of intangible capital is $C(I_{H,t+1}) = \frac{1}{2} I_{H,t+1}^2$. Innovators, then, create intangible capital until its marginal benefits are equal to its marginal costs and invest

$$I_t^* = \frac{\omega R_{t+1}^*}{\beta}$$

where $I_t^* = H_{t+1}^*$.

B.5 Proof of Lemma 5

Dividends of each firm f in period t are given by

$$\begin{aligned} d_t^* &= (1-\omega) R_t^* H_t^* \\ &= A^\rho \alpha \eta Y_t^{net\,1-\rho} H_t^{\alpha\rho} h^{(1-\alpha)\rho} \end{aligned}$$

Using logarithmic differentiation, the derivative of d_t^* to γ_t becomes

$$\frac{\partial d_t^*}{\partial \gamma_t} = \frac{1-\rho}{Y_t^{net}} \frac{\partial Y_t^{net}}{\partial \gamma_t} + \frac{\alpha\rho}{H_t} \frac{\partial H_t}{\partial \gamma_t}$$

which is smaller than 0.

Proof of Lemma 6

With adaptation, households' optimization problem becomes

$$\begin{aligned} \max_{c_{i,t+1}, x_{i,t}, L_{i,t}, S_{i,t}, s_{i,t}} \quad & \mathbb{E}\left(U(c_{i,t+1}, L_{i,t})\right) = \mathbb{E}_t\left(c_{i,t+1}\right) + v\left(L_{i,t}\right) \\ s.t. \quad & y_{i,t} \le \left(p_t + \frac{\theta}{2} x_{i,t}^2\right) L_{i,t} + s_{i,t} e_t + S_{i,t} \\ & c_{i,t+1} \le \max\left\{ y_{i,t+1} + p_{t+1}\left(1 - \xi_{i,t+1}\right) L_{i,t} + (e_{t+1} + d_{t+1}) s_{i,t} + (1 + \hat{r}_{t+1}) S_{i,t}, 0 \right\} \\ & c_{i,t+1}, x_{i,t}, L_{i,t}, s_{i,t} \ge 0 \end{aligned}$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

The same no-arbitrage condition as in Appendix B.1 can be used to find the expectation, formed at date t, of household i's consumption in period $t+1$, $c_{i,t+1}$, becomes

$$\mathbb{E}_t\left(c_{i,t+1}\right) = (1 + r_{t+1})(S_{i,t}) + p_{t+1}\left(1 - \mathbb{E}\left(\xi_{i,t+1}\right)\right) L_{i,t} + d_{t+1} s_{i,t}$$

With adaptation $\mathbb{E}\left(\xi_{i,t+1}\right) = \left(1 - x_{i,t}\right) \mu \gamma_{t+1}$ and the household optimization problem can be written as

$$\begin{aligned} \max_{c_{i,t+1}, x_{i,t}, L_{i,t}, S_{i,t}, s_{i,t}} \quad & \mathbb{E}\left(U(c_{i,t+1}, x_{i,t}, L_{i,t})\right) = (1 + r_{t+1})\left(S_{i,t}\right) + p_{t+1}\left(1 - (1 - x_{i,t}) \mu_L \gamma_{t+1}\right) L_{i,t} + d_{t+1} s_{i,t} + v(L_{i,t}) \\ s.t. \quad & y_{i,t} \le \left(p_t + \frac{\theta}{2} x_{i,t}^2\right) L_{i,t} + S_{i,t} + s_{i,t} e_t \\ & c_{i,t+1}, x_{i,t}, L_{i,t}, s_{i,t} \ge 0 \end{aligned}$$

and the budget constraint is substituted to obtain

$$\max_{c_{i,t+1},x_{i,t},L_{i,t},S_{i,t},s_{i,t}} \mathbb{E}\left(U(c_{i,t+1},x_{i,t},L_{i,t})\right) = (1+r_{t+1})\left(y_{i,t} - \left(p_t + \frac{\theta}{2}x_{i,t}^2\right)L_{i,t} - s_{i,t}e_t\right)$$
$$+ p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)L_{i,t} + d_{t+1}s_{i,t} + v(L_{i,t})$$
$$s.t. \quad c_{i,t+1}, x_{i,t}, L_{i,t}, s_{i,t} \geq 0$$

The FOC for $L_{i,t}$ is given by

$$-(1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^2\right) + p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right) + v'\left(L_{i,t}\right) = 0$$

and the demand for housing capital of each household i in period t is given by

$$L_t^* = v'^{-1}\left((1+r_{t+1})\left(p_t + \frac{\theta}{2}x_{i,t}^2\right) - p_{t+1}\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)\right)$$

and the price of housing capital in a given period, t, becomes

$$p_t = \frac{\left(1-(1-x_{i,t})\mu_L\gamma_{t+1}\right)p_{t+1} + v'\left(L_t^*\right)}{(1+r_{t+1})} - \frac{\theta}{2}x_{i,t}^2$$

B.7 Proof of Lemma 7

The degree to which household i adapts to climate change in period t follow from the FOC from the household problem in Section B.6 to $x_{i,t}$:

$$x_t^* = \frac{\mu_L\gamma_{t+1}p_{t+1}}{\theta(1+r_{t+1})}$$

B.8 Proof of Lemma 8

Low-skilled households maximize expected utility subject to their budget constraint and the credit constraint

$$\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \mathbb{E}\left(U(c_{l,t+1},L_{l,t})\right) = \mathbb{E}_t\left(c_{l,t+1}\right) + v\left(L_{l,t}\right)$$
$$s.t. \quad w_t \leq \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t} + S_{l,t}$$
$$c_{l,t+1} \leq \max\left\{y_{l,t+1} + p_{t+1}\left(1-\xi_{l,t+1}\right)L_{l,t} + (1+\hat{r}_{t+1})S_{l,t}, 0\right\}$$
$$-S_{l,t} \leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t}$$
$$c_{l,t+1}, x_{l,t}, L_{l,t} \geq 0$$

where $c_{i,t+1}$ is the consumption of household i in period $t+1$ and $\mathbb{E}_t$ denotes expectations formed at date t.

The same no-arbitrage condition as in Appendix B.6 can be used to find the expectation, formed at date t, of low-skilled household l's consumption in period $t+1$, $c_{l,t+1}$, becomes

$$\mathbb{E}_t\left(c_{l,t+1}\right) = (1+r_{t+1})(S_{l,t}) + p_{t+1}\left(1-\mathbb{E}\left(\xi_{l,t+1}\right)\right)L_{l,t} + d_{t+1}s_{l,t}$$

Using that $\mathbb{E}\left(\xi_{l,t+1}\right) = \left(1 - x_{l,t}\right)\mu\gamma_{t+1}$, the optimization problem of low-skilled households can be written as

$$\begin{aligned}
\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \quad & \mathbb{E}\left(U(c_{l,t+1},x_{l,t},L_{l,t})\right) = (1+r_{t+1})\left(S_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right) \\
s.t. \quad & w_t \leq \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t} + S_{l,t} \\
& -S_{l,t} \leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t} \\
& c_{l,t+1}, x_{l,t}, L_{l,t} \geq 0
\end{aligned}$$

and the budget constraint is substituted to obtain

$$\begin{aligned}
\max_{c_{l,t+1},x_{l,t},L_{l,t},S_{l,t}} \quad & \mathbb{E}\left(U(c_{l,t+1},x_{l,t},L_{l,t})\right) = (1+r_{t+1})\left(w_t - \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right) \\
s.t. \quad & -S_{l,t} \leq \left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t} \\
& c_{l,t+1}, x_{l,t}, L_{l,t} \geq 0
\end{aligned}$$

Define the Lagrangian for this constrained maximization problem as

$$\begin{aligned}
\mathscr{L} = {} & (1+r_{t+1})\left(y_{l,t} - \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t}\right) + p_{t+1}\left(1-(1-x_{l,t})\mu_L\gamma_{t+1}\right)L_{l,t} + v\left(L_{l,t}\right) \\
& - \lambda\left(\left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t} - w_t + \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t}\right)
\end{aligned}$$

where $\lambda \geq 0$ denotes the Lagrangian multiplier.

The FOC for the demand for housing by low-skilled household l, $L_{l,t}$ is given by

$$-(1+r_{t+1}+\lambda)\left(p_t + \frac{\theta}{2}x_{l,t}^{*2}\right) + p_{t+1}\left(1-(1-x_{l,t}^*)\mu_L\gamma_{t+1}\right) + v'\left(L_{l,t}^*\right) + \lambda\left(1-\mathbb{E}(\bar{x}_t)\right)\mu_L\gamma_{t+1}p_{t+1} = 0$$

Given that $\mathbb{E}(x_{l,t}) = x_{l,t}^*$ in a symmetric equilibrium, the optimal demand for housing by low-skilled household l, $L_{l,t}^*$ is defined in terms of the optimal degree to which low-skilled household l invests in adaptation, $x_{l,t}^*$

$$L_{l,t}^* = v'^{-1}\left((1+r_{t+1}+\lambda)\left(p_t + \frac{\theta}{2}x_{l,t}^{*2}\right) - p_{t+1}\left(1-(1-x_{l,t}^*)\mu_L\gamma_{t+1}\right) - \lambda\left(1-x_{l,t}^*\right)\mu_L\gamma_{t+1}p_{t+1}\right)$$

where

The FOC for the degree to which low-skilled household l invests in adaptation, $x_{l,t}$, is given by

$$-\theta(1+r_{t+1}+\lambda)x_{l,t}^*L_{l,t}^* + \mu\gamma_{t+1}p_{t+1}L_{l,t}^* = 0$$

and the optimal degree to which low-skilled household l invests in adaptation, $x_{l,t}^*$, is given by

$$x_{l,t}^* = \frac{\mu\gamma_{t+1}p_{t+1}}{\theta(1+r_{t+1}+\lambda)}$$

The FOC for the Lagrangian multiplier, λ is given by

$$\left(1-(1-\mathbb{E}(\bar{x}_t))\mu_L\gamma_{t+1}\right)p_{t+1}L_{l,t} = w_t - \left(p_t + \frac{\theta}{2}x_{l,t}^2\right)L_{l,t}$$

Given that $\mathbb{E}(x_{l,t}) = x^*_{l,t}$ in a symmetric equilibrium, this equation provides the corner-solution for the optimal demand for housing by low-skilled household, $L^*_{l,t}$ in terms of their optimal degree of adaptation, $x^*_{l,t}$

$$L^*_{l,t} = \frac{w_t}{\left(1 - \left(1 - x^*_{l,t}\right)\mu_L \gamma_{t+1}\right) p_{t+1} + \left(p_t + \frac{\theta}{2} x^2_{l,t}\right)}$$

Appendix C: Simulation Method

The simulation of the model is carried out under the parameterization described in Section 6.1.1. Climate risk is absent in the baseline model, and only η grows over time, following the growth path provided in Döttling and Perotti (2017). The steady-state equations are determined and used to solve the model backwards. The simulation assumes that the economy moves instantaneously to a new steady-state, and therefore shows the balanced growth path of the economy. Counterfactual analysis is performed to compare the outcomes of the baseline model to the results found under different projections of the Representative Concentration Pathways (RCP) provided by the Intergovernmental Panel on Climate Change (IPCC). The results are to be interpreted as deviations from the balanced growth path.

For each RCP trajectory, γ follows the path described in Section 6.1.2. Since estimates of projected global mean sea level rise are only provided up to 2100, γ is assumed to reach a steady-state by 2100 and remains constant thereafter. Likewise, η is assumed to stop growing after 2100. The simulation is extended to include the case *(i)* when households adapt to climate change, and *(ii)* when households face financial constraints.

C.1 Model Simulations with Climate Change

In the model with climate change, the steady state price of housing capital is given by

$$\begin{aligned} p_S &= \frac{(1-\mu_L\gamma_S)p_S + v'(\bar{L}_S)}{1+r_S} \\ &= \frac{v'(\bar{L}_S)}{r_S+\mu_L\gamma_S} \end{aligned}$$

where $\bar{L}^S = 1$. The steady-state price of housing capital pins down its price in the previous period, as

$$p_{S-1} = \frac{(1-\mu_L\gamma_S)p_S + v'(\bar{L}_{S-1})}{1+r_S}$$

where

$$\bar{L}_{S-1} = \frac{\bar{L}_S}{(1-\mu_L\gamma_S)}$$

Then, the model can be solved backwards and price of housing capital is any period $S-t$ is defined as

$$p_{S-t} = \frac{(1-\mu_L\gamma_{S-(t-1)})p_{S-(t-1)} + v'(\bar{L}_{S-t})}{1+r_{S-(t-1)}}$$

where

$$\bar{L}_{S-t} = \frac{\bar{L}_{S-(t-1)}}{(1-\mu_L\gamma_{S-(t-1)})}$$

In addition, the financial market clearing condition provides an expression for the tangible capital that is invested upon by firms and the level of tangible capital that remains available for production is given by

$$\tilde{K}_{S-t} = (1-\mu_K\gamma_{S-t})\big((1-\alpha)Y_{S-t} - p_{S-t}\bar{L}_{S-t} - e_{S-t}\big)$$

The investment in intangible capital follows from the optimization of entrepreneurs. The starting (here: end) values for $\tilde{Y}, \tilde{K}$ and H are chosen such that the model converges.

C.2 Model Simulations with Adaptation

A similar strategy is used to simulate the model with adaptation. When households can adapt to rising climate risk, the steady state price of housing capital is given by

$$p_S(x_S) = \frac{\left(1-(1-x_S)\mu_L\gamma_S\right)p_S(x_S)+v'\left(\bar{L}_S\right)}{(1+r_S)} - \frac{\theta}{2}x_S^2$$

where

$$x_S(p_S) = \frac{\left(\mu_L\gamma_S\right)\cdot p_S}{\theta(1+r_S)}$$

and again $\bar{L}^S = 1$. Therefore, p_S is defined implicitly by the above system of equations. As this system consists of two equation with two unknowns, there is exists a unique solution which pins down the price of housing capital as well as the degree to which households adapt to climate change. The solution is used to determine the price of housing capital in the previous period, which is given by

$$p_{S-1} = \frac{\left(1-\mu_L(1-x_{S-1})\gamma_S\right)p_S+v'(\bar{L}_{S-1})}{(1+r_S)} - \frac{\theta}{2}x_{S-1}^2$$

where

$$x_{S-1} = \frac{\left(\mu_L\gamma_S\right)\cdot p_S}{\theta(1+r_S)}$$

and

$$\bar{L}_{S-1} = \frac{\bar{L}_S}{(1-\mu_L(1-x_{S-1})\gamma_S)}$$

and the model is again solved backwards, and price of housing capital is any period $S-t$ is defined as

$$p_{S-t} = \frac{\left(1-\mu_L(1-x_{S-t})\gamma_{S-(t-1)}\right)p_{S-(t-1)}+v'(\bar{L}_{S-t})}{(1+r_{S-(t-1)})} - \frac{\theta}{2}x_{S-t}^2$$

and the degree to which households adapt to climate change in a given period $S-t$ is given by

$$x_{S-t} = \frac{\left(\mu_L\gamma_{S-(t-1)}\right)\cdot p_{S-(t-1)}}{\theta(1+r_{S-(t-1)})}$$

and

$$\bar{L}_{S-t} = \frac{\bar{L}_{S-(t-1)}}{(1-\mu_L(1-x_{S-t})\gamma_{S-(t-1)})}$$

The financial market clearing condition of the model with climate change adaptation provides an expression for the tangible capital that is invested upon by firms. Then, the level of tangible capital that remains available for production is given by

$$\tilde{K}_{S-t} = (1-\mu_K\gamma_{S-t})\left((1-\alpha)Y_{S-t} - \left(p_{S-t}+\frac{\theta}{2}x_{S-t}^2\right)\bar{L}_{S-t} - e_{S-t}\right)$$

The investment in intangible capital again follows from the optimization of entrepreneurs.

C.3 Model Simulations with Credit Constraints Currently being updated

Appendix D: The Discounting Debate in Climate Change

The private optimum for adaptation is efficient as long as the social planner maximizes utilitarian welfare where generations are weighted based on market discount rates. This brings forward what has been regarded as "the thorniest issues in all climate-change economics: how should we compare present and future costs and benefits?" (Nordhaus (2013)). While this question has received considerable attention within the economics of climate change literature, disagreements about the value of the appropriate discount rate continue to be at the heart of the climate policy debate (Heal and Millner (2014)). In particular, there is no consensus about the choice of r in the dynamic social welfare function of utilitarian form, i.e.

$$\int_{t=0}^{\infty} e^{-rt} u(c(t))dt$$

which, in discrete time, is equivalent to

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t u(c(t))$$

Stern (2007) argues that it is "immoral" to use a discount rate based on market discount rates when evaluating the welfare of future generations and favours a more a priori approach, where $r = 0.014$.[8] On the other hand, Nordhaus (2008) argues that economists have no particular expertise in what is morally right and should ensure that models replicate reality. Therefore, Nordhaus (2008) advocates a more market based approach where $r = 0.055$.[9] While the difference between the discount rates appear small, these small differences lead to large discrepancies between the estimated social cost of carbon and consequently the recommended intensity of climate mitigation policies (Heal and Millner (2014)). This paper shows that the choice of discount rates matters for the determination of the efficient level of adaptation. When the social planner were to evaluate the welfare of future generations based on a discount rate smaller than the market discount rate, the market outcome would no longer be efficient and households would underinvest in adaptation.

D.1 Proof of Corollary 2

Suppose that the social planner discounts the welfare of future generations at rate $r^{SP} \in [0,1]$. Then, the unconstrained social planner maximizes

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r^{SP}}\right)^t \left[-(1+r_{t+1})\frac{\theta}{2}x_t^2 L_t + v(L_t)\right]$$

subject to

$$L_j = L_t \prod_{i=t}^{j-1} (1-(1-x_i)\mu_L \gamma_{i+1})$$

[8]Following the Ramsey rule, the relationship between the equilibrium real return on capital, r^*, and the growth rate of the economy, g^* is given by $r^* = \varsigma + \sigma \cdot g^*$, where σ denotes the elasticity of consumption and ς denotes the generational rate of time preference (Nordhaus (2007)). Stern (2007) assumes that $\sigma = 1$, $\varsigma = 0.001$ and $g^* = 0.013$. This gives a real return of capital equal to $r^* = 0.014$..

[9]Nordhaus (2008) assumes that $\sigma = 2$, $\varsigma = 0.015$ and $g^* = 0.02$. This gives a real return of capital equal to $r^* = 0.055$.

The first order condition for x_t is

$$(1+r_{t+1})\theta x_t = \mu_L \gamma_{t+1} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r^{SP}}\right)^{j-t} \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{i=t+1}^{j-1} (1-(1-x_i)\mu_L\gamma_{i+1})$$

As the first order condition of the unconstrained household is given by

$$\begin{aligned}(1+r_{t+1})\theta x_t &= \mu_L \gamma_{t+1} \cdot p_{t+1} \\ &= \mu_L \gamma_{t+1} \sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \frac{1}{1+r_{\tau+1}}\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{i=t+1}^{j-1} (1-(1-x_i)\mu_L\gamma_{i+1})\end{aligned}$$

the private solution would not be efficient when $\frac{1}{1+r^{SP}} \neq \frac{1}{1+r_{\tau+1}}$, in particular

1. *underinvest* in adaptation when $\frac{1}{1+r^{SP}} > \frac{1}{1+r_{\tau+1}}$, where the size of the underinvestment is given by

$$\sum_{j=t+1}^{\infty} \left(\frac{1}{1+r^{SP}}\right)^{j-t} - \prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{i=t+1}^{j-1} (1-(1-x_i)\mu_L\gamma_{i+1})$$

2. *overinvest* in adaptation when $\frac{1}{1+r^{SP}} < \frac{1}{1+r_{\tau+1}}$, where the size of the overinvestment is given by

$$\sum_{j=t+1}^{\infty} \left(\prod_{\tau=t+1}^{j} \left(\frac{1}{1+r_{\tau+1}}\right) - \left(\frac{1}{1+r^{SP}}\right)^{j-t}\right) \left[-(1+r_{j+1})\frac{\theta}{2}x_j^2 + v'(L_j)\right] \prod_{i=t+1}^{j-1} (1-(1-x_i)\mu_L\gamma_{i+1})$$