Green Bonds & Certification: is getting certified always optimal?

Rebecca Cardot * Carole Bernard† and Jamil Jaballah‡§

This version: January 30, 2023

Abstract

In this paper, we focus on the certification cost for green bonds to understand the decision rationale of issuers when it comes to getting certified. We find that if this cost is null, requesting a certification is always optimal for the issuers, no matter the outcome of the process. However, with a strictly positive cost, we demonstrate the existence of a threshold above which the certification cost becomes too high and getting certified is not optimal anymore. This threshold depends on the distribution of the prior belief that potential buyers have about the commitment of the issuers to their "green" claim before the issuance announcement and the certification process.

Keywords: Green bonds, Certification decisions, Signalling

1 Introduction

According to the Climate Bonds Initiative (CBI), 2020 has been a record year for the issuance of green bonds, with USD 270bn. The CBI is an international non-for-profit organisation that provides one of the most widely-used lists of green bonds (Ehlers and Packer (2017)). Among the top ten issuers (in terms of amounts) listed for 2020 by the CBI, only four have been certified by the Climate Bonds Standard and Certification Scheme as a "Climate Bond", representing USD 23.4bn over a total issued of USD 72.2bn for the 10 biggest issuance.[1] While there are other certifications available on the market, this figure illustrates that getting a bond certified as green, and choosing which certification to get, is an actual choice that companies have to make.

Getting a bond certified is a cost for the issuers (and thus also for the buyers): to be certified, they have to pay a fee to the certification body. Keeping with our example of the CBI, registering a bond with them costs one tenth of a basis point of the total issued amount.

*Corresponding author: Rebecca Cardot, Department of Accounting, Law and Finance, Grenoble Ecole de Management (email: rebecca.cardot@grenoble-em.com).

†Carole Bernard, Department of Accounting, Law and Finance, Grenoble Ecole de Management, and Department of Economics and Political Sciences at Vrije Universiteit Brussel (VUB) (email: carole.bernard@grenoble-em.com).

‡Jamil Jaballah, Department of Accounting, Law and Finance, Grenoble Ecole de Management (email: Jamil.Jaballah@grenoble-em.com).

§For valuable comments, we thank Professors Ibrahim Abada and François Desmoulins-Lebeault at Grenoble Ecole de Management

[1] See https://www.climatebonds.net.

However, getting a certification is not only a cost. Empirical research has found that investors are more likely to pay a "greenium" - that is to accept lower yield - if the green bond has been certified as green by an independent third party. The certification brings credibility to the green claim (Kapraun, Latino, Scheins, and Schlag (2021)). There is an increasing demand from investors to check the "quality" of the green bonds issued in the market to avoid "greenwashing". The certification appears to offer a way to trust the issuers and to increase transparency on the real use of funds and its impact on climate. It is indeed hard for investors to do due diligence and know if issuers are really going to invest in a way that fulfills their promise of a green investment. Getting a certification increases the issuers' credibility and thus, one can expect, the attractiveness of the bond issue.

Overall, there appears to be pros to getting certified - for example higher credibility for the issuers - and cons - notably the cost of the certification. With this paper we aim at understanding the decisions made by issuers about certification and how they balance these pros and cons. What leads them to choose to get certified? Is getting certified always optimal or does the certification cost impact the issuers' decision to get (or not) certified? Our objective is to understand how the benefits of getting certified are counterbalanced by its cost and determine if there is a threshold for the cost beyond which getting certified is not optimal anymore.

To answer these questions, we first model the decision process with no certification fee, following Gill and Sgroi (2012). In their paper, they define two types of firms: one which produces high-quality products and the other one which produces low-quality. In order to broadcast to their potential customers the quality of their products, the firms can choose to get a review and the toughness of this review before launching the product on the market. There are thus three potential situations. The review can give either a positive opinion on the quality of the product or a negative one, or no review is asked and there is no additional information available to potential buyers regarding the quality of the product before it is launched. Gill and Sgroi prove that in this situation, all possible equilibria are pooling. Furthermore, they prove that a firm, no matter its type, will always choose to ask for a review. In our paper, we apply this model to two firms issuing a green bond. One, similar to the high-quality firm of Gill and Sgroi (2012), will really commit the funds to "green" projects. We define this firm as the high-quality firm in our model. The second type of firms is using the "green" designation mostly for marketing purposes, without committing the funds to "green" projects. This relates to the situation of greenwashing and we consider this type of firm to issue a low-quality green bond. Both firms can ask for a certification, which can either succeed ("pass") or fail. Issuers know the quality of their bond. However, potential buyers have no way to access this information and be totally sure of the type they are facing. Before the issuers decide or not to be certified, buyers have a prior belief about the probability that the bond is of high quality. They then observe whether the firm is getting certified and whether the certification is successful. Based on this, they update their beliefs after the certification process and decide whether to buy or not the bond.

We start by replicating the results of Gill and Sgroi (2012) when there is no certification fee. We find the same pooling equilibrium: in this situation, all firms will get certified. We then expand the model and introduce a certification cost strictly superior to 0. Without making any assumption on the distribution of the prior beliefs of buyers, we first determine a sufficient condition on the certification fee for the certification to be an optimal decision. We find that there exists a level of certification cost above which getting certified is not optimal anymore. In a second step, we assume a uniform distribution of prior beliefs. This is the case when people have no prior information on the company. We find a necessary and sufficient condition on the certification cost, which can be expressed as a function of the prior beliefs of buyers. In a final step, we study two special situations in which people have positive or negative prior bias toward the issuer. In both cases, we find a necessary and sufficient condition on the certification cost above which getting certified is not optimal anymore. Including the certification fee in the model nuances the findings of Gill and Sgroi (2012). Getting certified is not always optimal. It remains optimal as long as the certification cost does not become higher than a certain threshold which depends on the distribution of prior beliefs of buyers. Overall, these results shed light on the decision of green bond issuers to be certified or not by studying the impact of the cost of the certification.

Our paper is related to several important strands of the literature. Firstly, we relate to the literature focusing on green bonds and the motivations for firms to issue them. Past research has tackled this question from several points of view. When it comes to the stock value of companies, Tang and Zhang (2020) study 28 countries over a ten-year period and find a positive market reaction at the announcement of the green bond issue. Similarly, Flammer (2021) also finds a positive market reaction when a green bond is issued. Moreover, the reaction is stronger for bonds which have been certified by independent third-parties. Overall, issuing green bonds - especially certified ones - is received positively by the markets. Another reason that is considered as a driver for the issuance of green bonds is whether green bonds are more interesting than conventional ones when it comes to the cost of financing. Research finds conflicting results at first glance. Flammer (2021), Hachenberg and Schiereck (2018) and Tang and Zhang (2020) do not find evidence of a significant difference in yields between both types of bonds. On the contrary, Gianfrate and Peri (2019) and Zerbib (2017) for example finds that green bonds trade at a slightly lower price on the markets than comparable conventional bonds. Kapraun, Latino, Scheins, and Schlag (2021) explain the heterogeneity of results in research by putting into light the role of the green certification in the pricing of the bond. They find that investors are more likely to pay a green premium (that is, accept lower yield) when the certification is credible. Bachelet, Becchetti, and Manfredonia (2019) find similar results. The credibility of the green bond issuance - either though the issuer reputation or an independent certification - impact the cost of financing of the firm. The more credible the green claim is, the lower the yield. When it comes to the credibility of green claims, the ESG rating of firms has also been considered as a variable. Immel, Hachenberg, Kiesel, and Schiereck (2021) and Hachenberg and Schiereck (2018) found that the higher the rating, the lower the yield. This echoes the findings mentioned above from Flammer (2021): certification, by increasing the credibility of the signal, plays a key role in the value of the bond. Daubanes, Mitali, and Rochet (2021) base their work on these elements - positive market reaction, low to no difference in yield and significance of the certification - and study the reason why firms issue green bonds. They introduce the concept of managerial incentives to justify the recourse to certified green bonds. Issuing a certified bond will reinforce the market reaction, which aligns with the concern of managers for the stock price of their firm.

Overall, the above literature sheds light on the motivations for firms to issue green bonds and show that certifications are central in the optimization of the advantages of green bonds. However, the literature has not yet studied the specifics of the certification itself and how they impact these findings. With this paper, we contribute to this literature by modelling the weight of the certification on the issuance of green bonds. We aim at understanding under which cost parameters getting certified remains optimal.

Because of our focus on certification of green bonds, we also relate to the literature on certification. In this literature, the focus is on either of the three main actors: the firm getting certified, the potential buyer or the certification agency. For example, Lizzeri (1999) and Mathis, McAndrews, and Rochet (2009) focus mainly on the role of the certification agencies. Both these papers study whether these agencies have incentive to only partially disclose information or to be too lenient in their ratings. In our paper, on the contrary, we focus on the decisions made by the firm and the buyers. Like in Skreta and Veldkamp (2009), whether the certification agencies attempt to "lie" or really provide unbiased ratings does not impact our model. In their paper, Skreta and Veldkamp study how firms select their credit rating agencies. The focus is on firms getting certified and whether they tend to "shop" for ratings, that is to ask ratings from several agencies and select the most favorable one. They find that increased asset complexity (as observed before the financial crisis of 2008) is enough to drive rating discrepancy and whether rating agencies have an incentive to produce biased estimates of the ratings is of limited impact. In our paper, our focus is also on the firms' decisions and behaviour, and on the potential buyers of the bond. The decision of the certification agency is modelled by a probability to either grant the certification or refuse it. There exists a probability of "false-positive", that is a certification given to a "bad-quality" firm, and of "false-negative". This represents the level of expertise of the certification agency. In that, we follow the model of Gill and Sgroi (2012). Contrary to their model, we however consider a situation where getting certified is costly. Hvide et al. (2009) also consider a situation where the certification is not free. In their model, there are several certification agencies. They differentiate themselves through the toughness of their tests. The model focuses on which agency the firm getting certified chooses, based on their quality-level and the level of difficulty of the tests offered by the different agencies, which in turns determines the cost of the certification. In this paper, our stance is different. We focus here on the impact of the certification costs, to determine how it changes the decision to get certified or not. This relates to Bonroy and Constantatos (2015). They study the effect of labels for consumers and firms and find that a label is not always beneficial, notably when taking into account the cost of the certification process as a parameter.

Finally, closely related to the previous strand of literature, we relate also to the literature focused on how firms signal their specificity to their potential buyers. Mezzetti and Tsoulouhas (2000) studies the role of information gathering by the potential buyers to separate the "good" firms from the "bad". As in Gill and Sgroi (2012), this idea is what motivates firms in our model to ask for a certification. High-quality firms have no other way to signal their type to the potential buyers of their bond except the certification. The decision to ask for a certification and the obtainment of it lead potential buyers to actualize their belief on the quality of the firm they are facing, and the price they are willing to pay. Rysman, Simcoe, and Wang (2020) also study the role of certifications as signal. They consider certifications with different levels in the sector of environmental buildings. They show that if there are several certification levels available, builders will try to differentiate themselves and showcase the green characteristics of their building by targeting different certification levels. To signal the green aspect of a product, Ki and Kim (2022) show that pricing can also be a way for a firm to differentiate itself from its competitors. They find that high pricing signals eco-friendliness. Using a two-period game-theoretic model, Jiang and Yang (2019) study the pricing and quality decisions of firms to attract customers. They prove the existence of the relationship between both types of decisions in the situation where early buyers can communicate information to the second-stage buyers. Another element that has been studied is the manipulation of consumer beliefs. Chen and Papanastasiou (2021) use a model similar to ours in which the firms can influence the consumers' beliefs on the quality of the product, not with a certification, but through manipulation. A significant difference with our model is that manipulation of the information available to the customers is not costly in their model.

The rest of the paper is organized as follows. In Section 2, we present the model and its assumptions and demonstrate the existence of equilibrium strategies. In Section 3, we focus on the situation where the certification cost is null and we model the decisions of the firms. Then, in Section 4, we introduce a certification cost strictly positive and prove how the results of Section 3 are changed by this parameter. Finally, Section 5 concludes.

2 Model

Inspired by the model of Gill and Sgroi (2012), we propose the following model to study the optimal certification decision of green bonds issuers. In our setting, we have two types of firms issuing green bonds. They can either issue a high-quality green bond (meaning they will follow up on their green commitment) or issue a low-quality one (that is the case for example of greenwashing). The issuers choose whether or not to ask for a green certification to advertise the "greenness" of their bond issue to the potential buyers. The objective of the issuers is to maximize their expected revenues. Potential buyers, based on the decision they observe regarding the certification and what they know of the issuers, decide to buy or not the bond. They aim at maximizing their utility. With this model, we focus on determining what influences the choice of the issuers to get certified.

In the following section, we present the model in more details along with the notations and hypotheses we use.

2.1 Notation and assumptions

We consider a corporate issuing a green bond aimed at a unit mass of potential buyers with unit demands. Specifically, we denote by v the bond's quality (whether the issuers will commit to their green claim or not) and assume for simplicity that

$$v \in \{0, 1\},$$

where $v = 1$ refers to the high-quality green bond and $v = 0$ refers to the low-quality green bond. Because of our focus on green bonds and the role of the certification in the players' decision, we only consider bonds with the same credit ratings. Quality here refers specifically to whether the issuers will act on the green claims they made at issuance or not.

The objective of the issuers, no matter the quality of the bond they issue, is to maximize their expected revenues: the decision to ask for a green certification is studied through this perspective. To analyze the demand for a certification, we make use of signal theory. The only agents who know the quality of the bond with certainty are the issuers. However, they have no means to reveal it to the potential buyers on the market. To provide information to the buyers on the quality v of the bond, the issuers can decide to request a green certification for their bonds. There are two possibilities: choosing not to ask for a certification, denoted N, or getting certified, denoted T. Given that low-quality issuers can replicate the decision of the high-quality issuers, the choice of being certified is not informative per se. It is the result of the certification that matters. In case the issuers ask for a certification, we define the certification outcome as:

$$d \in \mathbb{D} := \{P, F\}$$

where P is a pass and F is a fail. We note q_v^d the probability that a bond of quality v gets a certification decision d:

$$q_v^d := P(decision\ d/quality\ v)$$

These probabilities are defined as such that:

$$q_v^P + q_v^F = 1 \tag{1}$$

In particular (1) implies that

$$q_1^P - q_0^P = q_0^F - q_1^F. \tag{2}$$

The values in (2) represent the level of expertise of the certification, that is the ability of the certification body to attribute a "pass" to a high-quality bond and a "fail" to a low-quality bond. We denote expertise as κ and define it as:

$$\kappa := q_1^P - q_0^P = q_0^F - q_1^F. \tag{3}$$

The higher κ is, the better the certification is able to distinguish between low and high quality green bonds (and thus attribute a "pass" and "fail" without mistake). The probability of having a "pass" if $v = 1$ has to be higher than the probability of having a "pass" if $v = 0$, meaning

$$q_1^P > q_0^P$$

and

$$\kappa > 0.$$

In the situation where the certification agency has incentive to give biased estimates on the greenness of the bond issue, in order to preserve its credibility on the markets, we assume that it would ensure that it keeps a lower rate of "false positive", which means giving a "pass" to a low-quality green bond, than "true positive", meaning giving a "pass" to a high-quality green bond (and conversely for failing bond issue at the certification). As such, $q_1^P > q_0^P$ and $q_0^F > q_1^F$ remain true in the situation where the certification agency is not always unbiased.

In the simplest situation, there is only one certification agency with one certification type. The choice of the issuers is either not to get certified, N, or to get certified, T. The set of decisions of the issuers is thus $\{N, T\}$. If the issuers ask for a certification, either it "passes", which situation we denote TP, or it "fails", denoted TF. The issuers set the initial price V of the green bond based on their decisions to get certified or not, and the results of the certification process. Their objective is to maximize their expected revenue. Because we are interested in the effect of the certification (and of its outcome), we consider only bonds with the same credit rating. In this situation, the price set will be based on the certification being taken and on its result d. Given that the low-quality issuers can replicate all the actions of the high-quality ones, the quality v of the bond does not influence the price.

Moreover, because high-quality issuers cannot differentiate themselves directly from the low-quality ones, buyers cannot know the quality of the bond for sure by simply observing their decisions. Before the issuers make any decision regarding their bond issue, the potential buyers receive private signals regarding the quality v, which differ in their level on informativeness.

These signals follow an independent and identically distributed (i.i.d) random draw from a continuous distribution F_v, dependent on the quality of the bond. They use this private signal in combination with the prior belief they have on the quality of the bond to form a private prior belief π. This private prior belief π, represents the probability potential buyers each give to the event $[v = 1]$:

$$\pi = P(v = 1).$$

This is the probability that the bond issue they observe is of high quality (which means the issuers will really follow up on their green commitments). We assume that the certification result is independent from the buyers' private prior signals. Importantly, no private prior signal received is informative enough to reveal for sure the quality of the bond. A specific case of prior belief is the situation of "fair prior", where all buyers believe that $[v = 1]$ with a probability of 0.5. In this case, without loss of generality the private signal outcome and the prior belief coincide (see Smith and Sorensen (2006)).

We denote by $G(\pi)$, the cumulative distribution function over π. We assume it is bounded over $[\underline{\pi}, \overline{\pi}]$, with $0 < \underline{\pi}, < \overline{\pi} < 1$. By definition of a cumulative distribution function, $G(\pi)$ represents the proportion of buyers who believe that $P(v = 1) < \pi$. By opposition, $1 - G(\pi)$ is the proportion of buyers believing that they observe a high-quality green bond issue with a probability higher than π. We assume that this distribution G is known to the issuers. Indeed, firms have access to market data and can build on their experience to form expectations about the market for their bond issue. Moreover, bond issues are planned with financial advisors who can also provide support in building these expectations.

Buyers aim at maximizing their utility when deciding to buy or not the bond. Given that we consider bonds with the same credit ratings, we assume that a green bond will represent more utility for a buyer than a "classic" one. We assume buyers are risk neutral. Because of this, the buyers' payoff from buying a bond is then given by

$$v - V$$

The buyers observe the price V, the decision to be certified or not and the outcome of the certification d. These parameters lead them to update their private belief π into a posterior belief, which we denote by π'. This posterior belief represents the probability that the bond is of high-quality ($v = 1$) after observing the certification decision and outcome, and the pricing decision. By definition, we can express π'_d, the posterior belief that the bond issue is of high quality after the buyers observes the outcome d of the certification as a function of π, using Bayes formula.

$$\pi'_d = P(v = 1|d) = \frac{P(v = 1 \cap d)}{P(d)} = \frac{P(d|v = 1)P(v = 1)}{P(d)}$$
$$\Leftrightarrow \pi'_d = \frac{P(d|v = 1)P(v = 1)}{P(d \cap v = 1) + P(d \cap v = 0)} = \frac{q_1^d \pi}{q_1^d \pi + q_0^d (1 - \pi)}. \tag{4}$$

It follows that $1 - G(\pi')$ is the proportion of potential buyers believing that they observe $[v = 1]$ after the certification decision is known with a probability at least equal to π'. Under the assumption that the buyer is risk neutral and because the buyers cannot know the quality of the bond for sure, the expected utility of a buyer of the green bond is given by

$$\pi' - V.$$

A buyer will only buy if

$$\pi' - V \geqslant 0 \Leftrightarrow \pi' \geqslant V$$

and the proportion of buyers who actually buy the bond can be defined as

$$1 - G(V).$$

If no certifications happen, similarly, the expected utility of a buyer is given by

$$\pi - V.$$

Finally, all buyers make their purchasing decision simultaneously.

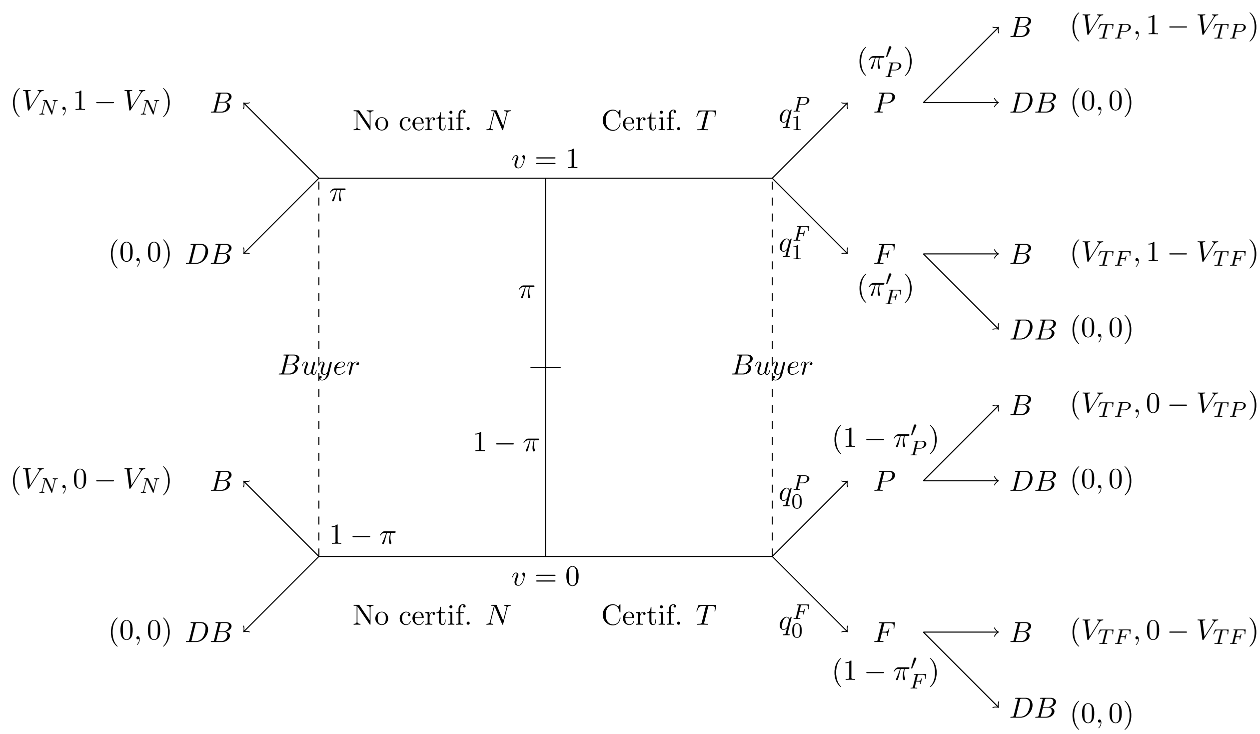

Figure 1: Game Situation Without Cost - $C = 0$

Figure 1 depicts the game situation using the players' decision tree and illustrates the setting explained above. Here, we have two types of firms, either high quality ($v = 1$, at the top of the figure) or low quality ($v = 0$, at the bottom). Both can choose to ask for a certification (certification $- T$) or not (no certification $- N$). The left side of the figure represents the situation where the firms choose not to get certified (N). In this case, the buyers observe this decision and decide to buy (B) or not to buy (DB). This decision depends on π, the private prior belief the buyers have that they are faced with a high-quality issue, and on their payoff. These payoffs are noted within brackets on the left side of the figure. The first payoff noted is the one of the issuers, the second is for the buyers. We denote V_N the price set by the issuers when they do not ask for the certification. There are two cases. Either the buyers buy the bond and they get $v - V_N$ or they don't and their payoff is 0 as they do not earn or lose anything. The payoff of the issuers depend on the buyers'. If the buyers actually buy the bond, the issuers get V_N for each bond. If they do not buy, the issuers do not get anything and their payoff is 0.

The right side of the figure presents the situation when there is a certification (T). Either it fails (F) with a probability q_v^F or it passes (P) with a probability q_v^P. The buyers observe the certification outcome and update their private prior belief π to their posterior belief π'. They then decide to buy (B) or not (DB) based on this new belief. Similar to the case where there is no certification, we denote the payoff within brackets, on the right side of the tree. The first payoff is the one of the issuers, the second is for the buyers. Here, we denote V_{TP}, the price of the bond in case the certification result is "pass" and V_{TF}, if the certification "fails". The payoffs follow the same logic as before. If the buyers do not buy, their payoff is 0. If they buy, they get $v - V_{TP}$ or $v - V_{TF}$ depending on the outcome of the certification and the quality of the bond. As before, the payoff of the issuers is directly related to the buyers'. If the buyers buy the bond, the issuers get V_{TP} or V_{TF} for each bond. If they do not buy, the issuers do not get anything and their payoff is 0.

2.2 Optimal Certification Decision: equilibrium strategies

Following Gill and Sgroi (2012), "we solve for perfect Bayesian equilibria, and we restrict attention to pure strategies". An essential factor to consider in the identification of equilibria is that a low-quality firm can replicate freely all the actions of a high-quality.

Based on this, we have the following proposition that is closely related to the results of Gill and Sgroi (2012):

Proposition 2.1. *The only possible equilibria are*

- *degenerate separating equilibria in which the issuer always earns 0 revenue*
- *pooling equilibria*

Proof. For $V > 0$, the proof follows by contradiction. Let us suppose a separating equilibrium. At its simplest, the strategy chosen by a bond issuer is either to be certified or not, which we refer to as T for choosing to be certified and N, for not being certified (see Figure 1). Assuming the issuers follow separating strategies, we have two cases: the firms with high-quality green bonds get certified and the ones with low-quality green bonds do not, written as (T N) and the opposite strategy, (N T).

Facing the strategy (T N), the buyers' optimal answer is to always buy when seeing a certification being taken, no matter if the certification is obtained or not, because observing a certification is synonymous to observing a high-quality bond. The payoff for buying a high-quality green bond is always strictly positive. It is strictly higher than 0, the expected payoff when not buying. So, the buyers' optimal response is to buy when seeing T (denoted B). When seeing N, however, the buyers know that this is a low-quality green bond and their optimal decision is not to buy (denoted DB), as their expected payoff when buying a low-quality bond is negative, compared to 0 when not buying.

Overall, if we write the set of strategies of the buyers as first their decision when there is a certification that "passes", then when there is a certification that "fails" and finally, when there is no certification, the buyers' optimal answer when observing (T N) is (B B DB). Indeed, the outcome of the certification, “pass” or “fail”, does not impact their optimal decision. Based on this optimal answer, a firm with a high-quality issue has no incentive to change its strategy. Indeed, choosing N would lead the buyers to think they are faced with a low-quality issue, and thus decide not to buy. This would lead to a payoff for the firm of 0, compared to the strictly positive payoff it gets when the buyers buy. So a high-quality issuer has no incentive to switch from getting certified.

A firm with a low-quality bond issue gets a payoff of 0 as long as it keeps not getting certified, given that the buyers do not buy. However, switching strategy and getting certified would lead the buyers to expect a high-quality issue, leading them to buy. In this case, the payoff from the issuer would be strictly positive and thus this issuer has an interest in switching strategy from N to T and choose to get certified. This means that ((T N), B B DB)) is not a Perfect Bayesian Equilibrium (PBE). By applying the same reasoning, we show that the strategy (N T) is not a PBE either. Separating equilibria are thus not possible in this situation and the only possible equilibria are pooling. Overall, in a separating equilibrium, issuers of low-quality green bonds would reveal their quality to be low the first time their choice of being certified or of price V for the bond differs from the optimal decision of issuers with high-quality green bonds. Given that this information is disclosed to the buyers, it is optimal for issuers with low-quality green bonds to deviate from their initial choice and replicate the strategy of the high-quality issuers.

Let us consider now consider the case where $V = 0$. In this situation, faced with the strategy (T N), the buyers make the same decision as previously: they will always buy when observing T, knowing it only happens if the firm is a high-quality issuer, and not buy when observing N. Their payoff for buying a high-quality green bond is indeed 1 (because utility is defined as $v - V$ with $V = 0$), which is higher than the payoff for buying a low-quality one, which is 0. The buyers' optimal answer when observing (T N) is thus still (B B DB). However, given that $V = 0$, knowing the buyers' set of decision, the issuers are indifferent between T and N. No matter their decision when faced with the buyers' decision, their payoff will remain 0. In this case, degenerate separating equilibria are thus possible.

Overall, the only possible equilibria are either degenerate seperating equilibria when the revenue from the bond is null or pooling equilibria otherwise, with both types of firms either getting both certified or not.

□

As a consequence from Proposition 2.1, the issuers cannot use the choice of asking for a certification or of price V to signal the quality of their green bond directly, and hence we focus attention on the direct role of the certification itself in transmitting information to the buyers, who learn only from the certification outcome. The low-quality issuers can replicate exactly the action of the high-quality issuers and thus are a priori indistinguishable from high-quality issuers. We can thus restrict ourselves to considering a high-quality green bond issuer and prices $V > 0$.

3 Requesting a certification: an optimal decision if it is free

3.1 Theoretical Developments

Theorem 3.1. *In the absence of a cost of certification, the issuer always opts to have a new green bond issued to be certified publicly.*

Proof. This result is similar to Proposition 1 of Gill and Sgroi (2012). Its proof is given in Appendix A of Gill and Sgroi (2012) but for completeness, we adapt it to our context and recall all details hereafter.

We denote $EP(B|TF)$ the expected payoff of the buyers if the certification fails and $EP(B|TP)$, their expected payoff if the certification passes. π' is the posterior belief of the buyers that the bond is of high quality. We denote π'_F, this belief if the certification results in a "Fail" and π'_P, if the certification results in a "Pass". Fig. 2 shows an illustration of the update of buyers' private beliefs from π to π', in case case where they are uniformly distributed over $[0.1; 0.9]$.

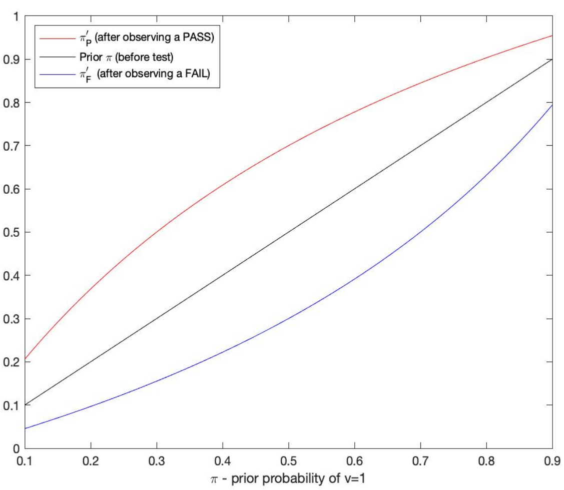

Figure 2: Illustration of the update of buyers' private beliefs in the case of a prior uniform distribution of the beliefs over [0.1;0.9]

Based on Figure 1, if the strategy followed by both type of issuers (high and low quality) is to get certified (T T) and if the certification fails, we define the expected payoff of the buyer as $EP(B|TF) = \pi'_F(1 - V_{TF}) + (1 - \pi'_F)(-V_{TF}) = \pi'_F - V_{TF}$ and thus it is optimal for the buyers to purchase the bond if and only if $\pi'_F \geqslant V_{TF}$.

By definition, π'_F is the posterior belief that the bond issue is of high quality after the buyers observes a "fail" and thus, using Bayes formula,

$$\pi'_F = P(v = 1|F) = \frac{P(v = 1 \cap F)}{P(F)} = \frac{P(F|v = 1)P(v = 1)}{P(F)}$$
$$\Leftrightarrow \pi'_F = \frac{P(F|v = 1)P(v = 1)}{P(F \cap v = 1) + P(F \cap v = 0)} = \frac{q_1^F \pi}{q_1^F \pi + q_0^F (1 - \pi)}. \tag{5}$$

We have

$$\pi'_F \geqslant V_{TF} \Leftrightarrow \frac{q_1^F \pi}{q_1^F \pi + q_0^F(1-\pi)} \geqslant V_{TF} \Leftrightarrow q_1^F \pi \geqslant V_{TF}(q_1^F \pi + q_0^F(1-\pi))$$
$$\Leftrightarrow q_1^F \pi - V_{TF} q_1^F \pi + V_{TF} q_0^F \pi \geqslant V_{TF} q_0^F \Leftrightarrow \pi(q_1^F - q_1^F V_{TF} + q_0^F V_{TF}) \geqslant V_{TF} q_0^F$$
$$\Leftrightarrow \pi \geqslant \frac{V_{TF} q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}. \tag{6}$$

Overall, if the certification fails, the buyers will buy if and only if their prior belief π fits with the condition found above in equation (6).

We follow the same reasoning in the case where the certification passes: $EP(B|TP) = \pi'_P(1-V_{TP}) + (1-\pi'_P)(-V_{TP}) = \pi'_P - V_{TP}$ and thus the buyers buy if and only if $\pi'_P \geqslant V_{TP}$.

By definition, π'_P is the posterior belief that the bond issue is of high quality after the buyers observe a "pass" and thus, using Bayes formula,

$$\pi'_P = P(v=1|P) = \frac{P(v=1 \cap P)}{P(P)} = \frac{P(P|v=1)P(v=1)}{P(P)}$$
$$\Leftrightarrow \pi'_P = \frac{P(P|v=1)P(v=1)}{P(P \cap v=1) + P(P \cap v=0)} = \frac{q_1^P \pi}{q_1^P \pi + q_0^P(1-\pi)}. \tag{7}$$

It follows that we have

$$\pi'_P \geqslant V_{TP} \Leftrightarrow \frac{q_1^P \pi}{q_1^P \pi + q_0^P(1-\pi)} \geqslant V_{TP} \Leftrightarrow q_1^P \pi \geqslant V_{TP}(q_1^P \pi + q_0^P(1-\pi))$$
$$\Leftrightarrow q_1^P \pi - V_{TP} q_1^P \pi + V_{TP} q_0^P \pi \geqslant V_{TP} q_0^P \Leftrightarrow \pi(q_1^P - q_1^P V_{TP} + q_0^P V_{TP}) \geqslant V_{TP} q_0^P$$
$$\Leftrightarrow \pi \geqslant \frac{V_{TP} q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}. \tag{8}$$

Overall, if the certification passes, the buyers will buy if and only if their prior belief π fits with the condition found above in equation (8).

The final possible case is if the buyers do not observe a certification. In this situation, they will buy if and only if $\pi \geqslant V_N$.

Based on these results, the expected payoff for a high-quality issuer requesting a certification is

$$EP_{v=1}(T) = q_1^P V_{TP} \left(1 - G\left(\frac{V_{TP} q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}\right)\right) + q_1^F V_{TF} \left(1 - G\left(\frac{V_{TF} q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}\right)\right). \tag{9}$$

The bond issuers will only keep this strategy and choose to be certified rather than not if $\max EP_{v=1}(T) > \max EP_{v=1}(N)$. We denote V_N^* the price V_N that maximizes $EP_{v=1}(N) = V_N(1 - G(V_N))$, the expected payoff of a firm not getting certified.

Equation (9) is true for any V_{TP} and V_{TF}. There exists a V_{TP} such as $\frac{V_{TP} q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}} = V_N^*$, which we define as V_X and a V_{TF} such as $\frac{V_{TF} q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}} = V_N^*$, which we call V_Y.

Considering the optimal choice of V_{TP} and V_{TF} such as $EP_{v=1}(T)$ is maximized, we have

$$\max[EP_{v=1}(T)] \geqslant q_1^P V_X \left(1 - G(V_N^*)\right) + q_1^F V_Y \left(1 - G(V_N^*)\right). \tag{10}$$

We denote EP_{XY} the right-hand side of (10).

To prove that the high-quality issuers have no interest to change strategy and prove Theorem 3.1 in the case where $v = 1$, we now prove that $EP_{XY} > \max EP_{v=1}(N)$. Indeed, proving this will show that there exists at least one set of V_{TF} and V_{TP} for which the expected payoff of the firm is always strictly higher when there is a certification than when there is none. The existence of such a set of prices means that getting certified is thus more profitable, no matter the result of the certification, than not getting certified at all.

Given the definition of the CDF G and of π, $V_N^* \in [\underline{\pi}, \overline{\pi}]$, with $0 < \underline{\pi} < \overline{\pi} < 1$. By definition, $V_N > 0$ and $1 - G(V_N) > 0$ for any V_N. Thus, the maximum expected revenue for the bond issuer choosing not to be certified, defined as $\max[EP_{v=1}(N)] = V_N^*(1 - G(V_N^*))$, is strictly positive.

By definition of V_N^*,

$$\begin{aligned} V_N^* = \frac{V_X q_0^P}{q_1^P(1 - V_X) + q_0^P V_X} &\Leftrightarrow V_N^* \left(q_1^P(1 - V_X) + q_0^P V_X\right) = V_X q_0^P \\ &\Leftrightarrow q_1^P V_N^* = V_X \left(q_0^P(1 - V_N^*) + q_1^P V_N^*\right) \\ &\Leftrightarrow V_X = \frac{q_1^P V_N^*}{q_0^P(1 - V_N^*) + q_1^P V_N^*}. \end{aligned}$$

We know that $0 < V_N^* < 1$, so we have:

$$\begin{aligned} 1 - V_N^* > 0 \Leftrightarrow q_0^P(1 - V_N^*) > 0 \Leftrightarrow q_0^P(1 - V_N^*) + q_1^P V_N^* > q_1^P V_N^* \\ \Leftrightarrow \frac{q_1^P V_N^*}{q_0^P(1 - V_N^*) + q_1^P V_N^*} < 1 \Leftrightarrow V_X < 1 \end{aligned}$$

We follow the same reasoning for V_Y and we can then reformulate EP_{XY} as

$$\begin{aligned} & EP_{XY} = q_1^P V_X \left(1 - G(V_N^*)\right) + q_1^F V_Y \left(1 - G(V_N^*)\right) \\ \Leftrightarrow\ & EP_{XY} = q_1^P \frac{q_1^P V_N^*}{q_0^P(1 - V_N^*) + q_1^P V_N^*} \left(1 - G(V_N^*)\right) + q_1^F \frac{q_1^F V_N^*}{q_0^F(1 - V_N^*) + q_1^F V_N^*} \left(1 - G(V_N^*)\right) \\ \Leftrightarrow\ & EP_{XY} = \sum_d \frac{(q_1^d)^2 V_N^* \left(1 - G(V_N^*)\right)}{q_0^d(1 - V_N^*) + q_1^d V_N^*}. \end{aligned} \tag{11}$$

By definition, $q_v^d \geqslant 0$ and $0 < V_N^* < 1$ so $q_1^d V_N^* \geqslant 0$ and $q_0^d(1 - V_N^*) \geqslant 0$. Moreover, following Eq. (1), q_1^d and q_0^d cannot be null at the same time so the denominators of the two elements of the sum in Eq. (11) are necessarily strictly positive.

We now prove that $EP_{XY} > \max[EP_{v=1}(N)]$. As a sidenote, V_N^* maximizes $EP_{v=1}(N)$ but not necessarily EP_{XY}.

$$\begin{aligned} EP_{XY} > \max[EP_{v=1}(N)] &\Leftrightarrow \sum_d \frac{(q_1^d)^2 V_N^* \left(1 - G(V_N^*)\right)}{q_0^d(1 - V_N^*) + q_1^d V_N^*} > V_N^* \left(1 - G(V_N^*)\right) \\ &\Leftrightarrow \sum_d \frac{(q_1^d)^2}{q_0^d(1 - V_N^*) + q_1^d V_N^*} > 1. \end{aligned}$$

Using (1), we can rewrite the above equivalence as follows

$$\Leftrightarrow (1-q_1^P)^2\left(q_1^P V_N^* + q_0^P(1-V_N^*)\right) + (q_1^P)^2\left((1-q_1^P)V_N^* + (1-q_0^P)(1-V_N^*)\right)$$
$$> \left((1-q_1^P)V_N^* + (1-q_0^P)(1-V_N^*)\right)\left(q_1^P V_N^* + q_0^P(1-V_N^*)\right).$$

Developing the two sides of the inequality and rearranging the terms, we obtain

$$\Leftrightarrow (1-q_1^P)\left(V_N^* q_1^P + q_0^P - V_N^* q_0^P - q_1^P q_0^P + V_N^* q_1^P q_0^P - (V_N^*)^2 q_1^P - V_N^* q_0^P + (V_N^*)^2 q_0^P\right)$$
$$+ (1-q_0^P)\left((q_1^P)^2 - V_N^* q_1^P - V_N^*(q_1^P)^2 + (V_N^*)^2 q_1^P - q_0^P + 2q_0^P V_N^* - q_0^P (V_N^*)^2\right) > 0$$
$$\Leftrightarrow (q_1^P)^2 - 2V_N^*(q_1^P)^2 + (V_N^*)^2(q_1^P)^2 - 2q_1^P q_0^P + 4V_N^* q_1^P q_0^P$$
$$- 2(V_N^*)^2 q_1^P q_0^P + (q_0^P)^2 - 2V_N^*(q_0^P)^2 + (V_N^*)^2(q_0^P)^2 > 0$$
$$\Leftrightarrow \left((q_1^P)^2 - 2q_1^P q_0^P + (q_0^P)^2\right)\left(1 - 2V_N^* + (V_N^*)^2\right) > 0$$
$$\Leftrightarrow (q_1^P - q_0^P)^2(1-V_N^*)^2 > 0.$$

By definition, $q_1^P > q_0^P$ and $V_N^* < 1$ so both terms of the products are indeed strictly positive. Thus, $EP_{XY} > \max[EP_{v=1}(N)]$ for any V_X and V_Y. Given Inequality (10), we can conclude that there is at least one set of V_{TP} and V_{TF} such as $\max[EP_{v=1}(T)] > \max[EP_{v=1}(N)]$. Firms setting their bond prices accordingly will have a higher expected payoff if they get certified than if they do not. Thus, high-quality bond issuers will always choose to get a certification for their green bonds if the certification is costless, which concludes the proof of Theorem 3.1. □

3.2 Numerical Illustration

The results in 3.1 do not depend on the choice of the distribution that G follows. We illustrate this point by showing for two different cumulative distribution functions that Theorem 3.1 holds. The CDF we study are shown in Fig. 3 and Fig. 4.

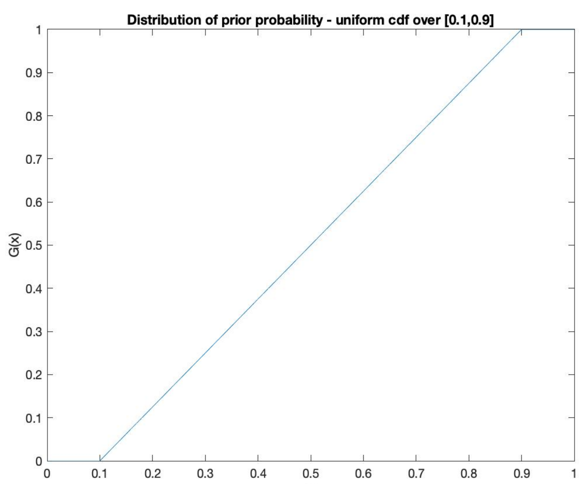

Figure 3: Numerical example for the optimization of the expected payoff of the issuer for the uniform CDF over [0.1;0.9]

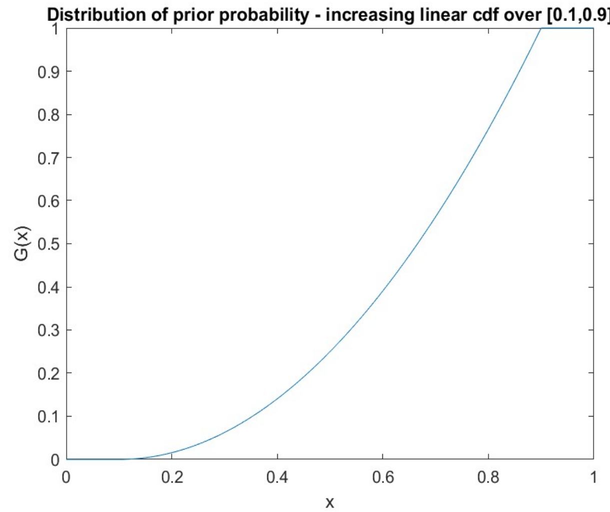

Figure 4: Numerical example for the optimization of the expected payoff linear increasing CDF of $G(\pi)$ over [0.1;0.9]

Fig. 5 is a numerical illustration of the proof, using uniformly distributed prior beliefs over $[0.1, 0.9]$. The black horizontal line is the maximal revenue of the issuer in case of certification (whether it "fails" or "passes"). It is always greater than the expected revenue in case of no certification (red curve). Fig. 6 illustrates how the expected payoff evolves depending on q_1^F, the probability of a high-quality issuer failing the test. On both figures, the black line shows that the expected payoff for an issuer getting certified is always higher than for an issuer choosing not to ask for a certification. Fig. 7 to 8 show the same for the increasing linear CDF.

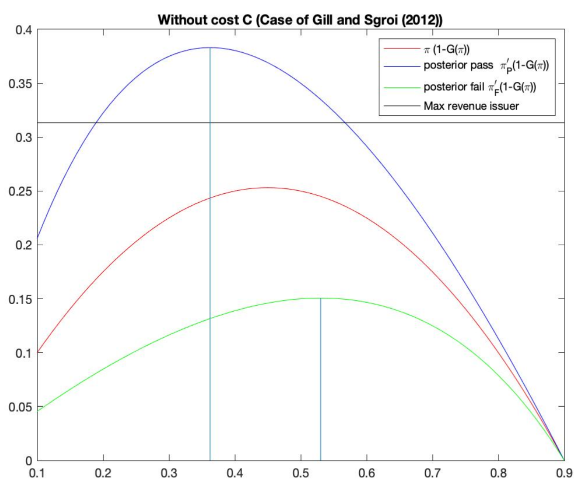

Figure 5: Numerical example for the optimization of the expected payoff of the issuer for the uniform CDF over [0.1;0.9]

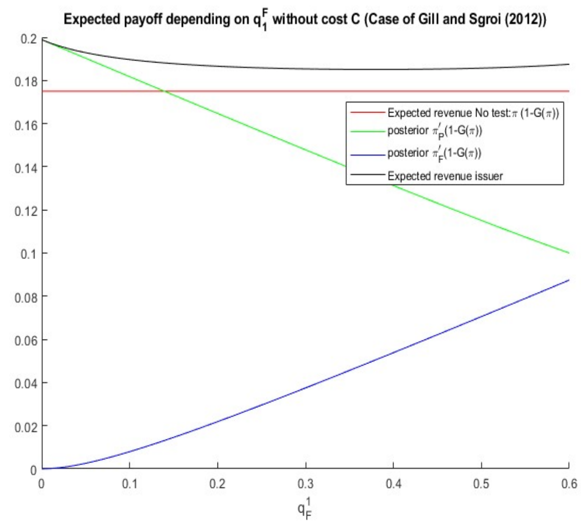

Figure 6: Expected Payoff using a uniform distribution over [0.1;0.9] as a function of q_1^F

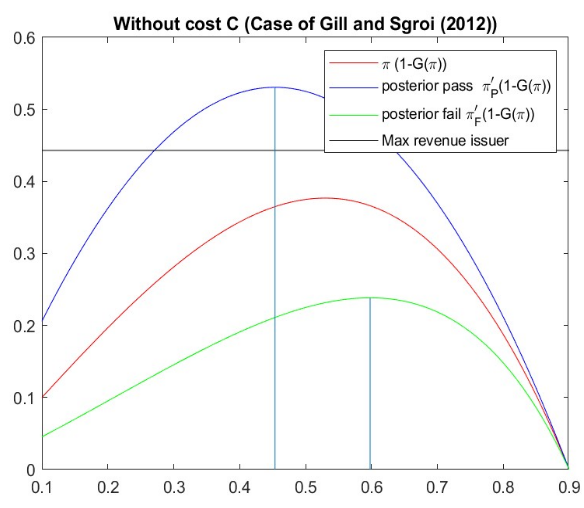

Figure 7: Numerical example for the optimization of the expected payoff of the issuer for the increasing linear CDF over [0.1;0.9]

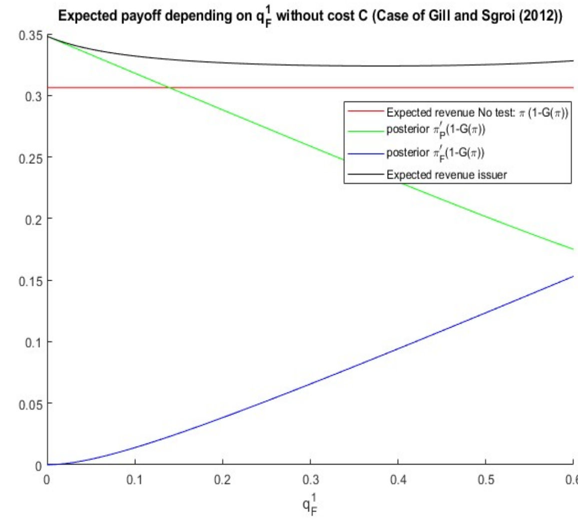

Figure 8: Expected Payoff using an increasing linear distribution over [0.1;0.9] as a function of q_1^F

4 Requesting a certification when it is costly: an optimal decision up to a certain point

In this section, the certification cost is strictly positive. We denote C, this cost. We consider C to be a fee proportional to the price V of the bond. We make this assumption because this fits with the information provided by the CBI on the way their fee is computed, which is the only pricing information for green bond certifications we were able to find. Passed a minimum fee, the certification fee is a tenth of a basis point of the total amount issued.[2] We focus here on the general case and consider thus a cost proportional to V. Figure 9 represents the game situation including this cost. The notations are the same as in Figure 1. The only difference comes into the expected payoffs. The firms' payoffs are affected by the certification cost that they pay when they request a certification. They request the certification before the buyers decide to buy or not. Because of that, the firm's payoff can be negative in case the buyers do not buy. In the following subsections, we study how this parameter affects their decisions.

4.1 Sufficient condition for an optimal certification

Theorem 4.1. *In the case where $C > 0$, there exists a level for the certification cost above which getting certified is not optimal anymore and issuers will choose not to ask for certification. This level depends on the distribution of prior beliefs of potential buyers defined as:*

$$C < \left(1 - \frac{1}{\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*}}\right)(1 - G(V_N^*)) \tag{12}$$

with $1 - G(V_N^*)$, the proportion of buyers in the case of no certification, determined by G, the cumulative distribution function of private prior beliefs.

[2]See `https://www.climatebonds.net`.

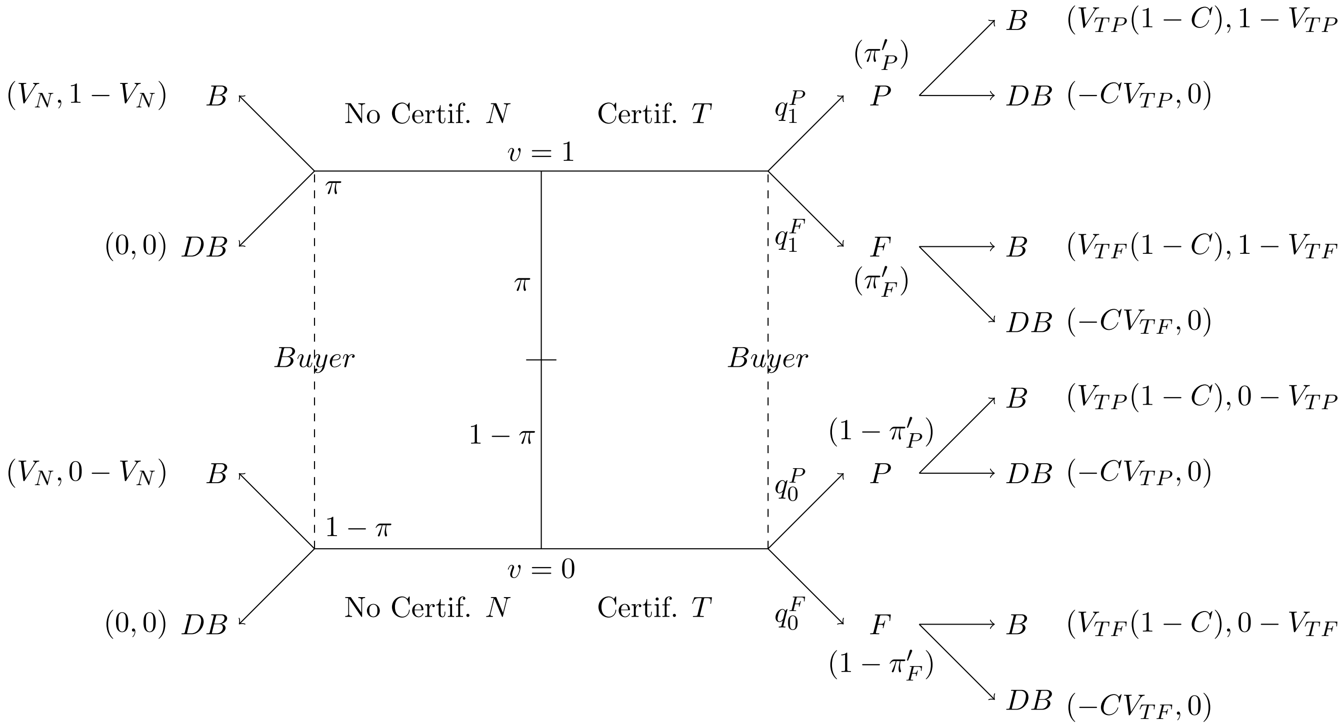

Figure 9: Game Situation With Cost - $C > 0$

Proof. The buyers are not directly affected by the cost of the certification. The cost is assumed by the company issuing the bond. Because of this, the setting of the model with $C > 0$ is similar to the one in the previous section. When observing a certification test resulting in a "pass" decision, the buyers will buy if and only if their prior belief π satisfies $\pi \geqslant \frac{V_{TP} q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}$. If observing a "fail" test, the buyers will buy if and only if $\pi \geqslant \frac{V_{TF} q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}$.

Based on this and Figure 9, the expected payoff of the company getting a certification for a high-quality bond issue is expressed as

$$EP_{(v=1)}(T) = q_1^P \left(1 - G\left(\frac{V_{TP} q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}\right)\right) V_{TP}(1-C) + q_1^P G\left(\frac{V_{TP} q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}\right) V_{TP}(-C)$$
$$+ q_1^F \left(1 - G\left(\frac{V_{TF} q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}\right)\right) V_{TF}(1-C) + q_1^F G\left(\frac{V_{TF} q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}\right) V_{TF}(-C),$$

which is equivalent to

$$EP_{(v=1)}(T) = q_1^P V_{TP} \left(1 - G\left(\frac{V_{TP} q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}\right) - C\right) + q_1^F V_{TF} \left(1 - G\left(\frac{V_{TF} q_0^F}{q_1^F(1-V_{TF} + q_0^F V_{TF}}\right) - C\right). \tag{13}$$

The payoff of a high-quality company choosing not to be certified is defined in the same way as in Section 3: $EP_{v=1}(N) = V_N(1 - G(V_N))$. We call V_N^* the price that maximizes $EP_{v=1}(N)$.

It is optimal for a company to get a certification if and only if

$$\max EP_{(v=1)}(T) > \max EP_{(v=1)}(N) = V_N^*(1 - G(V_N^*)). \tag{14}$$

We use a similar reasoning as in the previous section. We define V_X, a V_{TP} such as $V_N^* = \frac{V_X q_0^P}{q_1^P(1-V_X)+q_0^P V_X}$. Similarly, we denote V_Y, a V_{TF} such as $V_N^* = \frac{V_Y q_0^F}{q_1^F(1-V_Y)+q_0^F V_Y}$. We denote EP_{XY}, the expected payoff $EP_{(v=1)}(T)$ for such V_X and V_Y.

We then reformulate both expressions so that we isolate V_X and V_Y: $V_X = \frac{V_N^* q_1^P}{q_0^P(1-V_N^*)+q_1^P V_N^*}$ and $V_Y = \frac{V_N^* q_1^F}{q_0^F(1-V_N^*)+q_1^F V_N^*}$.

Let us find the conditions on C for which $EP_{XY} > \max EP_{(v=1)}(N)$. Using (13), we have

$$
\begin{aligned}
& q_1^P \frac{V_N^* q_1^P}{q_0^P(1-V_N^*)+q_1^P V_N^*}(1-G(V_N^*)-C) + q_1^F \frac{V_N^* q_1^F}{q_0^F(1-V_N^*)+q_1^F V_N^*}(1-G(V_N^*)-C) > V_N^*(1-G(V_N^*)) \\
& \Leftrightarrow \left(\frac{(q_1^P)^2}{q_0^P(1-V_N^*)+q_1^P V_N^*} + \frac{(q_1^F)^2}{q_0^F(1-V_N^*)+q_1^F V_N^*}\right)(1-G(V_N^*)-C) > 1-G(V_N^*) \\
& \Leftrightarrow \sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*}(1-G(V_N^*)) + \sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*}(-C) > 1-G(V_N^*) \\
& \Leftrightarrow \sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*}(1-G(V_N^*)) - (1-G(V_N^*)) > C\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*} \\
& \Leftrightarrow \left(\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*} - 1\right)(1-G(V_N^*)) > C\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*}. \qquad (15)
\end{aligned}
$$

From the proof of theorem 3.1, we have $\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*} > 1$ and so, $\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*} - 1 > 0$. By definition of G and V_N, $1-G(V_N^*)$ is strictly positive.

From (15), we can thus conclude that a sufficient condition on C for a company to get a certification is

$$
C < \left(\frac{\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*} - 1}{\sum_d \frac{(q_1^d)^2}{q_0^d(1-V_N^*)+q_1^d V_N^*}}\right)(1-G(V_N^*)), \qquad (16)
$$

which is equivalent to the threshold shown in Theorem 4.1, which ends the proof.

To conclude, we can define a level for C above which getting certified is not optimal anymore, which is dependent on the probabilities of success and failure of the certification, V_N^*, the price maximizing the issuer's revenue if there is no certification and $1-G(V_N^*)$, the proportion of buyers in the case of no certification, determined by G, the cumulative distribution function of private prior beliefs. □

In the specific case where $C = 0$, we can confirm that the condition found for Theorem 4.1 corresponds to the findings for Theorem 3.1. If C=0, we know from Section 3 that by definition of G, $1-G(V_N^*) > 0$. Moreover, from the proof of 3.1, we know that $\sum_d \frac{(q_1^d)^2}{(q_0^d(1-V_N^*)+q_1^d V_N^*)} > 1$ so we have $\frac{1}{\sum_d \frac{(q_1^d)^2}{(q_0^d(1-V_N^*)+q_1^d V_N^*)}} < 1$. Thus, we have $\left(1 - \frac{1}{\sum_d \frac{(q_1^d)^2}{(q_0^d(1-V_N^*)+q_1^d V_N^*)}}\right) > 0$. Overall, the right-hand side of the inequality (16) is always strictly positive. Our result for Theorem 3.1 holds. If C=0, requesting a certification is always optimal.

4.2 Necessary and sufficient condition with uniform prior beliefs

Theorem 4.2. *In the situation where $C > 0$ and potential buyers have no prior information about the issuer, there exists a level of certification cost above which asking for certification is not optimal anymore defined as:*

$$C < \frac{-(\overline{\pi})^2(q_1^P\pi + q_0^P(1-\pi))(q_1^F\pi + q_0^F(1-\pi))}{4(\overline{\pi}-\underline{\pi})\pi((q_1^F)^2(q_1^P\pi + q_0^P(1-\pi)) + (q_1^P)^2(q_1^F\pi + q_0^F(1-\pi)))} + 1 \tag{17}$$

Proof. From (13), we have

$$\begin{aligned} EP_{(v=1)}(T) = q_1^P V_{TP}&\left(1 - G\left(\frac{V_{TP}q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}\right) - C\right) \\ &+ q_1^F V_{TF}\left(1 - G\left(\frac{V_{TF}q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}\right) - C\right). \end{aligned}$$

As per (14), it is optimal to be certified if and only if

$$\max EP_{(v=1)}(T) > \max EP_{(v=1)}(N) = V_N^*(1 - G(V_N^*)).$$

We define V_{TP}^* and V_{TF}^* the V_{TP} and V_{TP} that maximize $EP_{(v=1)}(T)$.

In this section, we assume the prior beliefs have a uniform distribution over $[\underline{\pi}, \overline{\pi}]$, with $G(\pi)$ its cumulative distribution function over this interval (refer to Fig. 3 for illustration). From (13) and (14), it is optimal to ask for a certification if V_{TP}^* and V_{TF}^* satisfy:

$$\begin{aligned} & q_1^P V_{TP}^*\left(1 - G\left(\frac{V_{TP}^*q_0^P}{q_1^P(1-V_{TP}^*) + q_0^P V_{TP}^*}\right) - C\right) + q_1^F V_{TF}^*\left(1 - G\left(\frac{V_{TF}^*q_0^F}{q_1^F(1-V_{TF}^*) + q_0^F V_{TF}^*}\right) - C\right) \\ &\qquad > V_N^*(1 - G(V_N^*)) \\ &\Leftrightarrow -q_1^P V_{TP}^* C - q_1^F V_{TF}^* C > V_N^*(1 - G(V_N^*)) - q_1^P V_{TP}^*\left(1 - G\left(\frac{V_{TP}^*q_0^P}{q_1^P(1-V_{TP}^*) + q_0^P V_{TP}^*}\right)\right) \\ &\qquad - q_1^F V_{TF}^*\left(1 - G\left(\frac{V_{TF}^*q_0^F}{q_1^F(1-V_{TF}^*) + q_0^F V_{TF}^*}\right)\right) \\ &\Leftrightarrow C < \frac{V_N^*(1 - G(V_N^*)) - q_1^P V_{TP}^*\left(1 - G\left(\frac{V_{TP}^*q_0^P}{q_1^P(1-V_{TP}^*) + q_0^P V_{TP}^*}\right)\right) - q_1^F V_{TF}^*\left(1 - G\left(\frac{V_{TF}^*q_0^F}{q_1^F(1-V_{TF}^*) + q_0^F V_{TF}^*}\right)\right)}{-q_1^F V_{TF}^* - q_1^P V_{TP}^*}. \end{aligned} \tag{18}$$

For simplicity, let us call $A = \frac{V_{TP}^*q_0^P}{q_1^P(1-V_{TP}^*) + q_0^P V_{TP}^*}$ and $B = \frac{V_{TF}^*q_0^F}{q_1^F(1-V_{TF}^*) + q_0^F V_{TF}^*}$ in the remainder of this proof.

Using the fact that G follows a uniform distribution over $[\underline{\pi}, \overline{\pi}]$, we have

$$C < \frac{V_N^*\left(1 - \frac{V_N^* - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right) - q_1^P V_{TP}^*\left(1 - \frac{A - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right) - q_1^F V_{TF}^*\left(1 - \frac{B - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right)}{-q_1^F V_{TF}^* - q_1^P V_{TP}^*}. \tag{19}$$

We optimize first on V_N^*. We differentiate $f(x) = x\left(1 - \frac{x - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right)$ and have f'(x)=0 for $x = \frac{\overline{\pi}}{2}$.

It follows that

$$\max EP_{(v=1)}(N) = V_N^*(1 - G(V_N^*)) = \frac{\overline{\pi}}{2}\left(1 - \frac{\frac{\overline{\pi}}{2} - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right) = \frac{\overline{\pi}}{2}\left(\frac{\frac{\overline{\pi}}{2}}{\overline{\pi} - \underline{\pi}}\right) = \frac{\overline{\pi}^2}{4(\overline{\pi} - \underline{\pi})}. \tag{20}$$

From (19),

$$\begin{aligned} C &< \frac{\frac{\overline{\pi}^2}{4(\overline{\pi}-\underline{\pi})} - q_1^P V_{TP}^* \left(1 - \frac{A-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right) - q_1^F V_{TF}^* \left(1 - \frac{B-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)}{-q_1^F V_{TF}^* - q_1^P V_{TP}^*} \\ \Leftrightarrow C &< \frac{\frac{-\overline{\pi}^2}{4(\overline{\pi}-\underline{\pi})} + q_1^P V_{TP}^* \left(1 - \frac{A-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right) + q_1^F V_{TF}^* \left(1 - \frac{B-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)}{q_1^F V_{TF}^* + q_1^P V_{TP}^*} \\ \text{with } A &= \frac{V_{TP}^* q_0^P}{q_1^P(1 - V_{TP}^*) + q_0^P V_{TP}^*} \text{and } B = \frac{V_{TF}^* q_0^F}{q_1^F(1 - V_{TF}^*) + q_0^F V_{TF}^*} \end{aligned} \tag{21}$$

Now, we study the maximum for A and B. The reasoning is the same for both. We use the equivalences (6) and (8) between V_{Td} and π.

$$q_1^d V_{Td}^* \left(1 - \frac{\frac{V_{Td}^* q_0^d}{q_1^d(1-V_{Td}^*)+q_0^d V_{Td}^*} - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right) = q_1^d \frac{\pi_d^* q_1^d}{\pi_d^* q_1^d + q_0^d(1 - \pi_d^*)} \left(1 - \frac{\pi_d^* - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right) \tag{22}$$

We define f the function defined over $[\underline{\pi}, \overline{\pi}]$ such as:

$$\begin{aligned} f(\pi) &= q_1^d \frac{\pi q_1^d}{\pi q_1^d + q_0^d(1 - \pi)} \left(1 - \frac{\pi - \underline{\pi}}{\overline{\pi} - \underline{\pi}}\right) \\ \Leftrightarrow f(\pi) &= \frac{\pi (q_1^d)^2}{\pi q_1^d + q_0^d(1 - \pi)} \left(\frac{\overline{\pi} - \pi}{\overline{\pi} - \underline{\pi}}\right) \\ \Leftrightarrow f(\pi) &= \frac{\pi (q_1^d)^2 (\overline{\pi} - \pi)}{(\pi q_1^d + q_0^d(1 - \pi))(\overline{\pi} - \underline{\pi})} \end{aligned} \tag{23}$$

We differentiate on π and get:

$$\begin{aligned} f'(\pi) &= \frac{\left((q_1^d)^2\overline{\pi} - 2(q_1^d)^2\pi\right)\left(\pi q_1^d + q_0^d(1 - \pi)\right) - (q_d^1)^2\pi(\overline{\pi} - \pi)(q_1^d - q_0^d)}{\left(\pi q_1^d + q_0^d(1 - \pi)\right)^2 (\overline{\pi} - \underline{\pi})} \\ \Leftrightarrow f'(\pi) &= \frac{\pi^2 \left(q_0^d(q_1^d)^2 - (q_1^d)^3\right) + \pi\left(-2(q_1^d)^2 q_0^d\right) + q_0^d(q_1^d)^2\overline{\pi}}{\left(\pi q_1^d + q_0^d(1 - \pi)\right)^2 (\overline{\pi} - \underline{\pi})} \end{aligned} \tag{24}$$

By definition, we know that $0 < \underline{\pi} \leqslant \pi \leqslant \overline{\pi} < 1$ and $q_1^P - q_0^P = q_0^F - q_1^F = \kappa > 0$, so we have:

$$\begin{gathered} \overline{\pi} - \underline{\pi} > 0 \\ q_1^P > q_0^P \text{so even if } q_0^P = 0, \text{ we have } q_1^P > 0 \\ \text{and similarly, } q_0^F > q_1^F \text{so even if } q_1^F = 0, \text{ we have } q_0^F > 0 \end{gathered}$$

and we can conclude that $\forall\pi \in [\underline{\pi};\overline{\pi}]$, $\left(\pi q_1^d + q_0^d(1-\pi)\right)^2(\overline{\pi}-\underline{\pi}) > 0$. The sign of $f\prime(\pi)$ depends on its numerator. We find

$$\Delta = 4\left((q_1^d)^4(q_0^d)^2 - \overline{\pi}(q_0^d)^2(q_1^d)^4 + q_0^d(q_1^d)^5\overline{\pi}\right).$$

Given that $\overline{\pi} < 1$ and $q_v^d \geqslant 0$, we have $\overline{\pi}(q_0^d)^2(q_1^d)^4 \leqslant 1(q_0^d)^2(q_1^d)^4$. Overall, $\Delta > 0$ and f' has two roots noted

$$x = \frac{(q_1^d)^2 q_0^d \pm \sqrt{\left((q_1^d)^4(q_0^d)^2 - \overline{\pi}(q_1^d)^4(q_0^d)^2 + q_0^d(q_1^d)^5\overline{\pi}\right)}}{\left(q_0^d(q_1^d)^2 - (q_1^d)^3\right)}$$

We know that $q_1^p > q_0^P$ and $q_0^F > q_1^F$ by definition. The positive root for $d = P$, denoted π_P^* is

$$\pi_P^* = \frac{(q_1^P)^2 q_0^P - \sqrt{(q_1^P)^4(q_0^P)^2 - \overline{\pi}(q_1^P)^4(q_0^P)^2 + q_0^P(q_1^P)^5\overline{\pi}}}{\left(q_0^P(q_1^P)^2 - (q_1^P)^3\right)} \tag{25}$$

and the root for $d = F$, denoted π_F^*, is

$$\pi_F^* = \frac{(q_1^F)^2 q_0^F + \sqrt{\left((q_1^F)^4(q_0^F)^2 - \overline{\pi}(q_1^F)^4(q_0^F)^2 + q_0^F(q_1^F)^5\overline{\pi}\right)}}{\left(q_0^F(q_1^F)^2 - (q_1^F)^3\right)} \tag{26}$$

Using the equivalences (6) and (8) between V_{Td} and π, we can convert (25) and (26) into V_{Td}^*:

$$V_{TP}^* = \frac{(q_1^P)^3 q_0^P - q_1^P\sqrt{(q_1^P)^4(q_0^P)^2 - \overline{\pi}(q_1^P)^4(q_0^P)^2 + q_0^P(q_1^P)^5\overline{\pi}}}{(q_1^P)2q_0^P - (q_1^P)^3 - (q_1^P)2(q_0^P)^2 + (q_1^P)^3 q_0^P + (q_0^P - q_1^P)\sqrt{(q_1^P)^4(q_0^P)^2 - \overline{\pi}(q_1^P)^4(q_0^P)^2 + q_0^P(q_1^P)^5\overline{\pi}}} \tag{27}$$

and

$$V_{TF}^* = \frac{(q_1^F)^3 q_0^F + q_1^F\sqrt{(q_1^F)^4(q_0^F)^2 - \overline{\pi}(q_1^F)^4(q_0^F)^2 + q_0^F(q_1^F)^5\overline{\pi}}}{(q_1^F)2q_0^F - (q_1^F)^3 - (q_1^F)2(q_0^F)^2 + (q_1^F)^3 q_0^F + (q_1^F - q_0^F)\sqrt{(q_1^F)^4(q_0^F)^2 - \overline{\pi}(q_1^F)^4(q_0^F)^2 + q_0^F(q_1^F)^5\overline{\pi}}} \tag{28}$$

As stated in section 2.1, the firm choosing to be tested knows the distribution of the buyers' private beliefs, defined by π and $G(\pi)$.

Moreover, the optimal V_{TF} and V_{TP} are equal to the posterior beliefs of the buyers after the certification results are known, π_F^* and π_P^*.

From (5),we can express π^* as a function of π. In this case, we have: $V_{TF} = \pi_F^* = \frac{q_1^F\pi}{q_1^F\pi + q_0^F(1-\pi)}$ and $V_{TP} = \pi_F^* = \frac{q_1^P\pi}{q_1^P\pi + q_0^P(1-\pi)}$.

Finally, it follows from (4.2) that we have a necessary and sufficient condition on C, which can be expressed as a function of π, the prior belief of buyers in the setting where these beliefs are uniformly distributed over $[\underline{\pi},\overline{\pi}]$. Rewriting A and B based on this leads to $A = \pi$ and

$B = \pi$ and the necessary and sufficient condition on C can be expressed as:

$$C < \frac{\frac{-\overline{\pi}^2}{4(\overline{\pi}-\underline{\pi})} + \frac{(q_1^P)^2\pi}{q_1^P\pi+q_0^P(1-\pi)}(1-\frac{\pi-\underline{\pi}}{\overline{\pi}-\underline{\pi}}) + \frac{(q_1^F)^2\pi}{q_1^F\pi+q_0^F(1-\pi)}(1-\frac{\pi-\underline{\pi}}{\overline{\pi}-\underline{\pi}})}{\frac{(q_1^F)^2\pi}{q_1^F\pi+q_0^F(1-\pi)} + \frac{(q_1^P)^2}{q_1^P\pi+q_0^P(1-\pi)}}$$

$$\Leftrightarrow C < \frac{-(\overline{\pi})^2(q_1^P\pi + q_0^P(1-\pi))(q_1^F\pi + q_0^F(1-\pi))}{4(\overline{\pi}-\underline{\pi})\pi((q_1^F)^2(q_1^P\pi + q_0^P(1-\pi)) + (q_1^P)^2(q_1^F\pi + q_0^F(1-\pi)))} + 1 \tag{29}$$

□

4.3 Impact of "informative" prior beliefs

Two examples are used to illustrate the case where the buyers have a positive or negative bias on the quality of the green bond (based on the information they can gather on the corporation).

4.3.1 Impact of positive "informative" prior beliefs: G follows an increasing linear distribution

Theorem 4.3. *If $C > 0$ and potential buyers have a positive prior bias towards the issuer, there exists a level of C above which getting certified is not optimal anymore and issuers will choose not to ask for certification defined as:*

$$C < \frac{-\frac{(4\underline{\pi}+\sqrt{D_1})(36(\overline{\pi}-\underline{\pi})^2-(-2\underline{\pi}+\sqrt{D_1})^2)}{216(\overline{\pi}-\underline{\pi})^2} + q_1^P V_{TP}\left(1-\left(\frac{A-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right) + q_1^F V_{TF}\left(1-\left(\frac{B-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right)}{q_1^P V_{TP} + q_1^F V_{TF}} \tag{30}$$

$$\textit{with } A = \frac{V_{TP}q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}} \ \textit{ and } B = \frac{V_{TF}q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}$$

Proof. In this section, we assume that the distribution of the prior beliefs is linear and increasing. We define the density function of the prior beliefs as: $f(x) = \frac{2(x-\underline{\pi})}{(\overline{\pi}-\underline{\pi})^2}$. We integrate the function f to get the cumulative distribution function G, defined as: $G(x) = \left(\frac{x-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2$.

We follow a reasoning similar to section 4.2. From (13), we have

$$EP_{(v=1)}(T) = q_1^P V_{TP}\left(1 - G\left(\frac{V_{TP}q_0^P}{q_1^P(1-V_{TP}) + q_0^P V_{TP}}\right) - C\right)$$
$$+q_1^F V_{TF}\left(1 - G\left(\frac{V_{TF}q_0^F}{q_1^F(1-V_{TF}) + q_0^F V_{TF}}\right) - C\right).$$

As per (14), it is optimal to be certified if and only if

$$\max EP_{(v=1)}(T) > \max EP_{(v=1)}(N) = V_N^*(1-G(V_N^*)).$$

It follows that

$$C < \frac{V_N^*\left(1-\left(\frac{V_N^*-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right) - q_1^P V_{TP}\left(1-\left(\frac{A-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right) - q_1^F V_{TF}\left(1-\left(\frac{B-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right)}{-q_1^F V_{TF} - q_1^P V_{TP}} \tag{31}$$

with $A = \frac{V_{TP} q_0^P}{q_1^P(1-V_{TP})+q_0^P V_{TP}}$ and $B = \frac{V_{TF} q_0^F}{q_1^F(1-V_{TF})+q_0^F V_{TF}}$.

We optimize this inequality on V_N^*. We differentiate

$$f(x) = x(1-(\frac{x-\underline{\pi}}{\overline{\pi}-\underline{\pi}})^2) \Leftrightarrow f(x) = \frac{-x^3+2x^2\underline{\pi}+(\overline{\pi}^2-2\overline{\pi}\underline{\pi})x}{(\overline{\pi}-\underline{\pi})^2}$$
$$\Leftrightarrow f'(x) = \frac{-3}{(\overline{\pi}-\underline{\pi})^2}x^2 + \frac{4\underline{\pi}}{(\overline{\pi}-\underline{\pi})^2}x + \frac{\overline{\pi}^2-2\underline{\pi}\overline{\pi}}{(\overline{\pi}-\underline{\pi})^2} \tag{32}$$

and we solve $f'(x) = 0$ for this trinome, with

$$\Delta = \frac{16\underline{\pi}^2+12\overline{\pi}^2-24\underline{\pi}\overline{\pi}}{(\overline{\pi}-\underline{\pi})^4}.$$

We now study the sign of the numerator of Δ to determine the number of roots for this trinome.

$$16\underline{\pi}^2+12\overline{\pi}^2-24\underline{\pi}\overline{\pi} = (4\underline{\pi})^2+(3\overline{\pi})^2+3\overline{\pi}^2-2*4*3\underline{\pi}\overline{\pi} = (4\underline{\pi}-3\overline{\pi})^2+3\overline{\pi}^2$$

Given that $\overline{\pi} > \underline{\pi} > 0$, the numerator of Δ is strictly positive. Thus, $f'(X) = 0$ has two roots and given that V_N^* is a price, we focus only on the positive one:

$$x = \left(\frac{4\underline{\pi}}{(\overline{\pi}-\underline{\pi})^2} + \sqrt{\frac{16\underline{\pi}^2+12\overline{\pi}^2-24\underline{\pi}\overline{\pi}}{(\overline{\pi}-\underline{\pi})^4}}\right)\frac{(\overline{\pi}-\underline{\pi})^2}{6} = \frac{4\underline{\pi}+\sqrt{16\underline{\pi}^2+12\overline{\pi}^2-24\underline{\pi}\overline{\pi}}}{6}.$$

For simplicity in the following developments, we note $D_1 = 16\underline{\pi}^2+12\overline{\pi}^2-24\underline{\pi}\overline{\pi}$. Given this root, the maximum of $V_N^*\left(1-\left(\frac{V_N^*-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right)$ is equal to

$$\begin{aligned} V_N^*\left(1-\left(\frac{V_N^*-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right) &= \frac{4\underline{\pi}+\sqrt{D_1}}{6}\left(1-\left(\frac{-2\underline{\pi}+\sqrt{D_1}}{6(\overline{\pi}-\underline{\pi})}\right)^2\right) \\ &= \frac{4\underline{\pi}+\sqrt{D_1}}{6}\left(\frac{36(\overline{\pi}-\underline{\pi})^2-(-2\underline{\pi}+\sqrt{D_1})^2)}{36(\overline{\pi}-\underline{\pi})^2}\right) \\ &= \frac{(4\underline{\pi}+\sqrt{D_1})(36(\overline{\pi}-\underline{\pi})^2-(-2\underline{\pi}+\sqrt{D_1})^2)}{216(\overline{\pi}-\underline{\pi})^2}. \end{aligned} \tag{33}$$

From (33), in the case where $G(\pi)$ follows an increasing linear distribution on $[\underline{\pi}, \overline{\pi}]$, getting certified is optimal for a company issuing a bond if and only if

$$C < \frac{-\frac{(4\underline{\pi}+\sqrt{D_1})(36(\overline{\pi}-\underline{\pi})^2-(-2\underline{\pi}+\sqrt{D_1})^2)}{216(\overline{\pi}-\underline{\pi})^2} + q_1^P V_{TP}\left(1-\left(\frac{A-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right) + q_1^F V_{TF}\left(1-\left(\frac{B-\underline{\pi}}{\overline{\pi}-\underline{\pi}}\right)^2\right)}{q_1^P V_{TP} + q_1^F V_{TF}}. \tag{34}$$

□

4.3.2 Impact of negative "informative" prior beliefs: G follows a decreasing linear distribution

Theorem 4.4. *If $C > 0$ and potential buyers have a negative prior bias towards the issuer, there exists a level of C above which getting certified is not optimal anymore and issuers will* *choose not to ask for certification defined as:*

$$C < \frac{-\frac{(4\overline{\pi}+\sqrt{D_2})\left(8(3\underline{\pi}^2+\overline{\pi}^2-6\underline{\pi}\overline{\pi})-4\overline{\pi}\sqrt{D_2}\right)}{216(\overline{\pi}-\underline{\pi})^2} + q_1^P V_{TP}\left(1-\frac{2\overline{\pi}A-A^2}{(\overline{\pi}-\underline{\pi})^2}\right) + q_1^F V_{TF}\left(1-\frac{2\overline{\pi}B-B^2}{(\overline{\pi}-\underline{\pi})^2}\right)}{q_1^P V_{TP} + q_1^F V_{TF}} \tag{35}$$

$$\textit{with } A = \frac{V_{TP}q_0^P}{q_1^P(1-V_{TP})+q_0^P V_{TP}} \textit{ and } B = \frac{V_{TF}q_0^F}{q_1^F(1-V_{TF})+q_0^F V_{TF}}$$

Proof. For this subsection, we assume now that the distribution of the prior beliefs is linear and decreasing over $[\underline{\pi},\overline{\pi}]$. Under this assumption, we define the density function of the prior beliefs as: $f(x) = \frac{2(\overline{\pi}-x)}{(\overline{\pi}-\underline{\pi})^2}$. We integrate the function f to get the cumulative distribution function G, defined as

$$G(x) = \frac{2\overline{\pi}x - x^2}{(\overline{\pi}-\underline{\pi})^2}.$$

We follow a reasoning similar to section 4.3.1. From (13) and (14), we have

$$C < \frac{-V_N^*\left(1-\frac{2\overline{\pi}V_N^*-V_N^{*2}}{(\overline{\pi}-\underline{\pi})^2}\right) + q_1^P V_{TP}\left(1-\frac{2\overline{\pi}A-A^2}{(\overline{\pi}-\underline{\pi})^2}\right) + q_1^F V_{TF}\left(1-\frac{2\overline{\pi}B-B^2}{(\overline{\pi}-\underline{\pi})^2}\right)}{q_1^F V_{TF} + q_1^P V_{TP}} \tag{36}$$

with $A = \frac{V_{TP}q_0^P}{q_1^P(1-V_{TP})+q_0^P V_{TP}}$ and $B = \frac{V_{TF}q_0^F}{q_1^F(1-V_{TF})+q_0^F V_{TF}}$.

We optimize this inequality on V_N^*.

We differentiate

$$\begin{aligned} f(x) = x\left(1-\frac{2\overline{\pi}x-x^2}{(\overline{\pi}-\underline{\pi})^2}\right) &\Leftrightarrow f(x) = \frac{x^3-2x^2\overline{\pi}}{(\overline{\pi}-\underline{\pi})^2} + x \\ &\Leftrightarrow f'(x) = \frac{3}{(\overline{\pi}-\underline{\pi})^2}x^2 - \frac{4\overline{\pi}}{(\overline{\pi}-\underline{\pi})^2}x + 1 \end{aligned} \tag{37}$$

and we solve $f'(x) = 0$ for this trinome

$$\Delta = \frac{4\overline{\pi}^2 - 12\underline{\pi}^2 + 24\underline{\pi}\overline{\pi}}{(\overline{\pi}-\underline{\pi})^4}.$$

We now study the sign of the numerator of Δ to determine the number of roots for this trinome. From the development of Δ, we know that

$$4\overline{\pi}^2 - 12\underline{\pi}^2 + 24\underline{\pi}\overline{\pi} = 16\overline{\pi}^2 - 12(\overline{\pi}-\underline{\pi})^2$$

We know that $\overline{\pi} > \underline{\pi} > 0$, so $\overline{\pi} - \underline{\pi} > 0$ and

$$\overline{\pi} > \overline{\pi} - \underline{\pi} \Leftrightarrow \overline{\pi}^2 > (\overline{\pi}-\underline{\pi})^2 \Leftrightarrow 12\overline{\pi}^2 > 12(\overline{\pi}-\underline{\pi})^2 \Leftrightarrow 16\overline{\pi}^2 > 12(\overline{\pi}-\underline{\pi})^2$$

This proves that the numerator of Δ is strictly positive. Overall, $f'(x) = 0$ has two roots and given that V_N^* is a price, we focus only on the positive one:

$$x = \left(\frac{4\overline{\pi}}{(\overline{\pi}-\underline{\pi})^2} + \sqrt{\frac{4\overline{\pi}^2 - 12\underline{\pi}^2 + 24\underline{\pi}\overline{\pi}}{(\overline{\pi}-\underline{\pi})^4}}\right)\frac{(\overline{\pi}-\underline{\pi})^2}{6} = \frac{4\overline{\pi} + \sqrt{4\overline{\pi}^2 - 12\underline{\pi}^2 + 24\underline{\pi}\overline{\pi}}}{6}.$$

For simplicity in the following development, we note $D_2 = 4\overline{\pi}^2 - 12\underline{\pi}^2 + 24\underline{\pi}\overline{\pi}$. Given this root, the maximum of $V_N^*\left(1 - \frac{2\overline{\pi}V_N^* - V_N^{*2}}{(\overline{\pi}-\underline{\pi})^2}\right)$ is equal to

$$V_N^*\left(1 - \frac{2\overline{\pi}V_N^* - V_N^{*2}}{(\overline{\pi}-\underline{\pi})^2}\right) = \frac{4\overline{\pi} + \sqrt{D_2}}{6}\left(1 - \left(\frac{2\overline{\pi}\frac{(4\overline{\pi}+\sqrt{D_2})}{6} - (\frac{4\overline{\pi}+\sqrt{D_2}}{6})^2}{(\overline{\pi}-\underline{\pi})^2}\right)\right)$$
$$= \frac{4\overline{\pi} + \sqrt{D_2}}{6}\left(\frac{8(3\underline{\pi}^2 + \overline{\pi}^2 - 6\underline{\pi}\overline{\pi}) - 4\overline{\pi}\sqrt{D_2}}{36(\overline{\pi}-\underline{\pi})^2}\right). \quad (38)$$

From (38), in the case where $G(\pi)$ follows a decreasing linear distribution on $[\underline{\pi}, \overline{\pi}]$, getting certified is optimal for a company issuing a bond if and only if

$$C < \frac{-\frac{(4\overline{\pi}+\sqrt{D_2})\left(8(3\underline{\pi}^2+\overline{\pi}^2-6\underline{\pi}\overline{\pi})-4\overline{\pi}\sqrt{D_2}\right)}{216(\overline{\pi}-\underline{\pi})^2} + q_1^P V_{TP}\left(1 - \frac{2\overline{\pi}A-A^2}{(\overline{\pi}-\underline{\pi})^2}\right) + q_1^F V_{TF}\left(1 - \frac{2\overline{\pi}B-B^2}{(\overline{\pi}-\underline{\pi})^2}\right)}{q_1^P V_{TP} + q_1^F V_{TF}}. \quad (39)$$

□

5 Conclusion and further directions

In this paper, we aim at understanding the decision rationale of green bond issuers regarding certification by focusing on the cost of the certification as a core parameter influencing decisions. Similar to what Gill and Sgroi (2012) prove in the case of pre-launch product review, we find that if the certification cost is null, requesting a certification is always optimal for the issuers, no matter the outcome of the process. However, a strictly positive cost changes and nuances this finding. Indeed, in this case, we find that there is a threshold above which the certification cost becomes too high and getting certified is not optimal anymore. We moreover find that this threshold depends on the distribution of the prior beliefs that potential buyers have about the commitment of the issuers to their "green" claim before the issuance announcement and certification process. Our last numerical example notably hints at the fact that the more negative these beliefs are, the higher the acceptable certification cost for the issuers.

This last point is one of the element which we will develop in the next steps of our research. Indeed, one of the main limits of this paper is that our models are constrained by the assumptions we take. We will develop the role and impact of the private prior beliefs on our results in a more general context. Future research also include the study of the effect of risk aversion. Indeed, we hypothesized that potential buyers are risk neutral. This might be a strong assumption and it will be interesting to see the effect of having risk-averse potential buyers on our results. We expect it to be possible to study it at least numerically. Another parameter that we will work on is the toughness of the certification. Several types of certification are being offered on the markets nowadays. Some are closer to a "one-off" static analysis of the bond issuance and the claimed use of the proceeds. Others however are dynamic, with analysis at several points in time, notably after the bond has been issued, to control the use of the proceeds. One of the research questions we will tackle is to determine the optimal level of toughness. Another question closely related is to understand the effect of the certification toughness on the acceptable cost level. In this paper, we have modelled the certification cost as a percentage of the total issued amount based on the information we found about the practices in the business world. However, we were only able to find the pricing information for one certification agency, the CBI. It would be interesting to model the certification cost differently, notably using the toughness of the certification process, and see how our results adapt to various toughness levels. To complete our understanding of the significance of the certification cost, we will reinforce our findings through a sensitivity analysis to confirm our results. We also want to confirm them through an empirical perspective. Our focus here will be to investigate if and how avoiding certification is indeed less than optimal for issuers on the markets. This could notably bring elements to shed light on the sometimes conflicting results found by previous research on green bonds and their premium.

By focusing on the certification cost as a central parameter that drives the decisions of green bond issuers, we contribute to several strands of literature: the research on green bonds, on certification and on signalling. Moreover, this topic is also significant from a managerial perspective, given how important financing decisions are to the success of a company. Our findings shed light on how to measure the impact of a certification for new green bonds and to which extent it remains optimal to request it.

References

Bachelet, M. J., L. Becchetti, and S. Manfredonia (2019): "The green bonds premium puzzle: The role of issuer characteristics and third-party verification," *Sustainability*, 11(4), 1098.

Bonroy, O., and C. Constantatos (2015): "On the economics of labels: how their introduction affects the functioning of markets and the welfare of all participants," *American Journal of Agricultural Economics*, 97(1), 239–259.

Chen, L., and Y. Papanastasiou (2021): "Seeding the herd: Pricing and welfare effects of social learning manipulation," *Management Science*, 67(11), 6734–6750.

Daubanes, J. X., S. F. Mitali, and J.-C. Rochet (2021): "Why Do Firms Issue Green Bonds?," *Swiss Finance Institute Research Paper*, 21-97.

Ehlers, T., and F. Packer (2017): "Green bond finance and certification," *BIS Quarterly Review September*.

Flammer, C. (2021): "Corporate green bonds," *Journal of Financial Economics*, 142(2), 499–516.

Gianfrate, G., and M. Peri (2019): "The green advantage: Exploring the convenience of issuing green bonds," *Journal of cleaner production*, 219, 127–135.

Gill, D., and D. Sgroi (2012): "The optimal choice of pre-launch reviewer," *Journal of Economic Theory*, 147(3), 1247–1260.

Hachenberg, B., and D. Schiereck (2018): "Are green bonds priced differently from conventional bonds?," *Journal of Asset Management*, 19(6), 371–383.

Hvide, H. K., et al. (2009): "Oligopolistic certification," *BE Journal of Theoretical Economics*, 9(1), 20–35.

Immel, M., B. Hachenberg, F. Kiesel, and D. Schiereck (2021): "Green bonds: shades of green and brown," *Journal of Asset Management*, 22(2), 96–109.

Jiang, B., and B. Yang (2019): "Quality and pricing decisions in a market with consumer information sharing," *Management Science*, 65(1), 272–285.

Kapraun, J., C. Latino, C. Scheins, and C. Schlag (2021): "(In)-credibly green: which bonds trade at a green bond premium?," in *Proceedings of Paris December 2019 Finance Meeting EUROFIDAI-ESSEC*.

Ki, H., and J.-Y. Kim (2022): "Sell green and buy green: A signaling theory of green products," *Resource and Energy Economics*, 67, 101266.

Lizzeri, A. (1999): "Information revelation and certification intermediaries," *The RAND Journal of Economics*, pp. 214–231.

Mathis, J., J. McAndrews, and J.-C. Rochet (2009): "Rating the raters: Are reputation concerns powerful enough to discipline rating agencies?," *Journal of monetary economics*, 56(5), 657–674.

Mezzetti, C., and T. Tsoulouhas (2000): "Gathering information before signing a contract with a privately informed principal," *International Journal of Industrial Organization*, 18(4), 667–689.

Rysman, M., T. Simcoe, and Y. Wang (2020): "Differentiation Strategies in the Adoption of Environmental Standards: LEED from 2000 to 2014," *Management Science*, 66(9), 4173–4192.

Skreta, V., and L. Veldkamp (2009): "Ratings shopping and asset complexity: A theory of ratings inflation," *Journal of Monetary Economics*, 56(5), 678–695.

Smith, L., and P. N. Sorensen (2006): "Informational herding and optimal experimentation," *Working Paper*.

Tang, D. Y., and Y. Zhang (2020): "Do shareholders benefit from green bonds?," *Journal of Corporate Finance*, 61, 101427.

Zerbib, O. D. (2017): "The green bond premium," *Available at SSRN 2890316*.