Exclusive Portfolio Trading in OTC Markets*

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Abstract

We develop a novel theoretical framework that rationalizes exclusive portfolio trading in over-the-counter (OTC) markets where broker-dealers intermediate between sellers and buyers. With such an agreement, a seller grants a selected broker-dealer exclusive access to her entire portfolio of securities. We show that, when broker-dealers have an initially uncertain security demand, sellers face a trade-off between *ex ante* competition over the entire portfolio but *ex post* reduced purchases, and higher *ex post* competition only in those securities where two or more broker-dealers realize a positive demand. Consequently, sellers with access to few (many) broker-dealers prefer the former (latter). On a seller-level, exclusive access agreements reduce trade frequency by one-third. To obtain the aggregate market inefficiency, we structurally estimate the model for the European securities lending market. We show that exclusive security lending agreements reduce trading volume by up to €80.1 bn.

Key Words: Exclusive Dealing; Over-the-Counter Trading, Market (In-)Efficiency
JEL Classification: G14; G24; D43; D86

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1. Introduction

Almost all security classes are either partially or entirely traded over-the-counter (OTC). The opaque and decentralized nature of OTC markets has sparked a rich literature investigating who trades when and what with whom and under which conditions. Systematically overlooked in existing works is the wide presence exclusive dealing on portfolio level. For example, in the EU-based OTC security lending market 35% of lenders grant a single broker-dealer exclusive access to their equity portfolio (Kessler et al., 2022). As we show in this paper, exclusive access agreements are anti-competitive in nature and reduce trading volume by one-third on a portfolio-holder level. For the EU-based security lending market in particular, this aggregates to up to €80.1 bn in foregone trading.

The core contribution of this paper is the development of a novel 3-period partial equilibrium framework that reveals how the presence of exclusive portfolio access shapes OTC markets. The model insights apply to any OTC market with risk-neutral portfolio-holders selling securities to intermediating broker-dealers with private, independent demand. As ultimate demand is uncertain, the sellers must decide whether to grant exclusive access to the entire portfolio up front or wait until demand realizes for security-by-security competition to take place. Here, we rationalize why and when portfolio-holders grant exclusive access, and how this impacts transaction terms and frequencies.

We theoretically confirm our model environment nests the security lending market and empirically verify that both model inputs and outputs match empirical observations. In a structural estimation exercise, we showcase how the individual impact of exclusive access on trading frequency add up across all sellers to an substantial aggregate market impact in terms of total trading volume. Note that sufficient security lending supply is crucial for short sales to eliminate arbitrage opportunities, thereby ensuring that the law of one price holds in our financial markets. This highlights the practical importance of our paper in understanding potential sources of security mispricing.

The three-period model is populated by a representative, risk-neutral seller with exogenously given access to $N \geq 2$ competing broker-dealers seeking to maximize profits. At $t = 0$, the seller is endowed with a large security portfolio, where each security is of unit size and has a publicly known value. Further, the broker-dealers are each endowed with uncertain ask-prices (buying demand) that will be drawn independently from uniform distributions that are identical on the security level.[1] Thus, broker-dealers are symmetrical *ex ante*. At $t = 1$, before the borrowing demand realizes, the broker-dealers choose whether to compete in offering an exclusive access agreement that specifies a uniform bid-premium paid above security value and a lump-sum transfer. At $t = 2$, the borrowing demand realizes: For each security, each broker-dealer draws an independent ask-price from the same uniform distribution symmetric around the true value. If exclusive access was granted at $t = 1$, the agreement-holding broker-dealer buys all securities with a realized positive bid-ask-spread given the agreed-upon bid-premium. The lump-sum transfer is paid regardless of actual purchase quantities.[2] If no exclusive access was granted, either due to lack of offers or by rejection of such by the seller, the broker-dealers compete on a security-by-security level via second-price auctions.[3] Here, we assume that broker-dealers are unaware of each others' realized ask-prices and, thus, posses private information when submitting their auction bids.[4]

To derive equilibrium prices, transaction quantities, and profits, we apply the notion of termination-proof sub-game perfect Nash equilibrium (SPNE).[5] First, we derive the seller's and broker-dealers' expected profits assuming a separate second-price auction is held for each security at $t = 2$. Next, we derive the competitive uniform bid-premia and lump-sum

[1] Having two or more broker-dealers with separate borrowing demand ensures that we simultaneously capture the core-periphery structure documented in the Data Section 2 and have excess purchase demand for some but not all assets.

[2] This structure of the exclusive access agreement is inspired by the demand-boost theory of exclusive dealing by Calzolari, Denicolò, and Zanchettin, 2020.

[3] This is motivated by the fact that nowadays most most large scale trades is conducted over trading platforms where portfolio holders can announce individual stock level auctions. For security lending, examples of such trading platforms are the BlackRock Security Lending platform, Sharegain and FIS Securities Lending Platform. All three exclusively serve institutional clients

[4] This is in alignment with both Duffie, Malamud, et al., 2014 and Babus and Kondor, 2018, studying the implications of private information for competition in OTC market settings.

[5] The more common concept of renegotiation-proof equilibrium, see e.g. Segal and Michael D. Whinston, 2000, is not applicable in this setting as the lump-sum transfer ensures that one agent's gain results in an equivalent loss for the counterparty.

transfers, and the associated expected profit from exclusive access. We take into account that these auction-profits serve as exclusive access participation constraints for the seller *ex ante*. For the broker-dealers auction profits serve as a termination incentive for the agreement-holding broker-dealer *ex post* after ask-prices realize. Here, we assume that agreements are binding contracts, such that the termination mandates the broker-dealer to compensate the seller for losses. With this mind, we derive the broker-dealers' optimal exclusive access agreement offer and termination strategy.

For a seller with at least three broker-dealer connections, we find the auctions SPNE to be unique and exclusive access is never granted in equilibrium. For $N \geq 4$ the seller always reject exclusive access agreement offers as she can benefit from the increased competition via auctions. For $N = 3$, the seller initially grants exclusive access: Broker-dealers competing over the whole portfolio *ex ante* generate higher seller profits than security-by-security competition by the subset of broker-dealers with realized ask-prices above security value. However, we find that for $N = 3$, the agreement-holder is better off terminating as the termination fees are relatively low. Only for $N = 2$, there exists a SPNE with termination proof exclusive access agreements. Here, the seller strictly prefers the exclusive access while the chosen broker-dealer is indifferent between terminating the agreement or not. For completeness, we show that for $N = 1$, both seller and broker-dealer are indifferent between allocation with exclusive access or monopolistic pricing. In short, only sellers with one or two broker-dealers grant exclusive access.

We confirm that the auction SPNE meets the first-best benchmark: Every security with a positive bid-ask-spread is traded. The same holds for the exclusive access SPNE with $N = 1$. The exclusive access SPNE with $N = 2$, however, is inefficient relative to benchmark as total trading volume is reduced by one-third: Only 50% of the securities in the portfolio are traded while the uniform distribution of asked prices with a mean around true value dictates a 75% trading probability. To predict how this individual inefficiencies aggregate in an actual market, we rely on structural estimations.

For our structural estimation exercise, we utilize the newly available and confidential Security Transactions Financing Regulation (SFTR) data that contains all equity lending contracts entered involving at least one EU-based counterparty. A typical transaction report includes, among others, the counterparty LEIs, stock ISIN, stock quantity, loan value, lending fee (% of loan value), maturity (evergreen versus fixed term) and collateral type (none, cash, collateral basket). Most importantly, each contract comes with a flag indicating whether the trade was covered by an exclusive portfolio access agreement.

We aggregate all trades entered into and maturing in 2022 on a counterparty-pair level and identify five types of market participants[6]: lenders, broker-dealers, borrowers, traders and private clients. For the purpose of this paper, we mainly focus on transactions between lenders (sellers) and broker-dealers.[7] Here, we find that 35% of the lenders have active exclusive access agreements. Additionally, zooming in on transactions between broker-dealers and borrowers, we find that barely more than 4.5% of borrowers engage in exclusive dealing yet 88% engage with only one broker-dealer. Hence, our independent broker-dealer demand assumption seems justified. Without going into too much detail here, we also confirm the model fit by comparing model-implied and observed lending fee distribution of transactions with and without exclusive access.

In a final step, we predict the counterfactual trading volume in a market without any active exclusive agreements by applying a bootstrapping strategy. In simple terms, we randomly assign a share $x \in [0, 1]$ of lenders with exclusive access two broker-dealers in the counterfactual to the auction SPNE and the remaining share $1-x$ is assigned just one broker-dealer. We assign those lenders with two assigned broker-dealers a 50% increase in trading volume, while those with one broker-dealer remain unchanged. Repeating this for $100,000$ bootstraps, we obtain both a predicted increase in total trading volumes and the associated

[6]Due to the relative novelty of the data set, reporting before January 2021 is incomplete. Further, reporting standards changed on January 15$^{\text{th}}$, 2022. For consistency reasons we, thus, limit our sample to trades entered and matured in 2021.

[7]A detailed analysis on the transactions by the other types of agents than lenders, borrowers and broker-dealers can be found in an accompanying market analysis by Kessler et al., 2022.

confidence intervals. For all shares, we predict a significantly larger trading volume for the counterfactual case of prohibiting exclusive agreements. These trading volume gains increase linearly in the share of lenders with two counterparties with a total volume increase of €42.3 bn when $x = 1$. In a realistic upper bound estimate, where each lender with exclusive access gets three broker-dealers in the counterfactual, we find that gains in total volume would even grow to €80.1 bn.

Literature Review Rationalizing security pricing under exclusive access agreements, our paper is foremost an addition to the existing literature on the pricing in OTC markets. The seminal paper by Duffie, Garleanu, et al., 2005, studying pricing in a search-and-matching framework, has since sparked a rich debate on relevant pricing frictions. A full literature review goes beyond the scope of this paper and we focus on the papers closest to us without implying anything as to the relevance of omitted papers. Allowing for a choice between trading under exclusive access versus second-price auctions brings us close to the set-ups described by Babus and Kondor, 2018 and Dugast et al., 2019. Both study dealer choices between bilateral trading and trading via centralized platforms. Novel relative to their papers are both our focus on exclusive access agreements governing bilateral transaction and the introduction of asset portfolios rather than a single representative asset. We confirm their findings that less connected lenders prefer the bilateral option of exclusivity, while more connected lenders prefer the centralized auctions.

Both our model choices regarding the centralized trading platform to host a second-price auction and competition in prices for exclusive access are motivated by the idea that broker-dealers have private information regarding their borrowing demand (Babus and Kondor, 2018; Duffie, Malamud, et al., 2014). Similar to Duffie, Malamud, et al., 2014, we introduce auction pricing after (private) demand uncertainty has realized. They, however, focus on sophisticated traders only. We do not deem it reasonable in our setting to assume that lenders acquire knowledge over time. We, thus, opt to introduce second-price auctions instead of double-auctions to clear the market. In our model, we have additional ex ante uncertainty over broker-dealers private demand. Similar in set-up to Babus and Kondor, 2018, we follow their lead and allow for *ex ante* competition in prices over trading. We, however, abstract from quantity limitations on an individual security level and, rather, introduce price-competition over the entire portfolio in return for exclusive access. Ultimately, these changes allow us to rationalize why why some sellers grant exclusive access.

By explaining why some sellers voluntarily engage with a single broker-dealer we also contribute to the OTC paper network formation literature. Similar to our paper, papers by e.g. Colliard et al., 2021 and Gofman, 2014, take the network structure as exogenously given. While they study inventory costs shape the trading patterns, our focus lies on competitive mechanism. For this purpose, we instead zoom in on a single representative lender and her connected broker-dealers. Here, we let the lender endogenously choose the type of connection: with or without exclusice portfolio access. We, thus, are able to provide a complementary channel to endogenously explain the core-periphery structure of dealer markets to those described by Neklyudov, 2019 and Babus and Parlatore, 2022. Embedding our pricing-mechanism into a full network model is a natural next step and will, hopefully, provide rich new insights.

Via our application exercise, we also complement existing works specifically focusing on security lending transactions, such as for example D'Avolio, 2002 and Duffie, Gârleanu, et al., 2002. The seminal paper by Duffie, Gârleanu, et al., 2002 studies the lending price formation in a search-and-bargaining model, where pessimists are over time matched with both lenders and optimists, thereby being able to short sale. For tractability purposes, they abstract from the role of broker-dealers intermediating between lenders and borrowers, and profiting from the bid-ask-spread. In return, we simplify the interaction between broker-dealers and the ultimate borrowers (pessimists) in our paper. We, thus, take a complementary approach and focus specifically on the interaction between lenders and broker-dealers. We show that exclusive access agreements lead to lower transaction volume *relative* to competitive lending. By structurally estimating such aggregate quantity reduction over all lenders, we can quantify total market inefficiencies due to exclusive access. To the best of our knowledge we are the first to obtain a micro-founded empirical estimate of such inefficiencies. With this, we provide a micro-foundation short sale constraints, whose market impact has behind widely discussed (Asquith et al., 2005; Bai et al., 2005; D'Avolio, 2002; Gutierrez et al., 2018; Nagel, 2005; Nezafat et al., 2017).

Our evidence of substantial exclusive access induced market inefficiencies adds a counter-perspective to the theoretical literature on non-exclusive contracting in financial markets. Here, papers by for example Attar et al., 2014; Biais et al., 2000; Bizer and DeMarzo, 1992; Detragiache et al., 2000; Kahn and Mookherjee, 1998; Pauly, 1978; van Boxtel et al., 2020 either directly show or imply that exclusive financing is not obtainable in equilibrium, yet is desired and efficiency improving. Crucially, we deviate in set-up by assume that exclusive agreements are binding *ex post* in a sense that termination is costly. Other assumptional difference are that we consider: private information of value on the buyer (broker-dealer) rather than seller (lender) side (Biais et al., 2000), simultaneous trades rather than sequential transactions Bizer and DeMarzo, 1992; Detragiache et al., 2000; van Boxtel et al., 2020, non-divisible stocks instead of divisible goods Attar et al., 2011, the absence of moral hazard (Kahn and Mookherjee, 1998; Pauly, 1978). Unifying our model with one or more of the above mentioned alternative settings is a natural next step and, over time, will hopefully lead to a fruitful discussion in this line of research.

Modeling the exclusive access agreements as contracts spanning a portfolio of securities (goods), in limited capacity, relate to the literature on exclusive contracting between retailers and manufacturers (Bernheim and Michael D Whinston, 1998; Calzolari and Denicolò, 2013; Calzolari and Denicolò, 2015; Mathewson and Winter, 1987).[8] Most notably, we deviate from the substitute goods assumption common to this literature. Instead, we consider exclusive contracting over an entire equity portfolio, where each stock poses a distinct good.

[8] See Armstrong and Wright, 2007 for exclusive contracting by two-sided platforms, where the platform does not interpose as an intermediary and therefore does not become the direct counterparty to both the supply and demand side.

Following Calzolari, Denicolò, and Zanchettin, 2020, we allow the broker-dealers to compete by setting both a per-transaction fee and a lump-sum transfers. Considering a multi-product setting, we are able confirm their result that in equilibrium exclusive contracts have a zero lending fee to boost demand and full profit-pass-through via lump-sum transfers. In both our and their paper, this demand-boosting leaves the buyer (broker-dealer) with exclusive rights worse off than in the competitive equilibrium. Here, we would like to conclude by highlighting that in our setting price competition ensures demand boosting *within* an exclusive access agreement but reduces overall trading across the multiple securities (goods). Mostly focused on consumer welfare, this reduction in quantities is of secondary interest in the classic Industrial Organization literature but adds substantial value to the market micro-structure literature concerned with the efficiency of financial markets.

2. Model Environment

The model is populated by a representative, risk-neutral seller that is endowed with a security portfolio, where each security has an independent (net-present) value. Further, there exist N for-profit broker-dealers, each endowed with an independent and uncertain demand for each security in the portfolio.[9] In a first period, and before security demand realizes, the broker-dealers may compete to enter an agreement with the seller that grants them exclusive portfolio access. An exclusive access agreement specifies a uniform bid-premium paid above value for every security purchase and a lump-sum transfer paid independently of traded quantities. In a second period, the broker-dealers each draw independent ask-prices for each security from an identical uniform distribution with mean equal to the seller's values. If an exclusive agreement was granted, the chosen broker-dealer purchases all securities with a positive bid-ask-spread from the seller's portfolio. If no agreement was entered, broker-dealers bid via second-price auctions on a security level to purchase those securities where

[9] This is a reduced form approach to each broker-dealer having a unique set of captive buyers sourcing the asset via them.

they observe above value ask-prices.

Table 1: Model Timing

	Broker-Dealers	Seller
$t = 0$	• Endowed with independent and uncertain demand for each security.	• Endowed with security portfolio.
$t = 1$	• Anticipate their security demand. • Offer competitive exclusive access agreements: – Uniform bid-premium above value. – Lump-sum transfer.	• Anticipates profits with and without exclusive access. • Decides whether to enter an agreement or reject all offers.
$t = 2$	• Demand uncertainty realizes. • With exclusive access agreement: – Holder buys securities with positive bid-ask-spread and pays lump-sum transfer. – Other broker-dealers remain inactive. • Without exclusive access agreement: – Broker-dealers bid security-by-security in a second price auction. – Highest bidder gets to buy security.	• With exclusive access agreement: – Receives lump-sum transfers. – Receives uniform bid-price for all sold securities. • Without exxlcusive access agreemed: – Offers each security via a second price auction. – Sells security to highest bidder.

Seller The risk-neutral seller (she) holds an portfolio of S securities of unit size, each indexed with s. We assume S to be large, reflecting the size of typical OTC market participants, such as pension funds, hedge funds, large firms and small banks. At time $t = 0$, each security independently draws a value v_s from a continuous distribution V. The seller is assumed to be sufficiently liquid to theoretically hold the securities for the long run, but is willing to sell them to a broker-dealer when offered a favorable bid-price:

$$b_s \geq v_s \tag{1}$$

Broker-Dealers There exist $N \geq 2$ profit-maximizing broker-dealers (they/he), which we label with superscripts $n \in \{1, 2, \ldots, n, \ldots, N\}$. The broker-dealers intermediate sales between the sellers and (potential) buyers. Here, we abstract from a detailed analysis on the buyer side, and simply assume that for each security s each broker-dealer n draws an independent ask-price a_s^n from a security-specific uniform distribution at $t = 2$: $a_s^n \sim$

$U(v_s - a, v_s + a)$. To realize such ask-price, they must buy the security from the portfolio holder at competitive bid-price b_s^n (more below). For now, notice that broker-dealers are willing to buy and resell any security with a positive bid-ask spread:

$$\pi_s^b = a_s^n - b_s^n \geq 0. \tag{2}$$

Exclusive Access Agreements Anticipating their ask-prices, broker-dealers may compete for exclusive access to the seller's portfolio by offering competitive agreements at $t = 1$. All exclusive agreements specify a uniform bid-price premium p_E^n paid above value v_s.[10] On a security level, the bid-price $b_E^n(v_s)$ under exclusive access becomes:

$$b_E^n(v_s) = v_s + p_E^n \tag{3}$$

Additionally, broker-dealers promise a single lump-sum transfer T_E^n that is paid for the exclusive access rights. Being ex ante identical, broker-dealers jointly compete in prices both on the bid-premium and the lump-sum transfer. Throughout the paper, we refer to the broker-dealer that has offered and was granted exclusive access as the (single) agreement-holder.

At $t = 2$, the agreement-holder pays both the bid-prices and lump-sum transfer: While $b_E^n(v_s)$ is paid only for each *purchased* security, T_E^n is paid regardless of total trades. We assume that exclusive agreements are binding contracts, such that either non-payment of T_E^n or one-sided termination entitles the counterparty to receive compensation equal to the lost profits. Capturing their legal complexity, exclusive agreements can neither be offered nor entered at $t = 2$. Assuming no termination, the agreement-holder purchases every security at $t = 2$, where:

$$\pi_s^n = a_s^n - b_E^n(v_s) > 0. \tag{4}$$

[10]Note that the reasonability of the uniform bid-premium assumption crucially depends on the fact that the buyer-value uncertainty around true value is independent of the actual value. If one where to relax the latter assumption, one would also have to carefully evaluate the uniform bid-premium assumption.

Second-Price Auctions If no exclusive agreement is active at $t = 2$, all broker-dealers bid on a security-level via a second-price auction (Vickrey auction): After observing their realized ask-price a_s^n, they decide whether to participate and what bid-price b_s^n to submit. Assuming price setting via second-price auctions captures typical OTC-pricing via trading platforms, where broker-dealers only observe their own ask-price and not the others'. Simultaneously, second-price auctions capture that broker-dealers possess sufficient market knowledge to avoid paying more than necessary.[11]

With a small abuse of notation, we define $\max_{k \neq n} b_s^k$ as the largest value of all other k submitted bids or v_s in the absence of such. For any given auction s and a bid b_s^n, a broker-dealer n's profit (bid-ask-spread) can be characterized by the following step-function:

$$\pi_s^n = \begin{cases} a_s^n - \max_{k \neq n} b_s^k & b_s^n > \max_{k \neq n} b_s^k \geq 0 \\ 0 & \max_{k \neq n} b_s^k \geq b_s^n \geq 0 \cdot \\ 0 & b_s^n = \emptyset \end{cases} \tag{5}$$

Case one in the profit function (5) reflects that the bid-ask-spread is the difference between ask-price and second highest bid if the broker-dealer n submitted the winning bid. Here, we follow standard convention and assume that the probability of equal highest and second highest bid is zero. The second line in (5) reflects that the profits are zero if the broker-dealer n participated with a weakly positive bid, but did not have the highest bid. Finally, the third line in (5) reflects that the broker-dealer n receives zero profit from security s when refraining from submitting a bid.

Equilibrium Notion We apply the notion of sub-game perfect Nash equilibrium (SPNE). We start by assuming that no broker-dealer holds an exclusive agreement. Arriving immediately at $t = 2$, we derive each broker-dealers' S optimal auction bids and the con-

[11]The consequent bids and profits are equivalent to those under the assumption of Bertrand competition in prices, where broker-dealers observe all ask-prices. In OTC markets, assuming complete information is naive. However, using the model for other markets, Bertrand pricing may be more appropriate and can be assumed without loss of generality.

sequently total profits for sellers and broker-dealers. Subsequently, we derive the exclusive agreement terms offered at $t = 1$. Here, we take into account that sellers can reject all offers when expected payoffs are lower than the auction profits, which serve as her reservation utility. Finally, we identify the SPNEs characterized by the broker-dealers' strategic choices of offering exclusive access agreements and check their termination proofness (more below).

3. Equilibria

3.1. Second-Price Auctions

We first derive the optimal auction bids and resulting profits in the absence of any exclusive access agreement. Here, broker-dealers decide on a security-by-security level whether to participate in the auction and, conditional on participation, what to bid. For participation, recall a broker-dealer requires a positive bid-ask spread (see equation (6)). Simultaneously, the seller requires a bid-price weakly above value v_s to sell the security (see equation (7)). Combining these two conditions in equation (8), it is easy to see that broker-dealers participate in auction on if he draws an ask-price weakly larger than the security value:

$$\pi_s^n = a_s^n - b_s^n \geq 0, \tag{6}$$

$$b_s^n \geq v_s \tag{7}$$

$$a_s^n \geq b_s^n \geq v_s. \tag{8}$$

Reversely, broker-dealers refrain from bidding whenever they observe an ask-price below value v_s:

$$b_s^n = \emptyset \quad \text{if} \quad a_s^n < v_s. \tag{9}$$

As is standard in second-price auctions, the broker-dealer bids the entire ask-price upon participation, i.e. whenever observing an above value ask-price $a_s^n \geq v_s$. Bidding the entire ask-price is the highest possible bid that still ensures a weakly positive spread π_s^n, while simultaneously maximizing the chances of winning. A broker-dealer's optimal bidding strategy is, thus:

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq v_s \\ \emptyset & \text{otherwise} \end{cases} . \tag{10}$$

Given the optimal bidding strategy, we can derive the broker-dealers' expected profits from all S auctions. Note here that assuming a large S would in theory allow us to rely on the law of large numbers (LLN) stating that the expected and realized total profits should be approximately equal. However, as the auctions happen in the last stage of the game, deriving the expected auction profits is sufficient. For completeness, we nevertheless discuss the impact of slight deviations between the two on equilibrium outcomes in Appendix C.

We start by deriving expected profits π_s^n from a single auction for a representative broker-dealer n. First, the broker-dealer can only expect positive profits conditional participation. This happens with probability $Pr(a_s^n > v_s) = 0.5$. Next, recall that k denotes the number of other bidding broker-dealers out of the remaining $N-1$. Each having the same participating probability as n, it is easy to see that k follows a binomial distribution where we denote the probability of k with $Pr(k)$. For a given k, recall that $\max_{k \neq n} b_s^k$ denotes the highest other bid. Here, the participating broker-dealer n wins with probability $Pr(a_s^n \geq \max_{k \neq n} b_s^k \mid a_s^n \geq v_s)$. In case of participating and winning, the realizes the difference between highest and second highest bid. In all other cases, the broker-dealer makes zero profits. Thus, the expected profits from a single auction are:

$$\mathbb{E}_1 \pi_s^n = Pr(a_s^n > v_s) \sum_{k=0}^{N-1} Pr(k) Pr\left(a_s^n > \max_{k \neq n} b_s^k \mid a_s^n > v_s\right) \mathbb{E}_1 \left[a_s^n - \max_{k \neq n} b_s^k \mid a_s^n > \max_{k \neq n} b_s^k \geq v_s\right] \tag{11}$$

While the equation looks quite complex, we can simplify it using both properties from the uniform distribution of ask-prices and the binomial distribution of the number other participation. For one, the likelihood of submitting the highest of $k+1$ bids is simply $1/(k+1)$ as all ask-prices are independent draws from the same uniform distribution:

$$Pr\left(a_s^n > \max_{k\neq n} b_s^k \mid a_s^n > v_s\right) = \frac{1}{k+1}. \tag{12}$$

It is similarly straight forward to compute the expected difference between the highest and the second highest of $k+1$ draws exceeding v_s given the uniform distribution $U(v_s - a, v_s + a)$:

$$\mathbb{E}_1\left[a_s^n - \max_{k\neq n} b_s^k \mid a_s^n > \max_{k\neq n} b_s^k \geq v_s\right] = v_s + \frac{k+1}{k+2}a - \left(v_s + \frac{k}{k+2}a\right) = \frac{a}{k+2}. \tag{13}$$

These simplifications, and the fact that $Pr(a_s^n > v_s) = 0.5$, allow us to rewrite equation to equation (14). Here, we can apply the binomial theorem to arrive at a closed form expression:

$$\mathbb{E}_1\pi_s^n = \frac{a}{2}\sum_{k=0}^{N-1}\binom{N-1}{k}0.5^{N-1}\frac{1}{k+1}\frac{1}{k+2} \tag{14}$$

$$= \frac{a}{2^N}\frac{2^{N+1}-N-2}{N(N+1)} \tag{15}$$

Of course, the broker-dealers have access to S auctions. To obtain broker-dealers' total expected profits $\mathbb{E}_1\Pi^n$ we simply multiply equation (15) by S:

$$\mathbb{E}_1\Pi^n = \frac{Sa}{2^N}\frac{2^{N+1}-N-2}{N(N+1)}. \tag{16}$$

The seller's total expected profits $\mathbb{E}_1\Pi^S$ differ in two key characteristics from the broker-dealer profits. For one, it requires at least two participating broker-dealers for the seller to make a strictly positive profit in a single auction. Alternatively, the bid-price is set to v_s when only one broker-dealer participates in the auction and the trade fails when none participates. In either case, the seller makes zero profits. Second, the seller's profits, when two or more broker-dealers submit to the auction, are equal to the second highest bid. With a slight abuse of notation, let n denote the the index of the broker-dealer with the highest positive ask-price. Then, the seller's expected total profits Π^S from the S auctions at $t = 2$ are:

$$\mathbb{E}_1\Pi^S = S\sum_{n=2}^{N} Pr(n)\mathbb{E}_1\left[\max_{k\neq n} b_s^k \mid a_s^n > \max_{k\neq n} b_s^k\right] \tag{17}$$

$$= \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{18}$$

Similarly to broker-dealer profits above, profit function (17) contains S times the profit from a single auction. Here, we again utilize the distributional properties from ask-prices (uniform) and participation (binomial distribution). As such, the closed form expression (18) can be obtained by, after some manipulation, applying the binomial theorem. As a similar logic as in the broker-dealer case is applied, we refrain from providing further details here. The interested reader is kindly asked to refer to the Appendix for detailed derivations.

The seller profits conclude this sub-section, and we can now summarize the sub-game outcomes under S second-price auctions in Lemma 1. Here, we would like to highlight that both seller and broker-dealer profits do not depend on the underlying security value v_s but are simply determined by the ask-price uncertainty a around that value.

***Lemma* 1.** *The broker-dealers' optimal bidding strategy in a single second-price auction for security s at time $t = 2$ is:*

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq v_s \\ \emptyset & otherwise \end{cases}. \tag{19}$$

Aggregating the resulting profits over all S securities, the seller and broker-dealers expect the following total profits, respectively:

$$\mathbb{E}_{\mathbb{1}}\Pi^S = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \qquad \mathbb{E}_{\mathbb{1}}\Pi^n = \frac{Sa}{2^N}\frac{2^{N+1}-N-2}{N(N+1)} \quad \forall n \in N. \tag{20}$$

3.2. *Exclusive Access Agreements*

The seller compares the expected auction profits with the expected profits given the available exclusive access agreements at $t = 1$. We denote all prices and profits associated with an exclusive agreement with an additional subscript $_E$. We start with assuming that the seller has entered an agreement with broker-dealer n against a promised lump-sum transfer T_E^n and bid-price $b_E^n(v_s)$ that specifies an security-independent bid-premium p_E^n above value:

$$b_E^n(v_s) = v_s + p_E^n. \tag{21}$$

To ease the notation for the rest of the section, it is useful to notice that the ask-price a_s^n can similarly be decomposed into a value component and an uncertainty $\tilde{a}_s^n$ around that value:

$$a_s^n = v_s + \tilde{a}_s^n \quad \text{where} \quad \tilde{a}_s^n \sim U(-a, a). \tag{22}$$

Then, for a given security, the agreement-holder purchases a security at $t = 2$, if:

$$\pi_s^n = a_s^n - b_E^n(v_s) \geq 0. \tag{23}$$

$$\tilde{a}_s^n \geq p_E^n. \tag{24}$$

Accounting for the likelihood of purchase and size S of the portfolio, the seller's and agreement-holder's aggregate expected profits under an exclusive access are:[12]

$$\mathbb{E}_1\Pi_E^S = SPr(\tilde{a}_s^n > p_E^n)p_E^n + T_E^n = S\frac{a - p_E^n}{2a}p_E^n + T_E^n. \tag{25}$$

$$\mathbb{E}_1\Pi_E^n = SPr(\tilde{a}_s^n > p_E^n)\mathbb{E}_1[\tilde{a}_s^n - p_E^n \mid \tilde{a}_s^n > p_E^n] = S\frac{(a - p_E^n)^2}{4a} - T_E^n \tag{26}$$

Studying expression (25), we observe that the seller's expected profits are monotonically increasing in T_E^n. The profits are, however, non-linear in the bid-premium: A higher p_E^n

[12] Again the LLN applies, such that expected and realized profits are approximately.

increases the revenue from a single transaction, but reduces the probability of said transaction taking place. This non-linearity must be taken into account, when deriving the competitive prices for exclusive access.

To derive such prices, we start by arguing that any SPNE with an active exclusive access agreement must have two or more broker-dealers competing over it and that the ultimate holder makes zero profits (Bertrand Paradox). Let us initially assume that a single broker-dealer offers a profitable agreement for a given bid-premium p_E^n and lump-sum transfer T_E^n. Then any other broker-dealer has the incentive to offer the same bid-premium but a slightly higher lump-sum transfer to attract the seller instead. Else, he would make zero profits when loosing the agreement to a competitor. This applies for all bid-premia and transfer combinations, where the agreement-holder makes a positive profit. Hence, any two or more broker-dealer offering a competitive agreement must make zero profits in equilibrium.

For completeness assume that a single broker-dealer offers an agreement where he makes zero profits. Without competition, he then has an incentive to lower lump-sum transfers to keep some of the gains of trade. However, as just argued, such deviation is not an equilibrium either as then again the competitor(s) have incentive to offer slightly favorable terms, thereby attracting the seller. Hence, any SPNE with an active exclusive access agreements is necessarily characterized by at least two agreements, where the offered bid-premium and lump-sum transfer leave the broker-dealer with zero profits.

Inserting the zero-profit condition into agreement-holder's total expected profits (see equation (26)) yields the following equilibrium lump-sum transfer:

$$T_E^n = S\frac{(a - b_E^n)^2}{4a} \tag{27}$$

We then insert the equilibrium lump-sum transfer (27) into the seller profits. By equating the associated first-order-condition with respect to p_E^n to zero, we can show that $p_E^n = 0$ is the unique seller profit maximizing bid-price:

$$\mathbb{E}_1\Pi_E^S = S\frac{a - p_E^n}{2a}p_E^n + S\frac{(a - p_E^n)^2}{4a} \tag{28}$$

$$\frac{\partial \mathbb{E}_1 \Pi_E^S}{\partial p_E^n} = -2\frac{S}{4a}p_E^d = 0 \tag{29}$$

Inserting our optimal pricing into equations (25) and (26) result in the following expected total seller and broker-dealer profits, respectively:

$$\mathbb{E}_1 \Pi_E^S = \frac{Sa}{4} \tag{30}$$

$$\mathbb{E}_1 \Pi_E^n = 0 \quad \forall n \in N \tag{31}$$

In a final step, we must check that expected seller profits in equation (30) are greater than or at least equal to the expected profits from the S auctions in (20). Alternatively, the seller prefers to reject the agreement offers at $t = 1$ and simply lets the auction take place. Intuitively, the second-price auctions yield higher pay-offs the more broker-dealers compete. Due to the Bertrand Paradox, exclusive access profits are of course independent of N. Hence, it is no surprising that only sellers with few broker-dealer connections prefer to grant exclusive access:

$$\mathbb{E}_1 \Pi^S = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \leq \frac{Sa}{4} = \mathbb{E}_1 \Pi_E^S \tag{32}$$

$$N \leq 3 \tag{33}$$

***Lemma* 2.** *In any SPNE with exclusive access, the seller observes multiple agreement offers all leaving the broker-dealers with zero profits. However, a seller only excepts one of multiple agreement offers whenever $N \leq 3$. In that case the optimal fees and expected profits are:*

$$b_E^n = v_s \qquad T_E^n = \mathbb{E}_1 \Pi_E^S = \frac{Sa}{4} \qquad \mathbb{E}_1 \Pi_E^n = 0 \quad \forall n. \tag{34}$$

Whenever $N \geq 4$, the seller strictly prefers engaging in the second-price auctions.

Off equilibrium path, we may observe a single (monopolistic) broker-dealer offering an exclusive access agreement. We will see immediately below that this is a relevant potential deviation, wherefore we briefly here summarize these (off-path) expected profits.

In this case, the monopolistic broker-dealer simply charges the profit maximizing combination of p^n_E and T^n_E that maximizes his profits constrained by the lender's (binding) participation constraint. And thus, the seller always participates:

$$\mathbb{E}_1\Pi^n_E = \max_{p^n_E, T^n_E} S\frac{(a-p^n_E)^2}{4a} - T^n_E \tag{35}$$

$$s.t.$$

$$\mathbb{E}_1\Pi^l_E = \mathbb{E}_1\Pi^l \tag{36}$$

Solving the broker-dealer's constraint maximization in (35) above, we find that setting $p^n_E = 0$ is again optimal. Further, optimal lump-sum transfer are set just such that the seller's participation constraint just binds given $p^n_e = 0$. This ensures the lender agrees to exclusive access and all other broker-dealers are left empty handed. Lemma 3 below summarizes the bid-prices, transfer and resulting expected profits, whenever only a single broker-dealer n requests exclusive access.

***Lemma* 3.** *If offered a single ESLA, the lender always accepts and the optimal fee and expected profits are:*

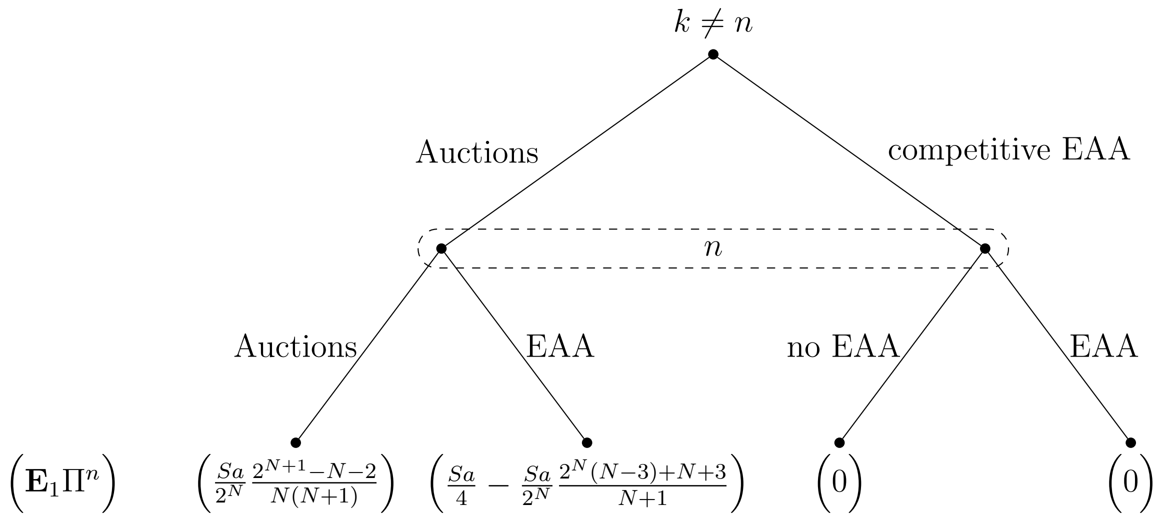
$$b^n_E = v_s \qquad T^n_E = \mathbb{E}_1\Pi^l_E = \mathbb{E}_1\Pi^l \qquad \mathbb{E}_1\Pi^n_E = \frac{Sa}{4} - T^n_E \qquad \mathbb{E}_1\Pi^k_E = 0 \quad \forall k \neq n \in N \tag{37}$$

3.3. The Termination Proof SPNEs

We now turn to deriving the termination proof SPNE in this market. Here, we start by deriving when, depending on N, the auctions versus exclusive access agreements are candidate SPNE. Consequently, we explain and apply the equilibrium refinement of termination proofness to any candidate exclusive access SPNE.

Candidate SPNE Comparing the above derived expected profits from auctions versus exclusive access, we now determine the broker-dealers' optimal agreement offer strategy in equilibrium. For this, we compare a representative broker-dealer n's expected profits when (correctly) anticipating none versus at least one other broker-dealer to offers an exclusive access agreement (EAA). Such expected profits are summarized in Figure 1 below. Using this comparison, we derive three types candidate SPNEs characterized by their frequency of agreement offers: auction SPNEs, single agreement SPNEs, and multiple agreement SPNEs. In the next paragraph, we check whether such candidate SPNEs are termination proof and, thus, truly sub-game perfect.

Figure 1: A Broker-Dealer's profits given the Others' Choices



Let us start assuming that a broker-dealer n anticipates that no other broker-dealer $k \neq n$ has offered an exclusive access agreement and instead prefer to move on to the auctions (left branch of the tree). Conditional on that, we obtain the optimal broker-dealer choice from comparing the two left-most profits in Figure 1: $\mathbb{E}_1[\Pi^n \mid \text{Auctions}]$ and $\mathbb{E}_1[\Pi^n_E \mid \text{Auctions}]$. After some manipulation, we reach the following two (in-)equalities:

$$\mathbb{E}_1[\Pi^n \mid \text{Auctions}] > \mathbb{E}_1[\Pi^n_E \mid \text{Auctions}] \qquad \forall\ N > 2 \tag{38}$$

$$\mathbb{E}_1[\Pi^n \mid \text{Auctions}] = \mathbb{E}_1[\Pi^n_E \mid \text{Auctions}] \qquad \text{if } N = 2 \tag{39}$$

Intuitively, the two inequalities state that a broker-dealer n, anticipating all other broker-dealers to prefer the auctions, also (weakly) prefers the auctions over offering a monopolistic exclusive access agreement. More formally, conditional on no other broker-dealer offering an agreement, a single broker-dealer has no incentive to deviate by offering a monopolistic agreement. Because all broker-dealers are symmetric, we can directly conclude from (38) and (39) that there, thus, exists an auction candidate SPNE where broker-dealer refrain from competing for exclusive access. Further, combining this with the insights from Lemma 2, we know that the auction SPNE is unique for $N \geq 4$.

***Lemma* 4.** *There always exists a candidate auction SPNE. It is sole candidate SPNE for* $N \geq 4$*.*

The second type of candidate SPNEs is characterized by the right branch in Figure 1: At least two broker-dealers offer a offer competitive exclusive access agreement in equilibrium. Comparing the profits in the right branch of Figure 1, a broker-dealer is always indifferent between offering an agreement on not, conditional on anticipating at least one other competitive agreement. Again relying on symmetry, this holds also for all other broker-dealers. Consequently, there exists at most four of such competitive agreement SPNEs: one for each broker-dealer pair and one for all three borker-dealers. Recall from from Lemma 2 that such exclusive access SPNEs only exist for $N \leq 3$, as else the seller rejects any agreement in favor of the auctions.

***Lemma* 5.** *For* $N \leq 3$*, there candidate SPNEs characterized by least two broker-dealers offering competitive exclusive access agreements.*

Terminations A common concern in the exclusive contracting literature is whether the candidate SPNE are renegotiation proof (Segal and Michael D. Whinston, 2000). In this model, any sure gain from the broker-dealer(s) always results in a sure loss to the seller. Thus, mutual beneficial re-negotiations are not possible.[13] A closely related and more relevant concept is that of termination proof SPNEs: No contract holder has an incentive to single-handedly terminate the contract.

Because agreement contracts always take the seller's participation constraint into account, the seller never has an incentive to terminate an agreement in SPNE. For the agree-

[13]Note that this depends on the presence of lump-sum transfers. If agreements could only specify uniform bid-premia, then expected seller profits are an u-shaped function in p_E^n. Thus, the seller-profit maximizing bid-premium may leave profits for the broker-dealer and, thus, renegotiation may be mutually beneficial ex post.

ment holding broker-dealer recall that any agreement termination requires the seller compensation of losses. And such, the agreement-holder does *not* terminate if:

$$\mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi_E^S - \mathbb{E}_1\Pi^S\right) \leq \mathbb{E}_1\Pi_E^n. \tag{40}$$

Inequalities (41) and (41) consider profit gains from termination when $N = 3$ and when $N = 2$ are respectively inserted in equation (40):

$$N = 3: \qquad \mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi_E^S - \mathbb{E}_1\Pi^S\right) = \frac{Sa11}{96} - \left(\frac{Sa}{4} - \frac{Sa3}{16}\right) > 0 = \mathbb{E}_1\Pi_E^n. \tag{41}$$

$$N = 2: \qquad \mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi_E^S - \mathbb{E}_1\Pi^S\right) = \frac{Sa}{6} - \left(\frac{Sa}{4} - \frac{Sa}{12}\right) = 0 = \mathbb{E}_1\Pi_E^n \tag{42}$$

As inequality (41) highlights, the multiple agreement SPNE is not termination proof for $N = 3$: The agreement-holder finds it profit maximizing to terminate the contract, thereby triggering an auction SPNE. And thus, for $N \geq 3$ the auction SPNE is the unique termination proof SPNE. For $N = 2$, however, a multiple agreement SPNE additionally exists. As equation 42 highlights, here the agreement-holder is just indifferent between entering the auctions and paying the punishment or not, and hence does not choose to terminate. In Appendix C, we show that the agreement-holders has a strict preference for not terminating in case he *ex post* observes higher ask-prices than anticipated *ex ante*. For completeness note that there exists a mixed strategy equilibrium for $N = 2$, where each broker-dealer offers an agreement with probability one-half.[14]

Of course, the auction SPNE is termination proof by assumption, as no agreement can be offered/entered at $t = 2$.

***Proposition* 1.** *There always exist a termination proof auction SPNE, which is unique for $N \geq 3$. For $N = 2$, there further exists a termination proof SPNE with both broker-dealers offering competitive agreements for exclusive access.*

[14] In the mixed strategy SPNE, the uniform agreement bid-price is always zero. Transfers are, however, conditional on the sellers total agreement offers.

Monopolistic Broker-Dealer For completeness, we also derive the SPNE with a single operating broker-dealer. For $N = 1$, the derivations are rather trivial as the broker-dealer is a monopolist and simply ensures that the seller just participates. Hence, the seller makes zero profits from lending and the broker-dealer realizes the entire ask-price for every security where such is positive. An agreement may be offered and granted, but neither changes profits nor fees. And, hence, an agreement comes at no benefit to the seller.

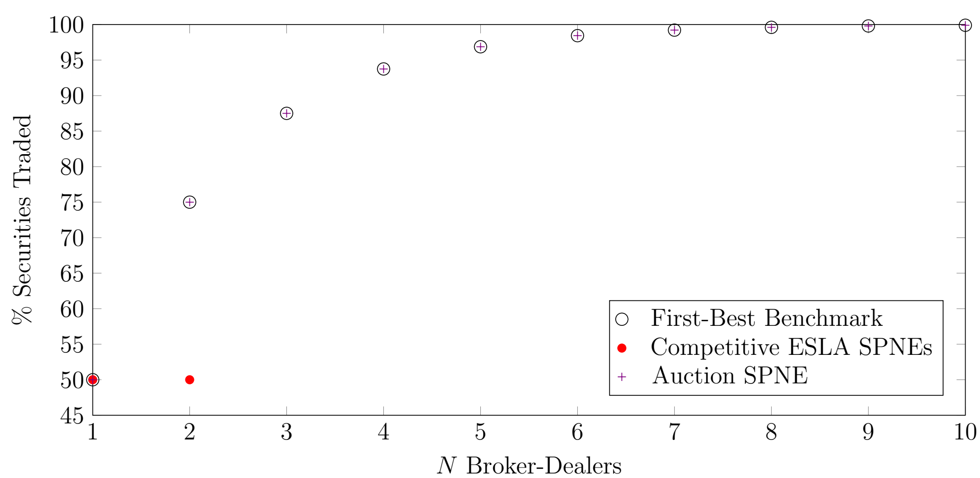
Remark 1. *For $N = 1$, the seller is indifferent between being offered an agreement or not, as the broker-dealer is a monopolist that always extracts all transaction surplus:*

$$b_s = b_E = T_E = \mathbb{E}_1 \Pi^S = 0 \qquad\qquad \mathbb{E}_1 \Pi^n = \frac{Sa}{4}. \tag{43}$$

3.4. Individual Level Inefficiency

From a market efficiency perspective, each security in the seller's portfolio where at least one of then N broker-dealers observes an above value ask-price should be traded. Hence, the first-best benchmark implies that a security is traded with with a probability $1 - 0.5^N$. And for a large portfolio S, this aggregates to a total of $(1 - 0.5^N) * 100$ percentage of securities traded. The hollow circles in Figure 2 below indicate such percentages as a function of N.

Figure 2: Percentage of Traded Securities in SPNE



The auction SPNE meets such first-best benchmark: Every broker-dealer with ask-prices weakly above value v_s participates in the auction. Thus, all securities with at least one realized ask-price of above v_s are traded. This is indicated by the blue crosses in Figure 2 overlapping exactly with the first-best. For $N = 2$, there additionally exists an exclusive access SPNE, where both broker-dealers offer an agreement, and a single one ultimately holds it. Upon realization, the agreement-holder purchases each security with probability one-half: the probability of a ask-price weakly above value. For the whole portfolio, this implies that 50% of all securities are traded, as indicated by the red dots in Figure 2. Here, it can easily be seen that the exlcusive access SPNE does not meet the first-best benchmark.

Corollary 1. *The auction SPNE always meets the first-best benchmark.*

The exclusive access SPNE experience a one-third lower trading volume relative to first-best for $N = 2$ but meets the first-best benchmark for $N = 1$.

4. Application: The Security Lending Market

In this section, we map the above described model to security lending market, where we have contract level data for all European Union (EU) based market participants. The section is organized in the following order: First, we show how the model nests the security lending market; next, we describe the security lending

4.1. A Model for Security Lending

A typical security lending transaction varies slightly from the above described set-up as, ultimately, the security is returned from the buyer to the broker-dealer and then the seller. Thus, the bid and ask prices reflect lending fees rather than purchase prices. Assuming away broker-dealer defaults for simplicity,[15] seller participating constraint becomes:

$$b'_s \geq 0. \tag{44}$$

[15] This is equivalent to assuming that correctly priced collateral is posted for every sale, generating the same expected lender pay-offs. Investigating potential collateral miss-pricing frictions in the security lending market is beyond the scope of this paper.

Further, the broker-dealer realizes a slightly modified ask-price $a_s^{n\prime}$ that is the difference between true value and buyer-perceived value:

$$a_s^{n\prime} = a_s^n - v_s \quad \text{where} \quad a_s^{n\prime} \sim U(-a, a). \tag{45}$$

Taking a closer look at equations (44) and (45), one can easily see that these are special case of the original model obtained by simply setting $v_s = 0$. Further, the seller and broker-dealer participation constraints are the only two equations, where v_s enters into the model. Thus, the baseline model nests the security lending market. Finally, note that neither the seller's nor the broker-dealers' expected profits depend on v_s, wherefore Proposition 1 hold without loss of generality.

Corollary 2. *The security lending market nests in the above model environment and captured by setting* $v_s = 0$. *Thus, all results hold without loss of generality.*

4.2. Security Lending Data

Applying the model to the EU-based security lending market is possible due to the recently available data collection via the Securities Financing Transaction Regulation (SFTR). The data set contains ca. 100 contract fields and 800 enrichment fields for every single repurchase agreement, margin lending, security buy-and-sell back, and securities or commodities lending transaction since mid 2020 whenever at least one counterparty was EU-based. It thus, provides, a rich source of information for both policy makers and researchers alike. For this paper in particular, the dataset is useful as market participants are required to flag whether the transaction was covered by an exclusive security lending agreement (ESLA).

For this exercise, we focus entirely on the subset of equity lending transactions. Simplified, the raw data contains both a daily stock and flow report from execution until maturity submitted for each of their individual transactions. First, we combine all daily reports in a single observation utilizing the unique transaction identifier and collect contract variables of particular interest: loan volume, quantity, prices and fees, collateral, realized maturity, both counterparty LEIs (lending and borrowing side) and, most importantly the ESLA-flag. Next, we limit our sample to transactions both entered and matured in 2021 to have consistent reporting standards throughout the sample. Subsequently, we drop all intra-group transactions, where the equity holder and the security receiver have the same parent LEI code. On the remaining transactions, we perform a series of additional quality checks and cleaning steps. A detailed description of the data set, the cleaning procedures and output generation process can be found in a complementary market analysis (Kessler et al., 2022).

Ultimately, we are left with 15,816,332 observations. Out of these, 13.89% are covered by ESLAs. As Table 2 highlights, transactions under ESLAs seem to have on average a 56% lower loan value, mainly due to lower quantities rather than stock prices.

Table 2: Transaction Statistics by ESLA Status

	% of transactions	Avg. Loan Value (€)	Avg. Stock Price (€)	Avg. Quantity
With ESLA	13.89	127837.71	42.43	9080.11
Without ESLA	86.11	291252.68	46.77	19742.78

4.3. Matching Model Inputs

At its core, we have a risk-neutral owners of a security portfolio and a small number of distinct intermediaries with private, independent demand. As demand is uncertain, the portfolio-owner must decide upfront whether to sell/lend the securities exclusively or competitively.

As we will show in the two paragraphs below, these assumptions match the security lending market quite well. The assumption of a risk-neutral owner is motivated by the presence of well-diversified lenders with rather large and frequent trades. Further, EEAs are predominately entered between lenders and broker-dealers. Yet, lenders without ESLAs trade only with a selected hand-full of broker-dealers and rarely more than three. In both cases, each individual lender makes up a small portion of any given broker-dealer's portfolio, validating the representative seller assumption. On the demand side, predominantly observe single homing: of 88% of all borrowers trade with only a single broker-dealer. Thus, the assumption of private borrowing demand of broker-dealers seems justified.

Solely, the timing difference between ESLA and auction offers cannot be inferred directly from the data. Here, we must rely on anecdotal evidence from practitioners highlighting the typically lengthy bilateral negotiations required to set up Master agreements that cover the exclusive access. This must be compared to the relative ease with which large scale lending is conducted over trading platforms where portfolio holders can announce individual stock level auctions.[16]

Counterparty Level Overview To showcase of who lends to whom and under which conditions, we assign each observed market participant one of five labels: borrower, broker-dealer, lender, private client and trader. Types are assigned based on their aggregate trading patterns across all observations in our sample. All types, except private clients, are corporate entities that are identified by their LEI code. For borrowers and lenders, we observe that 99% of all their transactions are borrowing and lending, respectively. Broker-dealers and traders engage in both lending and borrowing (see Table 3 below). Here, we find that traders are typically smaller agents with less than 100 counterparties that tend to lend more than they borrow. Broker-dealers on the other hand have larger trading volumes, more than a 100 counterparties and tend to borrow more than they lend.

Table 3: Lending and Borrowing Volumes

Type	Total Lending value (€ bn)	Total Borrowing Value (€ bn)	Ratio Lending-over-Borrowing
Broker-Dealer	1990.40	2867.86	0.69
Trader	590.09	383.49	1.54

For private clients, typically natural persons not subject to reporting requirements, we only observe a client ID assigned by the reporting counterparty. As assigned client IDs vary across reporting agents, we have no further information to the identify the private individuals behind the transactions. Therefore, we can not observe a private investors' total trading volume across all counterparties. Here, our analysis is limited to the specific reporting agent

[16]Examples of such trading platforms are the BlackRock Security Lending platform, Sharegain and FIS Securities Lending Platform. All three exclusively serve institutional clients.

and private client combination. However, the transaction volume involved is small relative to the total market size. For all other agents, we can report aggregated transaction statistics across all their counterparties (see Table 4 below).

Table 4: Trading Party Characteristics

Type	Nr. of Parties	Avg. Total Loan Value (€ mn)	Avg. Transaction Value (€ mn)	Avg. Nr. Transactions	Avg. Nr. of Counterparties
Borrower	469	2106.29	2.62	3305.13	1.54
Broker-Dealer	39	124570.89	1.23	423022.13	5859.36
Lender	6512	239.83	8.86	293.11	3.27
Private client	208214	0.54	0.01	51.73	1
Trader	442	2202.67	1.55	1893.40	6.07

Besides the obvious (and expected) differences in trading volume, Table 4 highlights a stark difference in the average number of trading parties given the type. Broker-dealers have on average more than 5500 different counterparties. Both lenders and borrowers are less connected, with on average 3.37 and 1.54 counterparties, respectively. Such core-periphery structure becomes quite apparent when studying the network visually in Figure 3. There, we have aggregated all borrowers, lenders and private clients with the same single counterparty as one trading party, and scaled the size of counterparties non-linearly to maintain confidentiality.

A noteworthy feature of the network, and a potential reason for fewer trading parties, is the presence of ESLAs. Indicated by the red connections, these can predominantly be found between lenders and broker-dealers. Upon further analysis, we find that 35% of all lenders grant an ESLA to a specific broker-dealer. 17% of all private clients agree to ESLAs, while only 4.5% of all borrowers do. Yet, the vast majority borrowers limit themselves to a single broker-dealer. Figure 4 illustrates these findings.

Transactions Between Lenders and Broker-Dealers Matching the model, we now restrict our sample to cover transaction between these two types of market participants from here on.[17] An important contract characteristic, determining among others the

[17]Given the explicit focus on ESLAs we perform an additional cleaning step, where we correct for misreporting of the ESLA-flag variable. For lenders with more than one counterparty, we replace ESLA="TRUE" with "FALSE", when less than 10% of the transactions are (falsely) reported to be covered by an ESLA. If

Figure 3: Network Plot of the EU Equity Lending Market

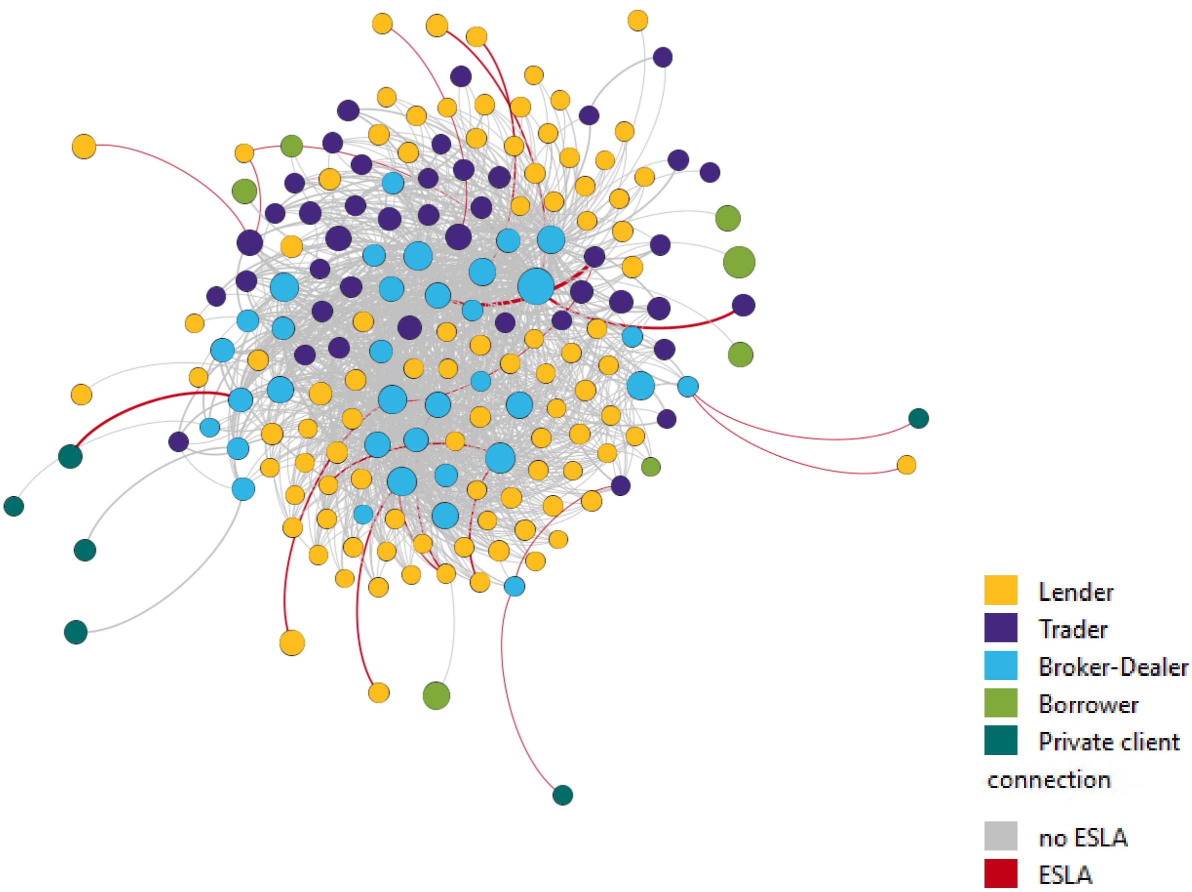
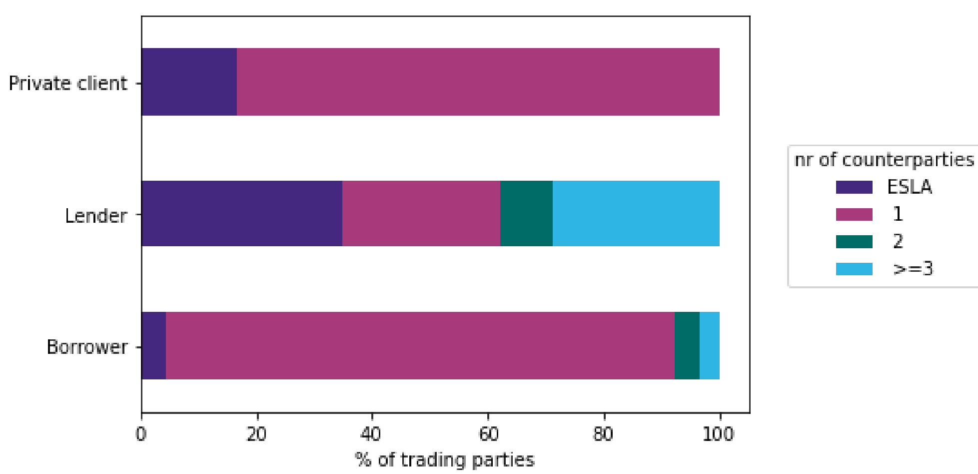


Figure 4: Relative Share of Trading Parties per Type



the reverse holds true, we drop the lender from the sample. For lenders with one single counterparty, we replace ESLA=”TRUE” with ”False” when less than 40% are covered by an ELSA. Vice-versa, we replace ESLA=”FALSE” with ”TRUE” when more than 60% of transactions are covered by an ESLA. For the remaining misreporting, where lenders have between 40%-60% covered by an ESLA and only one counterparty, we simply drop the lender.

fee structure, is the underlying type of collateral. Here, we observe three types of collateralization: none, basket, and cash.

Table 5: Collateral Usage

	Basket	Cash	None
Transactions	1,416,960.00	152,316.00	190,303.00
Total Loans (€ bn)	1,170.53	35.39	107.62
Avg. Loan Value (€)	826,086.42	232,377.67	565,520.65
Avg. Quantity	52,327.36	17,678.36	26,052.60

In the case of cash collateral, a net-rebate rate is reported and defines the difference between rebate rate paid by the lender for collateral re-use minus the lending fee paid by the borrower. Table 5 shows that only 10.5% of all lending transactions are cash-collateralized, making up 7.8% of the total transaction volume. Given the low market share, we abstract from further analysis of both the cash collateral and net-rebate rates in this paper.

Instead, we focus on the remaining transactions secured with either a collateral basket (89%) or no collateral (2.5%). In both cases, the borrowers pays the lender a fee, but no rebate rate is charged. Lending fees are typically reported as a percentage of loan value rather than in units of currency.

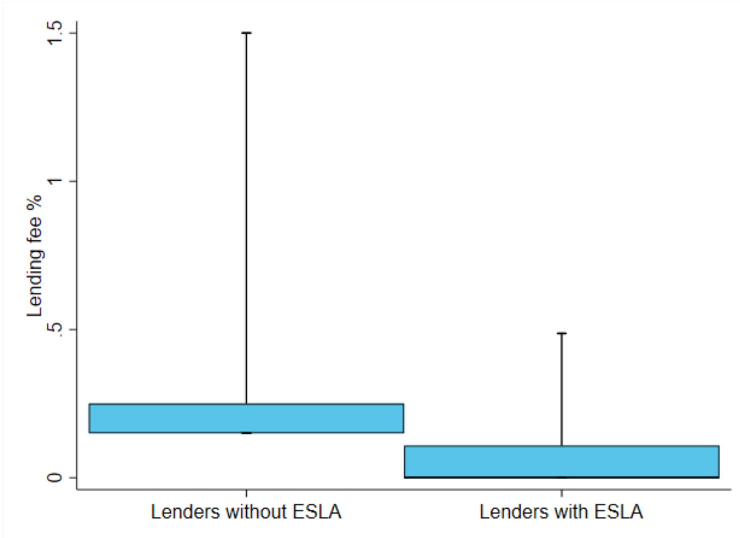
Table 6 and Figure 5 below, illustrate, respectively, the mean and median of the lending fees. Here, we distinguish between transactions covered by ESLAs and those that are not. Note here, that the whiskers of the two boxplots in Figure 5 indicate the 10th and 90th percentiles to ensure confidentiality.

Table 6: Average Lending Fee by ESLA

	without ESLA	with ESLA	Difference	SE	Obs.
Lending Fee (%)	13.82	0.79	13.03***	(4.11)	1513893

We can see that ESLAS on average yield higher lending fees. However, this is mainly

Figure 5: Lending Fee Distribution by ESLA



driven by a longer upper tail of the distribution. As indicated by the horizontal line, ESLAs have an median lending fee just above zero that is below the median fee of transactions not covered by ESLAs. Naturally, this leads to the next section: To which extend are reported fees fitting the model outcomes?

4.4. Matching Model Outcomes

To verify the model fit with the data, we focus only comparing equilibrium outcomes for on $N \leq 2$ with contract terms from lenders with at most two counterparties. For one, we have shown that ESLAs arise only sellers with two-or-less counterparties. Additionally, sellers with three or more counterparties are quite rare. Given this selection, we derive five testable hypotheses assuming a large number L of lenders. Recall that the lending fees (f_s) are typically reported as percentages of total loan value (v_s). The loan value in return is the stock price times the quantity (q_s). To match this with the model assumptions of each security being of unit-size, we transform the lending fee into a fee paid per-unit of borrowed stock:

$$b_s^* = \frac{f_s}{100} \frac{v_s}{q_s}. \tag{46}$$

We will subsequently use these per-unit fees to test the fit of our predicted prices with those of the realized prices in the data. Here, we continuously ensure data confidentiality and would like to highlight that each reported statistic contains at least three sellers and broker-dealers each, and no two market participant make up more than 85%. All Figures are produced using data-points between the 10th and 90th percentile. Tables, where only aggregated statistics are shown, are derived on data-points between the 1st and 99th percentile to ensure outliers are not driving the results.

Exclusive Access Agreements We start considering the subset of agreement granting sellers. From Lemma 3, we know that sellers with ESLAs should pay zero bid-prices. Of course, this simplifies reality as we assume in the model that sellers face zero marginal transaction costs. Softening the expectations slightly, we expect that lending fees should be close to zero under agreements.

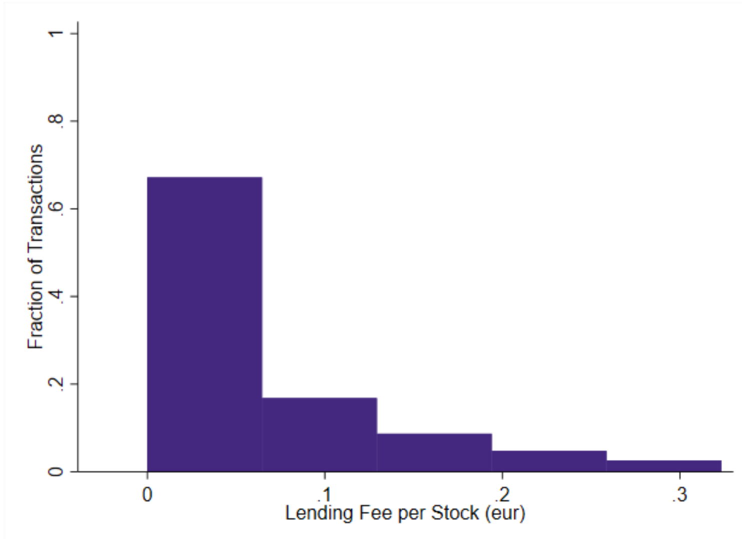
H1 Per-unit lending fees of transactions under ESLAs should be close or equal to zero.

Testing *H1* statistically using conventional methods is not possible, as lending fees do not follow a normal distribution: They have both a clear cut off and mass point at zero. To verify *H1*, we instead rely on visual output. As Figure 6 below shows, the transaction fees under exclusives are indeed very close to zero. In fact, 90% of all transactions are below five cents and all are less than 15 cents.

Auctions In the absence of agreements, the realized fee distribution depends on the sellers' number of counterparties. Hypotheses *H2* and *H3* below summarize the results for sellers with $N = 1$ and $N = 2$ respectively. We do not consider the case of sellers with three or more broker-dealers as predictions are cumbersome without any substantial additional insights. For $N = 1$, we predict that lending transactions have a zero lending fee as the single broker-dealer acts as a monopolist. Testing such hypothesis in the data is more challenging though, as we are only observing the realized number of counterparties and not the (hypothetically) available.

H2 Outside ESLAs, transactions of sellers with a single broker-dealer have more often

Figure 6: Distribution of the Per-Unit Fees under Exclusivity

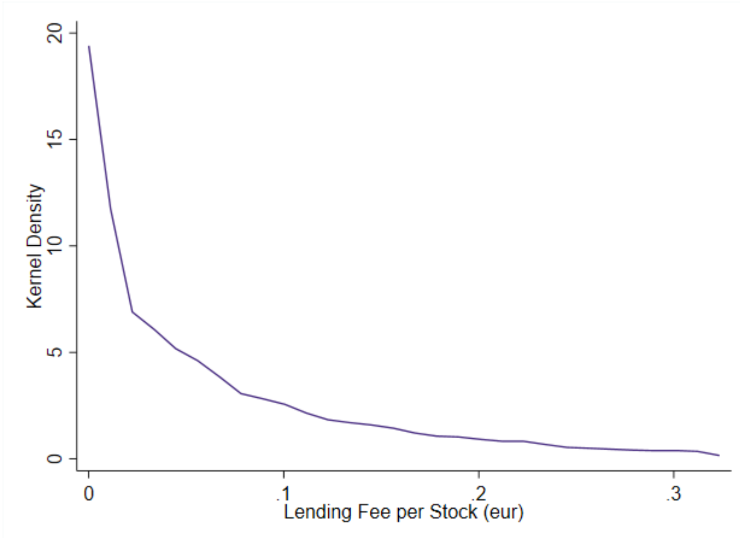


than not zero bid-prices.

To verify *H2*, we estimate the Kernel density over all per-unit lending fees paid by sellers with a single broker-dealer connection. Displayed in Figure 7, we observe that zero or close to fees are most likely. However, we also see that a non-negligible portion of sellers indeed makes a profit. This could be due to non-zero lending costs or that their restriction to one broker-dealer is by choice. A seller requesting offers from multiple broker-dealers, but ultimately selecting one, allows her to nevertheless enjoy the benefits of (some) competition without paying e.g. on-boarding costs. Unfortunately, the data does show how many counterparties they considered ex ante.

For sellers with two broker-dealers, obtaining the distribution of realized bid-prices, denoted with b_s^*, is more complex. It is derived from the likelihood that the second-largest price takes on a value (weakly) above zero when both broker-dealers experience a positive draw and exactly zero, when only one does. For ease of understanding, denote the other bid with b_s^o. Below, equation (47) defines the resulting probability density function (pdf) for all observed bid-prices in its general form.

Figure 7: Kernel Density of Per-Unit Fees without exclusives and $N = 1$



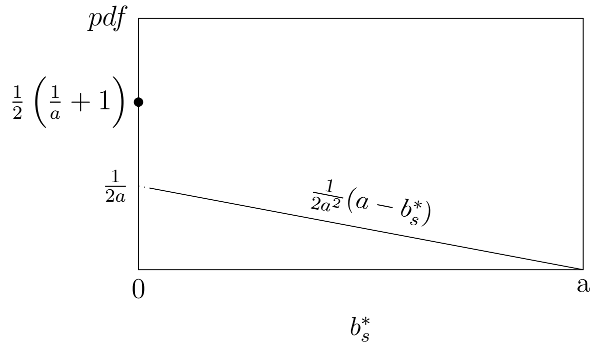
$$pdf(b_s^*) = \begin{cases} 2Pr(b_s^*)Pr(b_s^* < b_s^o) & b_s^* > 0 \\ 2Pr(b_s^*)Pr(b_s^* < b_s^o) + 2Pr(b_s^o = \emptyset)Pr(b_s > 0) & b_s^* = 0 \end{cases} \tag{47}$$

Inserting the properties of the uniform ask-price distribution, we can obtain a closed form expression for the pdf of bid-prices. The density function, displayed in Figure 8a below, takes on the value $1/2(a^{-1} + 1)$ at exactly zero. For marginally increasing bid-prices, we observe an immediate jump downwards to $1/2a$. This jump reflects that a zero bid-price is more frequent relative to all other positive bid-prices due to the absence of second bids whenever one of the two broker-dealers draws a negative ask-price. For every other value above zero, the density is a linearly decreasing function with a slope equal to $-1/2a^{-2}$. The hypotheses *H3* below summarizes such behavior.

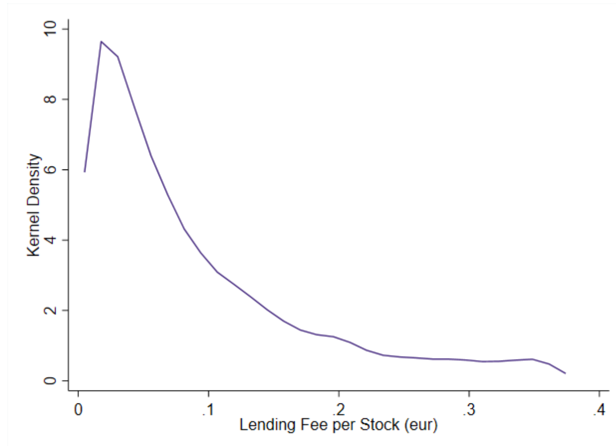
H3 *Trading with two broker-dealers, zero is the most frequent realized bid-price, followed by an immediate downward jump and consequent decrease in.*

Notice here that *H3* intentionally does not describe the decrease in density for increasing fees as linear, albeit the theoretical model predicting this. Again, this is motivated by the fact that we only observe the realized number of broker-dealers but not the initially available.

Figure 8: Bid-Price Distributions without agreement and $N = 2$



(a) PDF of Bid-Prices



(b) Kernel Density ($N = 2$)

In reality, some sellers could access offers from three (or more) broker-dealers, but chose to engage with two for simplicity. This increases the density of fees the closer they are to zero. Figure 8b displays the estimated Kernel densities using the transactions by sellers which trade with only one broker-dealer. As a small caveat, we did not estimate a discrete Kernel with a jump at zero, but rather relied on the standard assumption of continuity common to available statistical packages. Nevertheless, we can clearly see that values just above zero indeed have the highest density. Further, the density declines for higher values. Such decline is, however, convex rather than linear. This is additional evidence that some sellers may choose to let several broker-dealers compete but ultimately only engage with one.

5. Structurally Estimating Aggregate Inefficiency

Having hopefully convinced the reader that the security lending market is an appropriate application of our model, we can utilize the data to structurally estimate the aggregate market inefficiency stemming from exclusivity: How much are individual reductions in trading volumes impact aggregate trading volumes. We perform such counterfactual analysis by assuming away all EAAs in the data and predicting the aggregate increase in total lending.

A first challenge is to appropriately measure the *observed* lender-level trading volumes given that the model is one-shot while our data covers a trading year. To overcome this, we count each lender's traded International Securities Identification Numbers (ISINs) and her average trading volume per ISIN. Table 7 displays the average lender's Nr. of ISINs, avg. value per ISIN and total value given EAA status. Summing up all lenders' total loan values, we obtain a total market size of €998.18 bn.

Table 7: Observed Lending by EAA Status

	Without EAA		With EAA	
	Mean	SD	Mean	SD
Avg. Nr. of ISINs	94.85	183.23	51.05	133.43
Avg. Value per ISIN (€ mn)	6.54	17.84	3.48	8.68
Total Value (€ mn)	431.32	1393.86	193.90	667.57
Lenders	2216		441	

To obtain the counterfactual total lending, we want to predict how many ISINs each lender with an EAA would have lent out in the absence of such: The same with one counterfactual broker-dealer, 50% more ISINs with two counterfactual broker-dealer connections. A second challenges is here that neither model nor data allow us to infer which of the lenders would have one or two broker-dealer connections. We, therefore, simply obtain predictions for different shares of lenders having one versus two broker-dealer connections.

Then, for each share x in the range 0 to 1, we randomly assign lenders either one or two broker-dealers. The lenders assigned one counterparty remain untreated. The lenders with two randomly assigned broker-dealers are treated with an 50% increase in the number of ISINs. Simplified, assume that a lender is randomly indexed with $l \in \{1, \dots, l, \dots, L\}$ and recall that S denotes portfolio size (number of ISINs). Then:

$$\forall l \leq x \cdot L : \quad S^C = S \cdot 1.5, \tag{48}$$

$$\forall l > x \cdot L : \quad S^C = S. \tag{49}$$

Subsequently, we multiply each lenders' counterfactual size S^C with her (true) average loan value per ISIN to obtain the counterfactual total portfolio value. Summing over all lenders in the market and subtracting the original total market volume, we obtain the aggregate inefficiency. We repeat this process over 100,000 Monte Carlo simulations to obtain both average inefficiency and bootstrapped standard errors. A detailed description of the applied algorithm can be found in Appendix B.

Figure 9: Predicted Aggregate Inefficiency of ESLAs

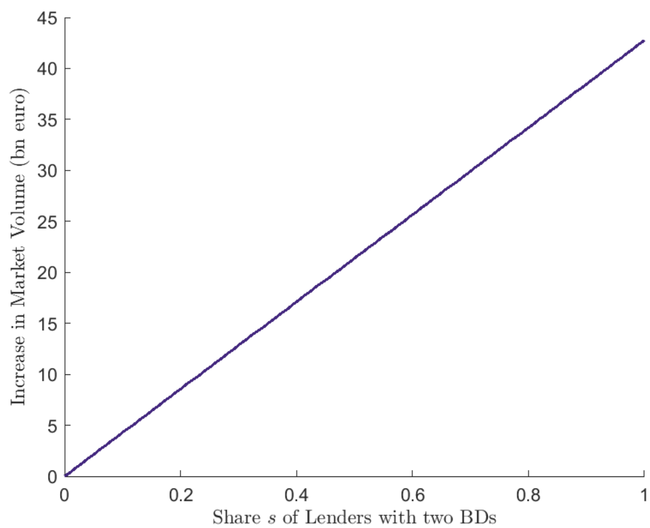


Figure 9 displays the structually estimated average reduction in trading volumes due to EAAs across the $100,000$ Monte Carlo draws. Bootstrapped confidence intervals are so narrow around the predictions that they would not be visible in the Figure. They are, thus, omitted with the comment here that all predictions are significant. Standard errors can be found in Appendix B.

We find that whenever at least some lenders with EAAs would instead trade with two broker-dealers, the overall trading volume increases significantly. Not surprisingly, this increase in total volume is bigger the more lenders are assumed to have two broker-dealer connections in the counterfactual. What is surprising is the magnitude. Even when only one-in-ten lenders are assigned two broker-dealers in the counterfactual, the total market volume increases by €4.28 bn . If all lenders with active EAAs had access to two competing broker-dealers market volume would even increase by €42.30 bn.

Conjecture 1. *In the counterfactual, ruling out ESLAs would likely results in additional lending in the billions of euro..*

We would like to acknowledge that the €42.30 bn are a not the largest sensible estimate of the predicted enlargement in traded portfolios. Motivated by the results of Proposition 1 and Remark 1, we assume in this exercise that EAAs only arise in equilibrium when lenders have one or two connected broker-dealers. The model environment, however, takes a lender's number of broker-dealers as exogenously given.

We could imagine a slightly modified model environment, where in a hypothetical period $t = -1$, a lender decides to only onboard with two rather than three available broker-dealers to force the EAA equilibrium to arise. This cannot be excluded in this model environment, as the following inequality shows:

$$\mathbb{E}_{-1}\Pi^l_E = \frac{Sa}{4} > \frac{Sa3}{16} = \mathbb{E}_{-1}\Pi^l(N = 3). \tag{50}$$

From Figure 2 in the individual level analyses above, we know that lenders with connected with three broker-dealers lend out more than those with two broker-dealers, holding the portfolio size constant. Studying all potential combinations of how lenders can be assigned one, two or three broker-dealer counterfactual connections goes beyond the scope of the study. To nevertheless give some indication, we additional compute the predicted increase assuming that all ESLA granting lenders had three counterparties instead. Under this extreme scenario, we predict an 8.03% increase trading volume or €80.16 bn.[18] Therefore, a more careful investigating into lenders' choice of broker-dealer connections seem like a natural extension of this paper.

Conjecture 2. *A reasonable upper limit of the increase in stocks being traded due to an out-ruling of ESLA agreements is €80.16 bn.*

[18] Note that because all lenders with ESLAs are assigned three broker-dealers, no bootstrapping of standard errors is possible, and hence no confidence intervals are obtained.

6. Conclusion

This paper has the two-fold objective of both rationalizing when exclusive access agreements arise in OTC markets and to quantify their impact on market efficiency. For these purposes, we develop a novel theoretical framework, where a representative seller has access to a small number of broker-dealers. The seller is endowed with a large equity portfolio, while the broker-dealers each are endowed with uncertain borrowing demand. Before the demand realizes, broker-dealers may compete for exclusive access to the lenders entire portfolio. If an agreement is granted, only the agreement-holder gets to buy those securities, where he will make a (weakly) positive bid-ask-spread. If no agreement was granted, broker-dealers compete on an equity level via second-price auctions for those securities, where they observe above value ask-prices.

The seller, thus, experiences a trade-off between higher *ex ante* competition over the entire portfolio but reduced lending *ex post* to only the holder, and higher *ex post* competition only in those securities that two or more broker-dealers demand. Due to the nature of price competition determining exclusive access terms, the *ex ante* competitive benefits of agreements are identical for all $N \geq 2$ (Bertrand Paradox). The benefits of the *ex post* competition, however, increases in the number of broker-dealers due to the increased likelihood of two or more positive demand realizations. Ultimately, this leads to exclusive access agreements only being granted by sellers with at most two broker-dealer connections. In these cases, exclusive access reduces transaction volume by one-third.

To quantity the aggregate inefficiency, we rely a structural estimation strategy that utilizes the newly available SFTR data covering security lending transactions with EU-based counterparties. We show theoretically, that the general model nests the security lending market. We, further, verify model assumptions by establishing that 35% of lenders have active exclusive agreements with broker-dealers. Ultimate borrowers, however, only engage in them 4% of the time, yet single home with a selected broker-dealer. Hence, the broker-dealer's independent demand assumption is likely justified. We also carefully show that model price predictions match realized fee distributions observed in the data.

In a final step, we provide an estimate for the aggregate in-efficiency induced by exclusive access agreements in the lending market. We find that access agreements jointly reduces trading by several billion euros: €4.28 bn if 10% of lenders would face two competing broker-dealers instead and €42.30 bn if 100% of lenders had access to two broker-dealer. However, a bank-of-the-envelope type calculation hows that if seller's choice of broker-dealer connections were endogenous, trade volume reductions could be as large as €80.1 bn annually. Thus, a natural extension is to expand the model to include an active choice of broker-dealer connections by the lender.

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Appendix A. Proofs

This section derives the results presented in the paper in consecutive order, starting with Lemma 1 deriving the optimal auction bid price.

Lemma 1 *The broker-dealers' optimal bidding strategy in a single second-price auction for security s at time $t = 2$ is:*

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq v_s \\ \emptyset & otherwise \end{cases}. \tag{51}$$

Aggregating the resulting payoffs over all S securities, the seller and broker-dealers realize the following respective total payoffs:

$$\mathbb{E}_1\Pi^S = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N=1} \qquad \mathbb{E}_1\Pi^n = \frac{Sa}{2^N}\frac{2^{N+1}-N-2}{N(N+1)}. \tag{52}$$

Proof.

0. For this proof, we assume to have arrived at $t=2$ and no EAA has been entered.

1. Recall that all a_s^n independent draws from the same uniform distribution with a mean at true value v_s: $a_s^n \sim U(v_s - a, v_s + a)$.

1.1. Thus, the probability of drawing above/vewlow value v_s in a single draw is 0.5.

1.2. The probability of having at least one out of N draws being above value is equal to one minus the probability of N below value draws. Relying on the symmetry of the uniform distribution around zero, this is equal to $1 - 0.5^N$.

1.3. Conditional on observing two above value draws, each broker-dealer is equally likely to have the higher draw.

2. Next, recall that a risk-neutral seller only provides the security, when receiving a weakly above value bid-price: $b_s \geq v_s$.

This lower limit on the price results in broker-dealers only participating when $a_s^n \geq 0$, as they would realize the a loss otherwise:

$$\pi_s^n = a_s^n - b_s^n \geq 0 \tag{53}$$

$$a_s^n \geq b_s^n \geq v_s. \tag{54}$$

.

Therefore, the optimal bidding strategy whenever $a_s^n < v_s$ is abstinence and $b_s^n = \emptyset$.

3. If $a_s^n \geq v_s$, truthful bidding is the optimal strategy.

3.1. First, assume that all other broker-dealers $k \neq n$ draw a below value ask-price $a_s^k < v_s$ and thus $b_s^k = \emptyset$. Then, n is indifferent between truthful bidding, overbidding and underbidding. In either case, he would always pay zero and make a profit $\pi_s^n = a_s^n - v_s \geq 0$.

3.2. Second, assume that at least one other broker-dealer k participates in the auction bidding a generic $b_s^k < a_s^n$. Here, the broker-dealer find truthful bidding optimal. Conditional on winning, n is indifferent between truthful bidding or bidding slightly less: He would win, pay b_s^k regardless, and make a positive return. However, underbidding below b_s^k is not optimal, as he would lose and forego a profitable transaction. Bidding truthfully maximizes the chance of winning without risking any losses or extra cost.

3.3 Finally, assume that n participates in the auction bidding a generic $b_s^k < a_s^n$. Then the broker-dealer n is indifferent between truthful bidding and underbidding: He would lose regardless making zero profits. With sufficient overbidding, the broker-dealer would win the auction. However, because $b_s^k < a_s^n$, he would make a loss. This is less optimal than the zero profits from truthful bidding.

3.4. With the same arguments applying for all other (symmetric) broker-dealers. We can conclude that the optimal bidding strategy is:

$$b_s^n = \begin{cases} a_s^n & a_s^n \geq v_s \\ \emptyset & otherwise. \end{cases} \tag{55}$$

4. With the optimal bids in mind, we now turn to calculating the seller's expected seller profits.

4.1. First note that the seller's profits from a single auction depend on the number of broker-dealers with a positive bid-price. Each broker-dealer independently draws an above value bid-price with probability one half. The total number of submitted bid-prices, thus, follows a Bernoulli distribution with N trials and $p = q = 0.5$.

4.2. Second recalls that the seller receives either the second highest bid, whenever two or more broker-dealers bid and value v_s otherwise. To capture this, denote the second-highest bid with $\max_{k \neq n} b_s^k \mid n$. With a slight abuse of notation, this operator automatically takes on the value zero, if strictly less than two broker-dealers participate.

4.3. Next, we rely on the distributional properties of the ask-prices and that S is large.

This allows us to apply the law of large numbers stating that expected and realized seller profits are approximately equal. Thus, the seller's profits Π^S from the S auctions at $t = 2$ are:

$$\Pi^S = S\sum_{n=2}^{N} Pr(n)\mathbb{E}_1\left[\max_{k\neq n} b_s^k \mid n\right] \tag{56}$$

$$= S\sum_{n=2}^{N}\begin{pmatrix} N \\ n \end{pmatrix} 0.5^N a\frac{n-1}{n+1} = S0.5^N a\left[\sum_{n=2}^{N}\begin{pmatrix} N \\ n \end{pmatrix}\frac{n}{n+1} - \sum_{n=2}^{N}\begin{pmatrix} N \\ n \end{pmatrix}\frac{1}{n+1}\right] \tag{57}$$

$$= S0.5^N a\left[\frac{N^2+3N-2^{N+2}+4}{2\left(N+1\right)} - \frac{(N-1)\left(N-2\left(2^N-1\right)\right)}{2(N+1)}\right] \tag{58}$$

$$= \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1}. \tag{59}$$

4.4. The total expected seller profits from S auctions with $N = 2$ are:

$$\mathbb{E}_1\Pi^S = SPr\left(a_s^n \& a_s^n > 0\right)\frac{a}{3} = \frac{Sa}{12}. \tag{60}$$

4.4. Notice that for seller's slight deviations between expected and realized profits to not matter for the model outcome. When the deviations realize at $t = 2$, entering an EAA is no longer possible. And hence, the seller is stuck with the auctions.

5. Next, we derive the seller profits from all S auctions, again we rely on the law of large numbers (LLN): For a large number of auctions, the realized total payoffs are close the expected payoffs. For simplicity, we assume exact equality for now and discuss slight deviations in Appendix C.

5.1. First, we denote the broker-dealer, whose perspective we take on, with n. Further, we let $k \neq n$ denote the index of the highest bidder of all other participating broker-dealers out of the remaining $N - 1$. Thus, $k \leq N - 1$.

5.2. Next, recall that every broker-dealer draws his own ask-price. We follow standard conventions and assume the probability of equal ask-prices (and thus bid-prices) to be zero:

$Pr(a^n_s = b^k_s) = 0$.

5.3. Then recall that a broker-dealer n expects a negative draw and zero profits with probability one half. With probability one half, he expects a positive draw. In that case, the number of other broker-dealers k that also have a positive ask-price draw, and thus participate, is again determined by the Bernoulli distribution.
The conditional likelihood of being the highest of $k = 1$ draws is $1/(k = 1)$.

5.4. Next, we let $\max_{k \neq n} b^k_s$ again denote the second highest bid, or zero in the absence of such. Conditional on winning, the broker-dealer then expects to make a profit between his realized ask-price and the second highest bid. Following the uniform distribution, the (expected) second highest bid out of $k + 1$ bids is:

$$\mathbb{E}[\max_{k \neq n} b^k_s \geq 0] = \frac{k}{k+2}. \tag{61}$$

Similarly, the n's expected ask-price, conditional on beating the second highest bid, is equal to having the highest of $k + 1$ draws:

$$\mathbb{E}_1 \left[a^n_s \mid a^n_s > \max_{k \neq n} b^k_s \geq 0 \right] = \frac{k+1}{k+2}. \tag{62}$$

5.5. Combining all above mentioned properties, a broker-dealer's expected profits are thus:

$$\Pi^n = SPr(a^n_s > 0) \sum_{k=0}^{N-1} Pr(k) Pr\left(a^n_s > \max_{k \neq n} b^k_s \geq 0 \right) \mathbb{E}_1 \left[a^n_s - \max_{k \neq n} b^k_s \mid a^n_s > \max_{k \neq n} b^k_s \geq 0 \right] \tag{63}$$

$$= S0.5 \sum_{k=0}^{N-1} \begin{pmatrix} N-1 \\ k \end{pmatrix} 0.5^{N-1} \frac{1}{k+1} a \left[\frac{k+1}{k+2} - \frac{k}{k+2} \right] \tag{64}$$

$$= \frac{Sa}{2^N} \frac{2^{N+1} - N - 2}{N(N+1)}. \tag{65}$$

5.5. In the case of two broker-dealers, total broker-dealer profits over all S auctions are:

$$\Pi^b = \frac{Sa}{8} + \frac{Sa}{24} = \frac{Sa}{6}. \tag{66}$$

Next, we move on to the proof for Lemmas 2 and 3. Before doing so, we briefly discuss the general properties of the seller's under EAAs. For this purpose, assume the seller has granted an EAA with a given uniform bid-price b_E^n. Then, the lending happens with probability:

$$Pr(a_s^n > b_E^n) = 1 - Pr(a_s^n < b_E^n) = 1 - \frac{b_E^n + a}{2a} = \frac{a - b_E^n}{2a} \tag{67}$$

Given this, the seller expects a total payoff:

$$\mathbb{E}_1\Pi^S = S(Pr(a_s^n > b_E^n)b_E^n + T_E^d = S\frac{a - b_E^n}{2a}b_E^n + T_E^d \tag{68}$$

The can similary dervie the EAA holders expected profits and show that thez are strictly decreasing in both b_E^n and T_E^d:

$$\begin{aligned}
\mathbb{E}_1\Pi_E^d =& S \cdot Pr(a_s^n > b_E^n)\mathbb{E}_1\left[a_s^n - b_E^n \mid a_s^n > b_E^n\right] - T_E^d & (69)\\
=& S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} - T_E^d & (70)\\
\frac{\partial \mathbb{E}_1\Pi_E^d}{\partial b_E^n} =& S\frac{b_E^n - a}{2a} < 0 & (71)\\
\frac{\partial \mathbb{E}_1\Pi_E^d}{\partial T_E^d} =& -1 & (72)
\end{aligned}$$

Lemma 2 *A seller only excepts one of multiple EAA offers whenever $N \leq 3$, in which case the optimal fees and expected profits are:*

$$b_E^n = 0 \qquad T_E^n = \mathbb{E}_1\Pi_E^S = \frac{Sa}{4} \qquad \mathbb{E}_1\Pi_E^n = 0 \quad \forall n. \tag{73}$$

Whenever $N \geq 4$, the seller strictly prefers engaging in the second-price auctions.

Sketch of Proof:

1. We start by establishing that both broker-dealers offer a bid-price and transfer combination that maximizes seller profit while neither resulting in losses nor gains for the broker-dealers. Any other combination prices cannot be sustained.

1.1. Assume that one broker-dealer sets a bid-price and transfer combination that allows him to make strictly positive profit. Then, any of the other broker-dealers can offer the same bid-price and marginally higher lump-sum transfer. This would allow him to attract the seller instead, and still making a positive, albeit slightly lower, profit.

1.2. Only if all broker-dealers make zero profit in equilibrium has no broker-dealer an incentive to deviate. Alternatively, every broker-dealer can simply attract the seller by offering part of said profit via a higher lump-sum transfer.

2. Next, we show that that there exists a single b_E^n and T_E^d combination that maximizes seller profits.

2.1. First note that the lump-sum transfer T_E^d must necessarily equate the chosen broker-dealer's expected profits to zero given the offered bid price b_E^n:

$$\mathbb{E}_1\Pi_E^d = S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} - T_E^d = 0 \tag{74}$$

$$T_E^d = S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} \tag{75}$$

2.2. Inserting this into the seller's profit function, we can see that the seller profits are highest at $b_E^n = 0$:

$$\mathbb{E}_1\Pi_E^S = \max_{b_E^n} S\frac{a - b_E^n}{2a}b_E^n + S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} \tag{76}$$

$$\frac{\partial \mathbb{E}_1\Pi_E^S}{\partial b_E^n} = -2\frac{S}{4a}b_E^n = 0 \tag{77}$$

$$b_E^n = 0 \tag{78}$$

2.2. Inserting the solution back into the broker-dealers zero profit condition yields the the following transfers:

$$T_E^d = S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} = \frac{Sa}{4} \tag{79}$$

3. Having derived optimal bid-price and lump-sum transfer, we can calculate the optimal broker-dealer seller profits: 3.1. Trivially, both broker-dealers make zero profits. This is regardless whether they are granted the EAA or not.

$$\mathbb{E}_1\Pi_E^n = 0 \quad \forall n \tag{80}$$

3.2. The seller then expects (and realizes due to LLN) the following profit:

$$\mathbb{E}_1\Pi_E^S = T_E^d = S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} = \frac{Sa}{4}. \tag{81}$$

4. Finally, notice that the auction pay-offs serve as a participation constraint for the seller. She may always reject an EAA offer to try her luck at those. Hence, EAAs are only every accepted then expected payoffs exceed those from the S auctions:

$$\mathbb{E}_1\Pi_E^S \geq \mathbb{E}_1\Pi^S \tag{82}$$

$$\frac{Sa}{4} \geq \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{83}$$

$$3 \geq N. \tag{84}$$

Hence, sellers accept competitive EAAs only if connected with less than four broker-dealers.

For completeness, we also derive optimal fee terms in case of a single EAA offer. Notice that those will be ruled out as potential SPNE.

Lemma 3*If offered a single EAA, the seller always accepts and the optimal fee and expected profits are:*

$$b_E^n = 0 \qquad T_E^n = \mathbb{E}_1\Pi_E^S = \mathbb{E}_1\Pi^S \qquad \mathbb{E}_1\Pi_E^n = \frac{Sa}{4} - T_E^n \qquad \mathbb{E}_1\Pi_E^k = 0 \quad \forall k \neq n \in N \tag{85}$$

1. We start by assuming only one broker-dealer offering an EAA and correctly anticipating the other broker-dealer not to.

2. Being a monopolist, the broker-dealer then sets the lowest combination of bid-price and transfer possible that still motivate the seller to grant the EAA. Hence, he equates the sellers participation constraint:

$$S\frac{a - b_E^n}{2a}b_E^n + T_E^d = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{86}$$

$$T_E^d = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} - S\frac{a - b_E^n}{2a}b_E^n \tag{87}$$

2.3. Inserting (87) into the sellers profit maximization problem yields:

$$\mathbb{E}_1\Pi_E^d = \max_{b_E^n \geq 0} S \cdot \frac{a - b_E^n}{2a}\frac{a - b_E^n}{2} - \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} + S\frac{a-b_E^n}{2a}b_E^n \tag{88}$$

And the FOC wrt. b_E^n is equal to:

$$\frac{\partial \mathbb{E}_1\Pi_E^d}{\partial b_E^n} = -\frac{S}{a4}2b_E^n = 0 \tag{89}$$

$$b_E^n = 0. \tag{90}$$

2.4. An optimal bid-price b_E^n implies a lump sum transfer:

$$T_E^d = \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{91}$$

2.5. The respective total payoffs are thus:

$$\mathbb{E}_1\Pi_E^S = \frac{Sa}{12} \tag{92}$$

$$\mathbb{E}_1\Pi_E^d = \frac{Sa}{4} - \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \tag{93}$$

3. Because the transfers take the participation constraint into account, the sellers always accept.

4. For the broker-dealers, the profits from EAA decrease in N. Here, offering a single EAA is only optimal as long it is better than the auction payoffs.

$$\mathbb{E}_1\Pi_E^d = \frac{Sa}{4} - \frac{Sa}{2^N}\frac{2^N(N-3)+N+3}{N+1} \geq \frac{Sa}{2^N}\frac{2^{N+1}-N-2}{N(N+1)} \tag{94}$$

$$N \leq 2 \tag{95}$$

Given the just derived pay-offs in the Lemmas 1-3, we now derive a set of candidate SPNEs.

Lemma 4 There always exists a candidate auction SPNE. It is sole candidate SPNE for $N \geq 4$.

Sketch of proof:

1. From Lemma 1, we know that both the seller and all broker-dealers make a strictly positive profit from the auctions. Hence, no one has an incentive to not participate, conditional on all other agents participating.

2. From Lemma 2, we know that the seller rejects any competitive ELSAs for $N \geq 4$.

3. From Lemma 3, we know that no monopolistic EAA is offered for $N \geq 3$.

4. Combining all three, there always exists a candidate auction SPNE and that it is the sole candidate for $N \geq 4$.

Lemma 5 *There exists no candidate SPNE with a single broker-dealer offering an EAA.*

Sketch of proof:

1. From Lemma 3, we know that a single broker-dealer only makes a monopolistic EAA offer for $N = 2$. In all other cases, enticing the seller to participate is too costly.

2. For $N = 2$, however, the other broker-dealer has no incentive to actually refrain from offering an EAA. Not offering an EAA results in zero profits for the competitor. As argued in Lemma 2, the other broker-dealer could alternatively offer a slightly higher lump-sum transfer, winning the seller over and still making a profit.

3. As Lemma 2 highlights, such logic can be continued ultimately leading to the zero profit Bertrand paradox typically found in price competition settings.

Lemma 6 *For $N \leq 3$, there exists several competitive EAA SPNEs, each characterized by either two or three broker-dealer competing via EAAs.*

Sketch of proof:

1. From Lemma 2, we know that the seller rejects any competitive ELSAs for $N \geq 4$, but accepts those for $N \leq 3$.

2. Here, conditional on some or all other broker-dealers making a competitive EAA, a single EAA is indifferent between making also a competitive EAA offer or not: he would be left with zero profits in either case.

3. As all broker-dealers are symmetric, this holds for every broker-dealer. Hence, an

candidate SPNE with competitive EAAs exist.

4. Note, here for $N = 2$ is requires both broker-dealers making a competitive EAA. For $N = 3$, it only requires two out of three making one.

Finally, we turn to test whether our candidate SPNEs are termination proof. This requires that the EAA holder has no incentive to terminate the EAA at $t = 2$, compensating the seller for potential losses.

Proposition 1

1. By assumption, the auction SPNE is termination proof. Once all agents have arrived at $t = 2$, no further EAA can be offered and/or accepted. Hence, arising at $t = 1$ it remains unchallenged at $t = 2$.

2. Assume now an EAA was granted at $t = 1$. The EAA holder can terminate the contract at $t = 2$, triggering the auction SPNE. However, he must compensate the seller for the profit losses due to breach of contact. The EAA holder thus terminates if:

$$\mathbb{E}_1\Pi^n - \left(\mathbb{E}_1\Pi_E^S - \mathbb{E}_1\Pi^S\right) \leq \mathbb{E}_1\Pi_E^n. \tag{96}$$

3. Let us start by assuming $N = 3$ and insert this into (96). It can easily be shown that the inequality is violated:

$$\frac{Sa11}{96} - \left(\frac{Sa}{4} - \frac{Sa3}{16}\right) > 0 = \mathbb{E}_1\Pi_E^n. \tag{97}$$

This implies that for $N = 3$, the EAA holder terminates the EAA and triggers the auction SPNE instead.

4. Inserting $N = 2$ into (96) instead yields in an equality:

$$\frac{Sa}{6} - \left(\frac{Sa}{4} - \frac{Sa}{23}\right) = 0 = \mathbb{E}_1\Pi_E^n. \tag{98}$$

Hence, for $N = 2$, the equilibrium is termination proof.

5. Just to provide some intuition, the punishment for termination decreases faster than the benefits from termination. And hence, for $N = 3$, the EAA holder gains slightly less

from termination than for $N = 2$, but pays substantially less punishment.

Finally, we conclude with studying the case of a single connected broker-dealer and $N = 1$.

Remark 1 *For $N = 1$, the seller is indifferent between being offered an EAA or not, as the broker-dealer is a monopolist that always extracts all transaction surplus:*

$$b_s = b_E = T_E = \Pi^S = 0 \qquad \Pi^n = \frac{Sa}{4}. \tag{99}$$

Sketch of proof:

1. Starting with the auctions at $t = 2$. Given that there is no competitive bid, the broker-dealer always pays a zero bid prices and realities the whole -ask price as a profit for every purchased security. This leads to:

$$\Pi^n = SPr(a_s^n > 0)\mathbb{E}_1[a_s^n \mid a_s^0] = \frac{Sa}{4} \tag{100}$$

$$\Pi^S = 0. \tag{101}$$

2. Because the auction profits serve as the sellers outside value, the broker-dealer can offer an EAA with zero bid-price and lump-sum transfer. The profits of both agents remain the same.

Appendix B. Computing the Aggregate In-Efficiency

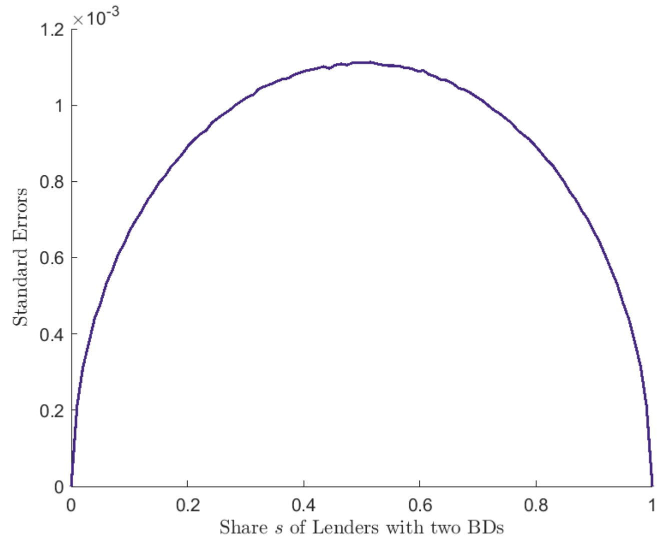
In this section, we describe our algorithm to compute the predicted increase in trading due to the counterfactual assumption of outlawing ESLAs. In a first step, we calculate the portfolio size of each lender in our data. Here, we decided to measure portfolio size by the number of distinct ISINs underlying the transactions. An alternative could have been to use the number of transactions or trading volume. However, such measures would significantly inflate the market size as some transactions are rolled over daily over an extended horizon. But de facto, no new equity has exchanged hands. With this, we can derive an initial total market size as the sum of all portfolios.

Next, we divide the lenders into those with ESLAs and those without. We would like to acknowledge that in the data, we observe a small number of lenders that enter an ESLA with their main broker-dealer but trade a small part of their portfolio with others. As we do not observe the exact contract terms here, we act conservatively and assume those behave equivalently to those lenders with multiple counterparties and no ESLA. The portfolios of the lenders without ESLAs are temporarily put aside. On those with ESLAs, we perform the following bootstrapping algorithm :

1. We define a grid of 100 values denoted x falling in equal steps between zero and one: $x \in [0, 1]$. Each x represents the share of lenders having two versus one connected broker-dealers in the counterfactual.
2. For each share x defined in the grid, we repeat for 100,000 times:
   (a) Draw a random number from a standard uniform distribution for each lender.
   (b) Sort the lenders by their draw.
   (c) Assign the first x share of lenders two counterfactual broker-dealers and the remaining $1 - x$ one.
   (d) For those, where we assign two broker-dealers, we follow Corollary 1 and predict a 50% increase in the traded portfolio.
   (e) We compute the counterfactual aggregate portfolio by summing the counterfactual portfolios of the ESLA holders and the actual portfolios of the non-ESLA holders.
   (f) We obtain the percentage increase in aggregate portfolio relative to the true total.
3. From the 100,000 bootstraps, we calculate the average predicted increase in total portfolio and the bootstrapped standard errors. Wih both, we can obtain the 5th and 95th confidence intervals.

Figure 1 in the main text shows the predicted increases in traded portfolios, but confidence intervals are omitted due to their small size. Figure 10 below plots the 5th to 95th percentile confidence range around the mean:

Figure 10: Confidence Intervals around the predicted increase



.

$$upper = +1.96 \cdot std, \tag{102}$$

$$lower = -1.96 \cdot std. \tag{103}$$

Here, we would like to point out that, as expected, the standard errors increase for medium shares. Here, there is simply higher variation between the bootstraps when assigning one versus two counterparties at random. Nevertheless, the standard errors always remain well below 0.02.

Appendix C. Extension: Ex Post Asymmetric Broker-Dealer

In the baseline model, the LLM ensures that all broker-dealers are both *ex ante* and *ex post* identical. In this extension, we check whether the ESLA SPNE is still termination proof, when the holder (unexpectedly) enjoys positively biased ask-price draws and, thus, on average can borrow more. Specifically, we assume that the cumulative probability of obtaining a draw above 0 for is ϵ above one-half. Denote such broker-dealer with biased draws with superscript b. Then:

$$Pr(a_s^b > 0) = 0.5 + \epsilon \qquad\qquad Pr(a_s^b \leq 0) = 0.5 - \epsilon. \tag{104}$$

To maintain tractability, we assume that conditional on having a positive or negative draw, the probability density is still uniform. Given this, the refined probability density function becomes:

$$Pr(a_s^b) = \begin{cases} \frac{0.5-\epsilon}{a} & a_s^b \leq 0 \\ \frac{0.5+\epsilon}{a} & a_s^b > 0 \end{cases}. \tag{105}$$

Assume that a broker-dealer arrives at $t = 2$ and observes the above described biased draws. In alignment with the private information assumption underlying second-price auctions, we assume that neither the lender nor the other broker-dealers are aware of such bias. Given other's oblivion, the expected profits from biased draws in the S auctions are:

$$\Pi^b = SPr(a_s^b > 0) \sum_{k=0}^{N-1} Pr(k) Pr\left(a_s^b > \max_{k \neq b} b_s^k \geq 0\right) \mathbb{E}_1\left[a_s^b - \max_{k \neq b} b_s^k \mid a_s^b > \max_{k \neq b} b_s^k \geq 0\right] \tag{106}$$

$$= \frac{Sa(0.5+\epsilon)}{2^{N-1}} \frac{2^{N+1} - N - 2}{N(N+1)} \tag{107}$$

Because within the interval $[0, a]$ the ask-price a_s^b is still uniformly distributed, the expected pay-off from an auction, conditional on participation, is not affected. However, the broker-dealers likelihood of participating in an auction has been increased by ϵ. Hence, the biased broker-dealer'expected auction profits can alternatively be described as:

$$\Pi^b = (1 + 2\epsilon)\Pi^n. \tag{108}$$

Now assume that we are in the ELSA SPNE and broker-dealer b was the one originally assigned the ESLA. Bid-price and transfers are thus:

$$b_E^b = 0 \qquad\qquad T_E^b = \frac{Sa}{4}. \tag{109}$$

Here, again the biased draw increases the likelihood of a positive ask-price but not the expected ask-price conditional on being positive. Given this, the broker-dealer's profits under an ESLA become:

$$\Pi_E^b = SPr(a_s^b > 0)\mathbb{E}[a_s^b \mid a_s^b > 0] - \frac{Sa}{4} = \frac{aS\epsilon}{2}. \quad (110)$$

To check whether the ESLA SPNE remains termination proof, we must again account for the compensation that the broker-dealer must pay the lender upon termination:

$$\Pi_E^b \geq \Pi^b - (\Pi_E^l - \Pi^l) \quad (111)$$

$$\frac{\epsilon Sa}{2} \geq \frac{Sa}{6} + \frac{Sa\epsilon}{3} - \left(\frac{Sa}{4} - \frac{Sa}{12}\right) \quad (112)$$

$$\epsilon \geq 0. \quad (113)$$

From inequality 86, we can conclude that when observing just a slightly biased draw, the broker-dealer strictly prefers not to terminate.