Imposing Time-series Restrictions on Cross-sectional Asset Pricing Regressions: A Note

By

Antoine Giannetti, Florida Atlantic University

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Corresponding author: Antoine Giannetti, Florida Atlantic University, College of Business, Department of Finance, 777 Glades Road, Boca Raton, FL, 33431
e-mail: giannett@fau.edu, phone: (561)297-3192

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Abstract

A fundamental implication of asset pricing theory is that investors must earn risk-premiums for bearing exposure to systematic risk. The two-pass cross-sectional regression is a popular approach for risk-premium estimation. The empirical literature has found that this approach often delivers estimates that significantly differ from their time-series counterparts. The paper explores a test of model misspecification that exploits the contrast between cross-sectional and time-series risk-premium estimates. The suggested approach complements the traditional inclusion of firm characteristics to detect model misspecification in cross-sectional regressions.

Keywords: Asset Pricing; Risk-premium; Model Misspecification; Simulations

1. Introduction

The main contribution of this work is to explore the testing of time-series restrictions imposed on cross-sectional risk-premiums. Traditionally, two empirical approaches have been used to identify risk-premiums in linear asset pricing models. One is the time-series regression approach pioneered by Black, Jensen and Scholes (1972) and further formalized by Gibbons, Ross and Shanken (1989). In this first approach, risk-premiums are identified as the time-series averages of factor realizations. Statistical significance of the estimated risk-premiums is assessed through a simple test of means.[1] The second approach is the two-pass cross-sectional regression introduced by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973). The first-pass, run in time-series, estimates the factor loadings. The second-pass, estimated in cross-section, delivers the risk-premium estimates. Researchers have long been interested in comparing the magnitudes of cross-sectional estimates to their time-series counterparts.[2]

Theoretically, it is well known that the time-series approach may only be utilized when risk-factors are traded assets (Cochrane (2005), Ferson (2019)). The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Black (1972) is one such model. While the two-pass approach is available irrespective of whether risk-factors are traded assets,[3] in practice, researchers often elect the time-series over the two-pass approach when factors are traded (see e.g. Fama and French (1993), Fama and French (2015)). The main benefit, as articulated by Lewellen and Nagel (2006) and Lewellen, Nagel and Shanken (2010), is that time-series regressions

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[1] Joint significance of the risk-premiums may be tested with a standard F-test (see e.g. Goyal (2012)).

[2] For instance, while testing the CAPM, Black, Jensen and Scholes (1972) report a zero-beta rate significantly larger than the average risk-free rate and a cross-sectional slope significantly smaller than the average market risk premium.

[3] When some or all the risk-factors included in the model are not traded assets (macroeconomic variables like inflation, GDP growth, etc. fill the bill), the researcher may elect to substitute mimicking portfolios of traded assets in lieu of the non-traded factors. See Breeden (1979).

automatically incorporate parameter restrictions on estimated risk-premiums. More specifically, risk-premiums estimated through time-series regressions are, by design, identical to the time-series averages of the factors. However, no such restrictions apply to second-pass cross-sectional estimates.[4] The magnitude of cross-sectional risk-premium estimates is statistically and economically unrestricted.

An added inference issue with the two-pass approach is the error-in-variables problem resulting from the use of first-pass estimated factor loadings as explanatory variables in the second-pass cross-sectional regression. Regardless, the two-pass approach avails itself to informative misspecification tests. In particular, the researcher may include control variables (like firm characteristics) on the right hand side of the second-pass cross-sectional regression to test whether the factor loadings account for all the explanatory power of the factor model (see e.g. Jagannathan and Wang (1998) and Jagannathan, Skoulakis and Wang (2010) for a detailed analysis of this technique). However, as factor loadings are estimated parameters and characteristics are not, measurement error is biasing, perhaps severely, the diagnostics of such misspecification tests (see e.g. Berk (2000)).

This paper explores an alternative test of misspecification when factors are traded assets. The proposed test exploits a well-known econometric link between the time series and the two-pass approach (Shanken (1992) and Cochrane (2005)). This link exists when the test-assets are augmented by the traded factors *and* the second-pass cross sectional regression is carried out using feasible GLS (Generalized Least Squares) with the estimated covariance matrix of first-pass residuals. When those sampling and testing requirements are met, it is well known that the GLS

[4] Lewellen, Shanken and Nagel (2010) argue that one may reasonably expect that "the risk premium associated with a factor portfolio should be the factor's expected excess return."

cross-sectional risk-premiums estimates are numerically identical to their time-series counterparts.[5] The latter observation suggests a test of contrast. Under the null hypothesis that a factor model is correctly specified, both the OLS (Ordinary Least Squares) and the feasible GLS estimators are consistent estimators of the second-pass cross-sectional risk-premiums; thus, under the null hypothesis, the numerical contrast between the OLS and GLS estimators must asymptotically be nil. Given a consistent estimator of the variance of the contrast, a researcher may test the null hypothesis by constructing a standard Wald test. Hausman (1978) provides the blue-print for such a procedure. Under the null hypothesis, the GLS estimator also is the efficient[6] estimator of the risk-premiums. In this context, Hausman shows that the variance of the contrast reduces to the contrast of the variances of the estimators. It is worth mentioning that the Hausman misspecification test is ubiquitous in empirical research. For instance, it is routinely applied in panel data analysis to decide whether fixed or random effects are better suited to capture unobserved heterogeneity. The test is also common in regression analysis to diagnose endogeneity problems and settle for either an asymptotically consistent instrumental variable or a more efficient least square estimation procedure.

This work extends recent empirical asset pricing literature that has focused on factor risk-premiums identification. Hou and Kimmel (2010) address the well-known Fama and French (1992) puzzle[7] that while the market risk-premium estimate is positive and statistically significant in time-series, it is often negative and insignificant in cross-sectional regressions. The former

[5] In their "Prescription 4", Lewellen, Nagel and Shaken (2010) argue that "a GLS cross-sectional regression, when a traded factor is included as a test-asset, is similar to the time-series approach of Black, Jensen and Scholes (1972) and Gibbons, Ross and Shanken (1989)"

[6] If the i.i.d. assumption for returns is violated, the GLS estimator is not fully efficient (see e.g. Cochrane (2005)). In this case, one may proceed by computing a robust Hausman test (see e.g. Cameron and Trivedi (2005).

[7] Roll and Ross (1994) is an early reference. Recently, in dealing with the omitted variable problem in large cross-sections, Giglio and Xiu (2021) emphasize the relevance of identifying a cross-sectional market risk premium close to its time-series average.

authors argue that the culprit is “extrapolation”, the fact that the risk-factors are unspanned. When risk-factors are unspanned (i.e. they are not included in the test-assets set), cross-sectional risk-premium estimates depend on the number and identity of all the risk-factors and test-assets. Conversely, when risk-factors are spanned, cross-sectional risk-premium estimates are unaffected by the inclusion/removal of risk-factors or test-assets. Lewellen, Nagel and Shaken (2010) advocate the imposition of time-series restrictions on cross-sectional risk-premium estimates. In their “Prescription 2: Take the magnitude of the cross-sectional slopes seriously”, they argue that time-series restrictions could be tested; however, they do not propose an actual test.

Shanken and Zhou (2007) introduce a test of specification characterized as a test of expected returns linearity. Their test obtains as a special case of their derivation of the asymptotic distribution of the two-pass risk-premium estimator under model misspecification. This asymptotic distribution is of interest when an asset pricing test performed on the residuals rejects the tested model, but the researcher is still interested in conducting inference on the estimated risk-premiums. While proceeding with their derivation, Shanken and Zhou (2007) observe that when an asset pricing model is correctly specified, all least square estimators (including OLS and GLS) must asymptotically converge to the same limit because they all are consistent estimators of the “true” risk-premiums. Computationally, the Shanken and Zhou test is a Wald test that relies on the greater efficiency of the GLS estimator under conditional homoscedasticity in the asset returns. In this paper, the use of GLS as the efficient estimator is not emphasized. Rather, the emphasis is on the role of GLS as the preferred estimation approach to economically pin down the cross-sectional risk-premium estimates. As such, even if the homoscedasticity assumption is not met, GLS estimation combined with test-assets augmentation is desirable. This is because irrespective of the considered test-assets and/or factors, GLS performed on the augmented test-assets allows for a unique identification of the cross-sectional risk-premiums as time-series averages of the risk-factors. This paper makes two distinct contributions. First, a novel derivation and interpretation of the Shanken and Zhou (2007) as a test of contrast à la Hausman (1978) is developed. Second, an alternative expression to compute the test of contrast is provided. This alternative expression is convenient because test-assets augmentation results in the singularity of the covariance matrix of first-pass residuals. This singularity, in turn, renders computation of the original test of contrast cumbersome.

The outline is as follows. In Section 2, the conceptual framework is introduced. In Section 3, a test of contrast is developed. Section 4 documents the statistical size and power of the test through simulations. Section 5 applies the test to a number of traded factor models and test-assets common in the empirical literature. In particular, it is found that the Fama and French (2015) five factor model is not rejected by a powerful test of contrast applied to the 32 portfolios formed on size, operating profitability and investment. Section 6 provides concluding remarks.

2. Conceptual Framework

Assume that a cross-section of asset returns $R_t = [R_{1,t}, \dots, R_{N,t}]'$ is driven by the following time-series linear process:

$$R_{i,t} = a_i + \beta_{i,1} f_{1,t} + \cdots + \beta_{i,K} f_{K,t} + \epsilon_{i,t} \tag{1}$$

where $i = 1, \dots, N$ denotes the i–th asset and t with $t = 1, \dots, T$ is the time index. Traded factors $f_t = [f_{1,t}, \dots, f_{K,t}]'$ with $k = 1, \dots, K$ account for systematic risk affecting asset returns $R_{i,t}$. The full-rank covariance matrix of the factors is denoted by Σ_f. Factor loadings $\beta_i = [\beta_{i,1}, \dots, \beta_{i,K}]'$ account for asset i exposure to the factors. Intercepts a_i are the so-called time-series alphas. As is common in the literature, disturbances $\epsilon_{i,t}$ are assumed to be independent over time and jointly distributed with mean zero and full-rank covariance matrix Σ_ε conditional on the factors.[8] A linear asset pricing model places the following restriction on the assets' expected returns:

$$E[R_t] = X\Gamma \tag{2}$$

For concreteness, two alternative specifications are entertained. In the first specification, one considers asset returns in excess of a designated zero-beta asset returns. In this case, R_t denotes a N-vector of returns in excess of this designated zero-beta asset returns,[9] $X = B$ with $B = [\beta_1, \dots, \beta_K]'$ is the $N \times K$ matrix of factor loadings and $\Gamma = \gamma$ with $\gamma = [\gamma_1, \dots, \gamma_K]'$ is the K-vector of market risk-premiums. Alternatively, one may consider the N-vector R_t of raw returns (see e.g. Jagannathan and Wang (1998)). In this specification, $X = [1_N\ B]$ and 1_N is a N-vector of ones. The zero-beta rate is denoted by γ_0 so that $\Gamma = [\gamma_0, \gamma']'$ in this specification. The second specification attempts to explain the levels of expected returns.

In sample, one may sequentially estimate expressions (1) and (2) by using the so-called two-pass cross-sectional regression approach pioneered by Black, Jensen and Scholes (1972). In the first-pass, one runs N time-series regressions to obtain factor loadings estimates $\hat{\beta}_i = [\hat{\beta}_{i,1}, \dots, \hat{\beta}_{i,K}]'$. In the second-pass, one estimates the factor risk-premiums by running the following cross-sectional regression:

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[8] This setting is referred to as conditional homoscedasticity as the asset returns are independent and identically distributed conditional on the time-series of the risk-factors (Shanken (1992) Assumption 1). Violations of conditional homoscedasticity are theoretically explored in Jagannathan and Wang (1998) and Kan Robotti and Shanken (2013).

[9] Cochrane (2005) argues that: "In fact, much asset pricing focuses on excess returns. Our economic understanding of interest rate variation turns out to have little to do with our understanding of risk premia, so it is convenient to separate the two phenomena by looking at interest rates and excess returns separately."

$$E_T[R_t] = \hat{X}\Gamma + \alpha$$

(3)

where $E_T[R_t]$ is the time-series average of the asset's returns and α is the vector of cross-sectional pricing errors. The explanatory variables are denoted by $\hat{X} = \hat{B}$ (with $\hat{B} = [\hat{\beta}_1 \dots, \hat{\beta}_K]'$) for the excess-returns specification and $\hat{X} = [1_N \ \hat{B}]$ for the raw-returns specification. Similarly, the sample estimate of the covariance matrix of the factors is estimated by $\hat{\Sigma}_f^* = \hat{\Sigma}_f$ for excess returns and by the "bordered" covariance matrix $\hat{\Sigma}_f^* = \begin{bmatrix} 0 & 0 \\ 0 & \hat{\Sigma}_f \end{bmatrix}$ for raw returns. Effectively, the excess-returns cross-sectional regression is estimated without an intercept while the raw returns' includes one to allow for estimation of the zero-beta return. Second-pass slope estimators are generically given by:

$$\hat{\Gamma} = \left(\hat{X}'\hat{A}^{-1}\hat{X}\right)^{-1}\hat{X}'\hat{A}^{-1}E_T[R_t]$$

(5)

where $\hat{A}$ is either a $N + K$ augmented matrix for the excess returns setting (N assets plus K traded factors) or $N + K + 1$ matrix for the raw returns setting (as the pseudo zero-beta asset is also added). For the OLS excess returns estimator, one uses $\hat{A} = I_{N+K}$ where I_{N+K} stands for the $N + K$ identity matrix ($\hat{A} = I_{N+K+1}$ is used for the raw returns). Computation of the feasible GLS estimator usually involves the covariance matrix of first-pass residuals. In this paper, even though Σ_ε, the covariance of time-series disturbances is assumed full-rank, Σ_a the augmented disturbances covariance matrix is necessarily singular (see Appendix A for further details).

Regardless, it is well known that the GLS risk-premium estimator may also be computed by using $\hat{\Sigma}$, the estimated covariance matrix of asset returns[10] (in lieu of first-pass residuals).

As pointed out by Shanken and Zhou (2007), risk-premium estimates obtained through the two-pass cross-sectional regression approach are unrestricted. When risk-factors are traded assets, however, this is not a tenable position. Because the time series is long and risk-factors are traded, the law of large numbers kicks in so that risk-premiums are consistently estimated by averaging out factor realizations over time. Thus, the sample vector:

$$\hat{\gamma} = \left[E_T\left(f_{1,t}\right), \dots, E_T\left(f_{K,t}\right)\right]' \tag{4}$$

is a consistent estimator of γ the population vector of "true" risk-premiums. The latter implies a constraint on the cross-sectional estimates of risk-premiums. This constraint has been recognized in the literature (see e.g Lewellen and Nagel (2006), Daniel and Titman (2012)). In their Proposition 2, Lewellen, Nagel and Shanken (2010) explicitly recommend to "take the magnitude of the cross-sectional slopes seriously" by incorporating time-series restrictions into cross-sectional risk-premium estimation. The next section heeds their recommendation by exploring a test of misspecification that assesses whether time-series constraints are binding on the cross-sectional risk-premiums estimates.

[10] See Cochrane (2005) pp. 238 or Lewellen, Nagel and Shanken (2010) Appendix A Result 1. The matrix of augmented test-asset returns is assumed full-rank.

3. A Test of Contrast

There are several empirical procedures available to estimate the second-pass cross-sectional regression in (3). In particular, since the returns' disturbances are assumed correlated, it is natural to favor a GLS cross-sectional regression over the traditional OLS cross-sectional regression. When the asset pricing model is correctly specified, Shanken (1992) proves that the GLS risk-premiums estimator is asymptotically efficient. Shanken and Zhou (2007) propose a test they characterize as a test of expected returns linearity. Their argument is, if the linear specification in (3) is correct, then all least-square estimators must asymptotically deliver consistent estimates of the risk-premiums.[11] In particular, under the null hypothesis in (3), OLS and GLS estimators must converge to the same limit as the number of periods gets large. Consequently, relying on the greater efficiency of GLS over OLS under the null hypothesis, they suggest a test of contrast between the two estimators (Shanken and Zhou (2007) Proposition 2).

This paper explores a related approach to the testing of time-series constraints applied to cross-sectional risk-premiums estimates. The proposed approach relies on i) augmenting the test-assets by including the traded factors for the case of excess returns and the traded factors plus the designated zero-beta asset in the case of raw returns; ii) performing both an OLS and a GLS regression on the augmented test-assets set; and iii) testing the contrast between OLS and GLS estimates. When traded factors are included as test-assets, a GLS cross-sectional regression estimates risk-premiums that are identical to the time-series averages of the traded factors.[12] Importantly, the latter occurs in any sample, irrespective of estimation error in the matrix of first-

[11] More specifically, the authors point out that, under the null hypothesis, risk-premiums estimators are independent of the weight matrix used to perform second-pass estimation.

[12] See also Lewellen, Nagel and Shanken (2010) Proposition 4: "If a proposed factor is a traded portfolio, include it as one of the test-assets on the left-hand side of the cross-sectional regression".

pass residuals. In the excess-returns setting, it is imperative to observe that time-series regressions of the traded factors on themselves result in a perfect fit. The ensuing GLS cross-sectional regression picks up the latter from the estimated matrix of first-pass residuals, and consequently, places maximal weights on the noiseless factors. Thus, cross-sectional slopes are identical to the factors' time-series averages. In the raw returns setting, the GLS cross-sectional slopes are also equal to the factor time-series averages. Details of the proof are deferred to Appendix A. In either case, excess returns or raw returns, estimating a GLS cross-sectional regression enforces the time-series constraints in expression (4) while an OLS cross-sectional regression does not. As such, the OLS estimator is an unrestricted estimator while the GLS estimator is the restricted estimator that enforces the time-series constraints on cross-sectional risk-premium estimates.

A test of whether time-series restrictions are statistically binding is constructed by evaluating the contrast between the OLS and GLS estimators and is commonly referred to as a Hausman (1978) test. Under the null hypothesis that the asset pricing model is correctly specified, both the OLS and GLS estimators are consistent estimators of the "true" risk-premiums. Additionally, under conditional homoscedasticity, the GLS estimator is more efficient. Under the alternative hypothesis, neither estimator needs be consistent as long as they converge to different limits in probability.[13] Because the test-assets have been augmented as previously described, the proposed approach does not actually test the linearity of the expected returns relationship. Linearity is assumed under both the null and the alternative hypothesis. The alternative hypothesis merely consists in the unrestricted linear projection of the asset returns onto the factor loadings; it

[13] Technically, from expression (8.34) in Cameron and Trivedi (2005), the Hausman test requires that $plim(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}) = 0$ under the null hypothesis and $plim(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}) \neq 0$ under the alternative. The stronger, classic requirement of Hausman (1978) is that one of the estimators is consistent and efficient under the null hypothesis but inconsistent under the alternative hypothesis, while the other estimator is consistent under both alternatives.

can fit all sorts of underlying economic models for returns. As such, the suggested procedure may be construed as a misspecification test rather than a specification test. [14] Ultimately, the goal is to test the magnitude of the cross-sectional premium deviations from their time-series counterparts. If the deviations are statistically "small," then the time-series constraints are binding on the cross-sectional risk-premium estimates, and the researcher fails to reject the asset pricing model. From Greene (2012) pp. 235, let H denote the following statistic:

$$\mathrm{H} = \left(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}\right)' \left[Var\left(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}\right)\right]^{-1} \left(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}\right) \tag{4}$$

where $\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}$is the contrast of the estimated risk-premiums using OLS (respectively GLS), and $Var\left(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}\right)$ denotes the asymptotic variance of the contrast. The evaluation of the variance of a contrast between an efficient estimator (GLS) and a consistent but inefficient estimator (OLS) is the main Hausman (1978) contribution.[15] Appendix B shows that in the current setting, the asymptotic variance of the contrast is given by:

$$Var\left(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}\right) = T^{-1}(1+\hat{c})\left[\left(\hat{X}'\hat{X}\right)^{-1}\hat{X}'\,\hat{\Sigma}_{\alpha}\hat{X}\left(\hat{X}'\hat{X}\right)^{-1} - \left(\hat{X}'\,\hat{\Sigma}_{\alpha}{}^{-1}\hat{X}\right)^{-1}\right] \tag{5}$$

where $(1+\hat{c})$ is the Shanken (1992) correction for EIV in the estimation of the factor loadings $\hat{X}$ and $\hat{\Sigma}_{\alpha}$ is the estimate of the second-pass pricing errors covariance matrix. Expression (5) is intuitive since the first term on the right-hand side represents the covariance matrix of the OLS estimator, while the second term accounts for the covariance matrix of the GLS estimator. The

[14] Harvey (1990) pp. 148 argues "unlike a test of specification, therefore, a test of misspecification is constructed with no clear alternative in mind. As such, it is a procedure designed for assessing the goodness of fit of a model implied by a particular maintained hypothesis."
[15] See Greene (2012) pp. 235 for details.

greater efficiency of GLS under the null hypothesis implies that the latter covariance matrix must be "smaller" than the former. Finally, from standard results on the distribution of quadratic forms,[16] the H statistic is asymptotically Chi-squared distributed $\chi(K)$ for excess-returns and $\chi(K+1)$ for raw-returns when a zero-beta intercept is added. The next section explores the finite sample properties of the proposed misspecification test.

4. Simulation Results

This section documents the size and power of the proposed Hausman statistics for various linear factor models and test-assets. All data comes from the Ken French webpage and spans the time-period from July 1963 to December 2018 (666 months). The models considered are the CAPM, the Fama and French 3 factor model (FF 3) and the Fama and French 5 factor model (FF 5). For the CAPM, test-assets are the Fama and French 25 portfolios (25 P), formed on size and book-to-market, the 25 P augmented by the 5 industry-sorted portfolios (25 P + 5 IND), and lastly, the 30 industry-sorted portfolios (30 IND). For the FF3 model, the same portfolios as for the CAPM are utilized. Finally, for the FF 5 model, the 32 portfolios, formed on size, operating profitability and investment (32 P), are substituted to the 25 P. Using those test-assets along with time series for the factors[17] Rm-Rf, SMB, HML, RMW, CMA and the one-month Treasury bill return for the zero-beta asset return, Monte Carlo simulations are carried out following Cochrane (2005) pp. 287. More specifically, each aforementioned factor model/test-assets are used to run first-pass regressions that estimate the time-series alphas and factor loadings as well as the

[16] See e.g. Greene (2012) pp. 130 Theorem 5.1
[17] Details about the factors are found in Fama and French (1993), Fama and French (1996), and Fama and French (2015).

covariance matrix of the factors and residuals. Based on those historical estimates, 10,000 sets of simulated test-asset returns are generated for T=60 to T=960. Factors and residuals are jointly drawn from two distinct statistical distributions; a multivariate i.i.d. normal distribution in the homoscedastic case and a multivariate *t*-distribution with 8 degrees of freedom in the heteroscedastic case. As Shanken and Zhou (2007) argue, the latter conveniently accommodates heteroscedasticity in returns as the residuals variance depends on the factor realizations. [18]

Under the null hypothesis that a linear factor model is correctly specified, excess-returns samples are generated by multiplying estimated factor loadings by randomly drawn factor realizations and further adding randomly drawn residuals. Raw returns samples are similarly generated but for the fact that $E_T(Rf)$, the time-series average of the risk-free rate is further added. Under the alternative hypothesis, first-pass estimated time-series alphas are added to deliver either excess returns or raw returns. Those alphas are "catch-all" parameters that capture the impact of missing factors and/or characteristics under the alternative hypothesis. As in Shanken and Zhou (2007), the alphas are kept fixed over time; thus, the estimated covariance matrix of the asset returns remains the same under both the null and the alternative hypothesis. The latter is consistent with the local alternative approach advocated by Hausman (1978). Under the local alternative approach, the same estimated variance of the contrast is used under both the null and the alternative hypothesis. Using the local alternative approach, Hausman shows that the test of contrast is asymptotically Chi-squared distributed under the null-hypothesis and non-central Chi-squared distributed under the alternative hypothesis.

For each of the 10,000 sets of simulated test-asset returns, first-pass regressions are performed and the estimated factor loadings as well as the estimated residuals covariance matrix

[18] Also see Jagannathan and Wang (1998)

are used to run the second-pass cross-sectional regression under both OLS and GLS. Lastly, following Cameron and Trivedi (2005), for each simulated set of returns, the H statistics in expression (4) are calculated by using the following estimator for the variance of the contrast:

$$Var\left(\hat{\Gamma}_{OLS}-\hat{\Gamma}_{GLS}\right)=\frac{1}{M-1}\sum_{m}\left(\hat{\Gamma}_{OLS}-\hat{\Gamma}_{GLS}-\bar{\Gamma}_{diff}\right)\left(\hat{\Gamma}_{OLS}-\hat{\Gamma}_{GLS}-\bar{\Gamma}_{diff}\right)' \tag{6}$$

with $\bar{\Gamma}_{diff}=(1/M)\sum_{m}\left(\hat{\Gamma}_{OLS}-\hat{\Gamma}_{GLS}\right)$ where M is the number of simulations and $\bar{\Gamma}_{diff}$ is the mean contrast over simulations. Noticeably, this simulated H statistics calculation does not rely on the full efficiency of the GLS estimator.[19]As such, it addresses the Cochrane (2005) observation[20] that the GLS estimator may not be the fully efficient estimator when one abandons the homoscedasticity assumption for the return generating process. Under the null hypothesis, size is assessed by comparing the simulation rejection rates to the nominal 5% level of the Chi-squared test. Under the alternative hypothesis, power is computed as the rejection rates for a 5% nominal level size of the Chi-squared test.

Table 1 reports size and power of the test of contrast under homoscedasticity. Panel A shows the excess-returns results with no-intercept in the cross-sectional regression while Panel B reports the raw returns case with an intercept in the cross-sectional regression. Overall, for large T, Chi-squared tests appear properly sized, close to the 5% nominal level, irrespective of whether the cross-sectional regression includes an intercept. With respect to power, however, there are substantial differences across models and data sets. As expression (4) reveals, power is functionally related to the magnitude of the contrast between the OLS and GLS risk-premium

[19] Explicitly, it does not rely on the main Hausman (1978) result that $Var\left(\hat{\Gamma}_{OLS}-\hat{\Gamma}_{GLS}\right)=Var\left(\hat{\Gamma}_{OLS}\right)-Var\left(\hat{\Gamma}_{GLS}\right)$ when the GLS estimator is fully efficient. See Cameron and Trivedi (2005) pp. 378,

[20] Cochrane (2005) pp. 241-242 raises this point in the context of GMM estimation of linear asset pricing models under less restrictive assumptions about the return generating process. Shanken and Zhou (2007) have numerically verified that the GLS is the optimal GMM estimator in the sequential sense of Ogaki (1993).

estimators under the alternative model, commonly referred to as the effect-size. Additionally, power is increasing proportionally to the inverse of the variance of the contrast between the two estimators. Based on those observations, there are three takeaways from Table 1. First, adding a risk-free asset to identify the zero-beta rate substantially increases the power of the misspecification tests. For instance, from Table 1 Panel A with T=960, the CAPM tested on the 25 P displays a 0.657 rejection rate when the null hypothesis is false. From Table 1 Panel B, the rejection rate jumps to 0.998 for the same model and portfolios. Intuitively, this is because the effect-size is much larger when the CAPM must fit both the 25 equity portfolios and the risk-free asset returns. Second, adding the 5 IND portfolios to either the 25 P or the 32 P also increases power, regardless of whether the zero-beta asset is included in the test-assets. The reason is, portfolios sorted by industries allow for a breakdown of the factor structure resulting from the characteristics-based sort. Increased effect-size results as the factor model struggles to jointly explain both characteristics and industry portfolios. This point has previously been noted by Lewellen, Nagel and Shanken (2010) as they recommend to "expand the set of test-assets beyond the size-B/M portfolios". Third, the case of the 30 industry-sorted portfolios is instructive. Daniel and Titman (2012) advocate the use of industry-sorted portfolios to test factor models. They argue that "industry portfolio exhibit variations in factor loadings relative to a number of macroeconomic factors, but this variation is largely unrelated to book-to-market ratios". From Table 1 Panel A with T=960, when testing the CAPM, the 30 IND portfolios power is only 0.253 compared to 0.657 for the 25 P. The latter should come as no surprise because the CAPM loadings on the industry-sorted portfolios are loosely correlated with average returns. For those portfolios, a model that solely includes an intercept fits better than any of the entertained factor models. When no intercept is included in the cross-sectional regression, the market slope must compensate for the lack of an intercept. Low power does not occur when the risk-free rate is added to the test-assets and/or when the FF 3 or FF 5 factor model are tested. The reason is, the effect-size on either the risk-free asset or the characteristics-constructed factors are large confirms the Daniel and Titman (2012) point.

Table 2 repeats the simulations of Table 1 by substituting a joint *t*-distribution with 8 degrees for the multivariate i.i.d. normal assumption of the $\epsilon_{i,t}$ disturbances. It is important to observe that this assumption applies to the unconditional distribution of the returns. There are many alternative ways to generate heteroscedasticity (for instance, by conditioning disturbances on state variables that describe the evolution of the opportunity set). Still, the results from Table 2 allow assessment of the finite sample statistical properties of the proposed H statistics when the normal assumption does not hold. Overall, there are two main takeaways from Table 2. First, misspecification tests tend to be oversized (i.e. the null hypothesis is rejected at a higher rate than the nominal 5 % level). Second, misspecification tests tend to have less power than in the multivariate normal setting (i.e. the alternative hypothesis that the model is false is not rejected as often as in the multivariate normal setting).

5. Application

This section reports estimation results for the linear factor models and test-assets set used in the previous section. To deliver a test of contrast, the first step is to estimate second-pass regression slopes by running both OLS and GLS regressions. As discussed in Section 2 for the augmented test-assets, GLS slope estimates are identical to the time-series averages of the risk-factors. Hou and Kimmel (2010) show that GLS factor risk-premiums are not impacted by the inclusion of additional factors/test-assets (as long as a those factors are also included in the test-assets). Monthly time-series risk-premium estimates are reported in Table 3 along with standard *t*-

tests for significance of the means. Aside from SMB, which is only marginally significant over the testing period from July 1963 to December 2018, all other risk-premiums are statistically significant at the usual levels.

Table 4 exclusively displays OLS estimation results for the excess returns when no-intercept is imposed in the second-pass cross-sectional regression. GLS estimates are not reported because, due to the empirical design, they are identical to the time-series averages reported in Table 3. As opposed to GLS, OLS risk-premium estimates vary depending on which specific factors/test-assets are being used. For the CAPM, the market risk-premium of 0.63 is statistically significant,[21] irrespective of the test-assets used. For the FF 3 factor and 5 factor models, aside from SMB, OLS estimated risk-premiums are significant on their "native" portfolios (i.e. the 25 P for the FF 3 factor model and the 32 P for FF 5 factor model). Adding the 5 IND portfolios to those "native" portfolios reduces statistical significance. Finally, as expected for the 30 IND portfolios, none of the characteristics-based factor risk-premiums are significant. For all tested models, the cross-sectional OLS R^2s are equal[22] or larger than 0.90. These results are unsurprising given previous research by Daniel and Titman (2012) that the OLS R^2s are usually large and not necessarily good indicators of model fit. The GLS R^2s, ranging from 0.06 to 0.40, are much lower across all models/test-assets. Theoretically, as established by Kandel and Stambaugh (1995), the GLS R^2 is a superior statistics. The GLS R^2 measures the proximity of the risk-factors to the minimum-variance boundary of the test-assets. As such, the GLS R^2 is close to unity only when the factors are nearly mean-variance efficient. The OLS R^2, on the other hand, may be close to unity even when the factors are grossly inefficient. For those reasons, Lewellen, Nagel and

[21] Those risk premium point estimates are similar in magnitude to those reported by Kan, Robotti and Shanken (2013b) Table IA.I.

[22] For a regression without a constant, the cross-sectional R^2 uses weighted sums of squared values of the dependent variable in the denominator and not squared deviations from the cross-sectional average.

Shanken (2010) recommend reporting the GLS R^2; they also argue that the latter is likely to be lower than the OLS R^2. The asset pricing tests[23] (AP-Test column) further reveal that all models are rejected, with the exception of the CAPM on the 30 IND portfolios. The exception is likely due to the low power of the CAPM asset pricing test on the 30 IND portfolios when no intercept is included in the cross-sectional regression. When the asset pricing test is rejected, current literature (Kan and Robotti (2012)) advocates the use of misspecification robust t-statistics to conduct cross-sectional slope inference. Misspecification robust standard errors (Shanken and Zhou (2007)) are derived by projecting the expected returns of the assets onto the factor loadings. The projection disturbances account for deviations from exact linear pricing in (2). It is important to observe that those are population disturbances and not mere sampling errors like the α's in (3). Economically, this approach best characterizes a factor model with pervasive priced disturbances (perhaps resulting from missing factors and/or characteristics). While a full-fledged discussion of misspecification robust standard-errors is beyond the scope of this work, it is important to note that the analysis developed in Shanken and Zhou (2007) relies on the concept of fixed (global) alternative. Under fixed alternative, the estimates of the first-pass residual covariance matrix are different under the null and the alternative hypothesis. Therefore, under the alternative hypothesis, a misspecification adjustment term is required for the calculation of consistent standard errors. On the other hand, Hausman (1978) relies on the concept of local alternative. Under this concept of model misspecification, a sequence of local alternatives[24] is constructed to remain asymptotically "close" to the null hypothesis. Under local alternatives, the estimates of the first-pass residual covariance matrix are asymptotically equal under the null and the alternative hypothesis. Thus, when local alternatives are entertained, no misspecification adjustment term is needed. Inference

[23] In this setting, the cross-section asset pricing test is equivalent to the Gibbons, Ross and Shanken (1989) test.
[24] Davidson and MacKinnon (1987) refers to those as "implicit alternative hypothesis".

may be conducted using either Shanken (1992) under homoscedastic returns, or Jagannathan and Wang (1998) under heteroscedastic returns. For traded assets, as Kan, Robotti and Shanken (2013) report, misspecification robust t-statistics are quantitatively not much different from the standard Shanken (1992) t-statistics adjusted for first-pass EIV.[25]

Finally, the last column of Table 4 displays estimation results for the Hausman H statistics proposed in this paper. For the CAPM, the time-series constraints are marginally binding for the 25 P and the 25 P + 5 IND at usual confidence levels. For the 30 IND portfolios, time-series constraints appear more strongly binding as the CAPM p-value stands at 0.29. However, as revealed by the simulation results from Table 1 Panel A, power is low for industry-sorted portfolios (p-value= 0.253 for T=960). For the FF 3 factor model, the time-series constraints are not rejected on the 25 P with a p-value of 0.12, while adding the 5 IND portfolios significantly decreases the p-value to 0.01. This is to be expected as the effect-size grows due to the inability of the characteristics-based factors to explain the cross-sectional variation in industry returns. The FF 5 factor model is less affected by the addition of the industry-sorted portfolios. For the 32 P + 5 IND portfolios, the p-value stands at 0.11, similar to the 32 P alone. The simulation results from Table 1 Panel A show that the test of contrast on the 32 P + 5 IND portfolios has lower power (p-value=0.773 for T=960) than on the 25 P + 5 IND portfolios (p-value=0.904 for T=960). Larger effect-size due to the addition of the industry portfolios appears to be "diluted" when more characteristics-sorted portfolios are present within the test-assets. Lastly, the time-series constraints placed on the cross-sectional risk-premiums are strongly rejected for both the FF 3 factor and the FF 5 factor model when tested on the 30 IND portfolios. Aside from the market

[25] See Kan, Robotti and Shanken (2013) pp. 2633.

slope, which compensates for the absence of an intercept, the remaining characteristics-based slopes have little explanatory power across industries.

Table 5 reports OLS raw returns regression results when an intercept is added to the asset pricing models being tested. In this paper, the identification of the cross-sectional intercept relies on the imposition of the time-series constraint on the zero-beta asset. This constraint is enforced by adding the one-month Treasury bill to the test-assets. Even though the historical one-month Treasury bill rate displays a small amount of systematic risk,[26] it is construed as a pseudo zero-beta asset. Because of the mild factor sensitivity of the one-month Treasury bill, the GLS estimate of the zero-beta rate $\hat{\gamma}_0$ (slightly) differs from the time-series average of the risk free rate $E_T(Rf)$. The first data column of Table 5 reports those GLS slope estimates for the zero-beta rate. For all models/test-assets, the GLS point-estimates are virtually identical to 0.38, the time-series average of the risk-free rate reported in Table 3. Confronting the OLS to GLS intercepts highlights the glaring empirical failure of the CAPM for all considered test-assets. The OLS zero-beta rate estimates stand around 0.71 vs 0.38 for GLS. Additionally, the OLS estimated market risk-premium is too low compared to the market time-series average. Those results fit the traditional findings, first reported in Black, Jensen and Scholes (1972), that the estimated SML (security market line) is too “flat”. In the typical empirical setting (see e.g. Kan, Robotti and Shanken (2013)), where the test-assets are not augmented by the factors, the OLS SML is even more downward tilted. The zero-beta rate is usually much higher and the market slope is negative. The latter is also documented for the FF 3 factor model (see e.g. Lewellen, Nagel and Shanken (2010) Table 1). The reason why estimated risk-premiums in this paper differ from the aforementioned

[26] This is trivially true by construction since Rf must be (mildly) correlated with Rm-Rf.

results is due to the augmentation of the test-assets by the traded factors and the risk-free asset[27]. Furthermore, the considered test-assets are paramount. For the 30 IND portfolios, the OLS zero-beta rate estimates stand near 0.70, regardless of the tested model. As a matter of fact, only the intercept shows up statistically significant for the 30 IND portfolios. For test-assets sorted by characteristics, the multi-factor models with characteristics-based factors fit better, so the OLS zero-beta rate estimate inches lower, closer to its GLS counterpart (and hence, closer to its time-series average). For instance, for the FF 5 factor model tested on the 32 P, the OLS zero-beta rate estimate is 0.37 (vs 0.38 for GLS), the market slope is 0.51 and the SMB, HML, RMW, CMA slopes are respectively 0.22, 0.36, 0.31 and 0.30. All slopes but SMB's are significantly different from zero at the usual confidence levels.[28] Of course, with characteristics-based factors included in the augmented portfolios, statistical significance of the OLS slopes is to be expected. The OLS R^2s confirm that the multi-factor models best fit is achieved on their "native" portfolios with the FF 3 factor model delivering a 0.74 R^2 on the 25 P and the FF 5 factor model a 0.83 R^2 on the 32 P. Adding industry portfolios to the characteristics-sorted test-assets somewhat reduces those OLS R^2s. The GLS R^2s as well as the asset pricing tests in Table 5 are identical to those in Table 4. Though no formal proof is offered, this is intuitively the case because the GLS regression does "repackage" the test-assets to achieve the same minimum-variance boundary. As in Table 4, the asset pricing test rejects all the models tested with the exception of the CAPM on the 30 IND portfolios. Finally, the test of contrast H also rejects the null hypothesis that the time-series

[27] In their Table 3, Hou and Kimmel (2010) display related results. However, their GLS risk-premium estimates are not exactly identical to the factors time series averages. The reason is, while they include an intercept in their second-pass cross-sectional regression, this intercept is a free parameter that is not identified by the inclusion of a zero-beta asset in the test-assets.

[28] When spread-portfolios like SMB and HML are used as factors, Ferson, Sarkissian and Simin (1999) argue that a market factor is needed to capture the grand-mean of the asset returns.

constraints are binding[29] on the cross-sectional risk-premiums for all but the FF 5 factor model tested on the 32 P. The non-rejection of the null hypothesis for the FF 5 factor model tested on the 32 P is an interesting finding because the simulated power of the test is respectable. Indeed, while the test of contrasts p-value is equal to 0.03 from Table 1 Panel B, its simulated power stands at 0.837 for T=960. Adding the industry-sorted portfolios results in the rejection of the H-test as the effect-size is larger when industry portfolios are added.

6. Concluding Remarks

This paper explores a simple test of misspecification for the two-pass cross-sectional regression in linear asset pricing. Economically, the entertained test amounts to assessing whether time-series constraints are binding on cross-sectional risk-premium estimates. Econometrically, the proposed test is best characterized as a Hausman (1978) test of contrast between OLS and GLS risk-premiums estimated from the original test-assets augmented by the traded factors and (possibly) a pseudo zero-beta asset. Such an augmentation of the test-assets set is computationally cumbersome as it results in the singularity of the first-pass residuals covariance matrix. To handle this singularity, the paper provides a convenient alternative expression for the test of contrast. Compared to the traditional inclusion of control variables in the second-pass of the cross sectional regression, the proposed test is attractive because it does not require identification of specific control variables to use.

Lastly, a couple of observations are in order. First, when an intercept is included in the cross-sectional regression, it is often the case, but not always, that both the traditional asset pricing

[29] Shanken and Zhou (2007) Table 13 fail to reject a test a contrast that OLS and GLS risk-premium estimates are equal for the FF 3 factor model tested on the 25 P.

test and the test of contrast will reject the asset pricing model being tested. The mere fact that the asset pricing test and the test of contrast commonly deliver similar inference results is important. Indeed, in cross-sectional asset pricing, risk-premiums are often found to be significantly different from zero; at the same time though, the asset pricing test performed on the residuals does reject the tested factor model (Ang, Liu and Schwarz (2020)). The test of contrast imposes that estimated risk-premiums must not only be statistically non-zero, but actually equal to their time-series counterparts. This is a higher bar to meet, and therefore, it leads to increased model rejection. In this respect, a noticeable empirical result of the paper is that, while the asset-pricing test does reject the FF 5 factor model tested on the augmented 32 P, the test of contrast does not. As Fama and French (1993, 2015) point out "the most serious problems of asset pricing models are in small stocks." Those portfolios (like "small growth") that cause the rejection of an asset pricing model may be plagued by idiosyncrasies in liquidity, trading patterns, etc. As such, their returns may simply not be reconcilable within the confines of a linear factor model. This may explain the divergence between the asset pricing test and the test of contrast.

Second, the paper emphasizes the importance of the choice of test-assets in the testing of time-series restrictions imposed on cross-sectional risk-premiums. The reason is, even though GLS risk-premiums estimates are unaffected by such a choice, OLS risk-premium and therefore contrast estimates definitely are affected. The reported simulation results reveal that power widely varies with test-assets selection. Lewellen, Nagel and Shanken (2010) also argue that the choice of test-assets can impact the power of asset pricing tests. Within their "skeptical appraisal of asset pricing tests," they suggest that explaining the size and book-to-market anomalies on the "usual" 25 portfolios is "a bit too easy." For the sake of robustness, they recommend raising the bar by including test-assets that are not directly related to the characteristics-based factors used in the model. This is achieved in this paper by adding 5 industry-sorted portfolios to the characteristics-sorted portfolios. While such an addition is certainly improving the power of the test of contrast, it may mechanically lead to the "easy" rejection of an otherwise economically interesting factor model. [30]On other hand, it is also well known that using characteristics-sorted portfolios blurs any cross-sectional variation in factor loadings unrelated to the chosen characteristics. In principle, asset pricing models should be tested on the entire universe of returns (Barrillas and Shanken (2016)). Unfortunately, given the many technical issues related to analyzing a large, random sample of stock returns, the existing literature suggests this is not a trivial endeavor.

[30] Daniel and Titman (2012) assert, " the advantage of using characteristics-sorted portfolios is that their returns exhibit a large spread in both factor loadings and realized returns. Of course, if a proposed model is correct, portfolios that generate a large spread in factor loadings should generate a spread in realized returns."

Table 1: Simulated Size and Power under Multivariate Joint Normal Distribution

This table reports rejection percentage rates under the null hypothesis that a linear asset pricing model is correctly specified (size) and that rejection percentage rates under the alternative hypothesis that the model is mispecified (power) for a 5 % nominal level of the Chi-squared distribution. 10,000 return samples are drawn from an i.i.d. joint normal distribution (homoscedasticity). Panel A (B) reports results for excess-returns (raw returns).

Panel A: Constrained Zero-Beta Rate						
Model/Assets	T=60	T=120	T=240	T=360	T=480	T=960
CAPM						
FF 25 P	0.051	0.050	0.050	0.050	0.050	0.048
	0.089	0.126	0.218	0.297	0.390	0.657
FF 25 P + 5 Ind. P	0.049	0.049	0.052	0.048	0.052	0.050
	0.094	0.143	0.253	0.343	0.439	0.718
FF 30 Ind. P	0.050	0.051	0.049	0.048	0.048	0.052
	0.061	0.070	0.096	0.123	0.145	0.253
FF 3 Factor						
FF 25 P	0.053	0.050	0.051	0.050	0.050	0.052
	0.074	0.108	0.188	0.278	0.368	0.661
FF 25 P + 5 Ind.	0.053	0.053	0.051	0.051	0.051	0.049
	0.112	0.176	0.316	0.466	0.594	0.904
FF 30 Ind.	0.060	0.059	0.056	0.054	0.050	0.052
	0.194	0.418	0.795	0.943	0.988	1.000
FF 5 Factor						
FF 32 P	0.062	0.057	0.054	0.053	0.052	0.053
	0.071	0.091	0.180	0.269	0.372	0.718
FF 32 P + 5 Ind.	0.063	0.056	0.054	0.050	0.054	0.050
	0.092	0.125	0.209	0.316	0.426	0.773
FF 30 Ind.	0.078	0.070	0.064	0.061	0.067	0.058
	0.292	0.581	0.919	0.988	0.999	1.000
Panel B: Estimated Zero-Beta Rate						
Model/Assets	T=60	T=120	T=240	T=360	T=480	T=960
CAPM						
FF 25 P	0.057	0.054	0.052	0.053	0.051	0.052
	0.180	0.338	0.617	0.818	0.905	0.998
FF 25 P + 5 Ind. P	0.057	0.051	0.051	0.054	0.048	0.049
	0.203	0.391	0.689	0.883	0.953	1.000
FF 30 Ind. P	0.055	0.054	0.051	0.054	0.052	0.050
	0.137	0.219	0.405	0.572	0.701	0.948
FF 3 Factor						
FF 25 P	0.063	0.061	0.055	0.054	0.052	0.054
	0.127	0.216	0.415	0.602	0.751	0.981
FF 25 P + 5 Ind.	0.064	0.057	0.056	0.056	0.051	0.052
	0.209	0.415	0.742	0.906	0.976	1.000
FF 30 Ind.	0.070	0.064	0.061	0.058	0.058	0.054
	0.239	0.500	0.833	0.970	0.994	1.000
FF 5 Factor						
FF 32 P	0.079	0.074	0.065	0.064	0.059	0.057
	0.098	0.131	0.235	0.352	0.495	0.837
FF 32 P + 5 Ind.	0.081	0.072	0.064	0.061	0.062	0.056
	0.132	0.216	0.382	0.564	0.710	0.967
FF 30 Ind.	0.089	0.083	0.082	0.075	0.074	0.068
	0.300	0.586	0.921	0.989	1.000	1.000

Table 2: Simulated Size and Power under Multivariate Joint *t*-Distribution

This table reports rejection percentage rates under the null hypothesis that a linear asset pricing model is correctly specified (size) and that rejection percentage rates under the alternative hypothesis that the model is mispecified (power) for a 5 % nominal level of the Chi-squared distribution. 10,000 return samples are drawn from a jointly t-distribution with 8 degrees of freedom (heteroscedasticity). Panel A (B) reports results for excess-returns (raw returns).

Panel A: Constrained Zero-Beta Rate						
Model/Assets	T=60	T=120	T=240	T=360	T=480	T=960
CAPM						
FF 25 P	0.052	0.052	0.054	0.053	0.050	0.056
	0.076	0.107	0.160	0.221	0.294	0.532
FF 25 P + 5 Ind. P	0.054	0.051	0.053	0.052	0.056	0.055
	0.081	0.109	0.185	0.255	0.340	0.619
FF 30 Ind. P	0.054	0.053	0.055	0.050	0.054	0.053
	0.059	0.066	0.083	0.096	0.111	0.173
FF 3 Factor						
FF 25 P	0.072	0.070	0.072	0.069	0.068	0.070
	0.083	0.100	0.142	0.199	0.259	0.527
FF 25 P + 5 Ind.	0.073	0.071	0.073	0.071	0.072	0.072
	0.106	0.146	0.235	0.348	0.459	0.817
FF 30 Ind.	0.076	0.074	0.070	0.069	0.071	0.070
	0.148	0.290	0.666	0.886	0.964	0.999
FF 5 Factor						
FF 32 P	0.089	0.089	0.081	0.086	0.081	0.083
	0.088	0.103	0.142	0.204	0.270	0.570
FF 32 P + 5 Ind.	0.091	0.088	0.084	0.085	0.080	0.079
	0.105	0.116	0.163	0.238	0.295	0.631
FF 30 Ind.	0.098	0.092	0.087	0.089	0.088	0.086
	0.203	0.407	0.812	0.967	0.990	1.000
Panel B: Estimated Zero-Beta Rate						
Model/Assets	T=60	T=120	T=240	T=360	T=480	T=960
CAPM						
FF 25 P	0.067	0.065	0.065	0.063	0.061	0.062
	0.146	0.243	0.461	0.691	0.836	0.986
FF 25 P + 5 Ind. P	0.066	0.064	0.061	0.061	0.061	0.063
	0.151	0.283	0.569	0.775	0.890	0.994
FF 30 Ind. P	0.066	0.065	0.064	0.067	0.064	0.065
	0.111	0.166	0.290	0.434	0.551	0.886
FF 3 Factor						
FF 25 P	0.086	0.081	0.079	0.085	0.074	0.076
	0.120	0.169	0.308	0.458	0.622	0.946
FF 25 P + 5 Ind.	0.080	0.079	0.077	0.078	0.079	0.074
	0.165	0.293	0.589	0.818	0.931	0.999
FF 30 Ind.	0.089	0.086	0.084	0.082	0.079	0.078
	0.179	0.354	0.715	0.922	0.972	1.000
FF 5 Factor						
FF 32 P	0.099	0.092	0.092	0.088	0.094	0.089
	0.110	0.126	0.186	0.264	0.357	0.722
FF 32 P + 5 Ind.	0.100	0.098	0.091	0.092	0.093	0.088
	0.128	0.163	0.271	0.437	0.575	0.915
FF 30 Ind.	0.112	0.101	0.097	0.097	0.100	0.094
	0.216	0.408	0.797	0.962	0.993	1.000

Table 3: Descriptive Statistics

This table reports the monthly time-series average and standard deviation of the percentage returns on the risk-free rate Rf as well as the risk-factors Mkt-Rf, SMB, HML, RMW and CMA from July 1963 to December 2018 (666 months). Standard *t*-test assessing, if the sample averages are equal to zero, are reported. The time-series average of each factor is identical to its cross-sectional GLS risk-premium for the augmented returns sets.

	Rf	**Mkt-Rf**	**SMB**	**HML**	**RMW**	**CMA**
Average	0.38	0.51	0.24	0.32	0.26	0.28
St. Dev.	0.26	4.39	3.02	2.80	2.17	2.00
t-Stat	(37.23)	(3.01)	(2.04)	(2.99)	(3.06)	(3.64)

Table 4: Estimation Results, Excess Returns Specification

This table reports second-pass estimation results for the excess returns specification. Only OLS point estimates are reported. GLS point estimates (not reported) are identical to the time-series averages reported in Table 3. T-statistics (in parenthesis) are adjusted for error-in-variable (EIV) using Shanken (1992). Cross-sectional GLS R^2 are calculated using the test-assets covariance matrix. The asset pricing test (AP-Test) adjusted for EIV is asymptotically distributed $\chi(N)$. The Hausman H-test, adjusted for EIV, is asymptotically distributed $\chi(K)$. P-values for the AP-Test and the H-Test are reported in brackets.

Model/Assets	Variables					OLS R^2	GLS R^2	AP-Test	H-Test
CAPM	**Mkt-Rf**								
FF 25P	0.63					0.90	0.07	117.47	3.88
	(3.50)							[0.00]	[0.05]
FF 25P + 5 Ind. P	0.63					0.90	0.06	150.23	4.57
	(3.52)							[0.00]	[0.03]
FF 30 Ind. P	0.56					0.91	0.17	42.96	1.13
	(3.19)							[0.06]	[0.29]
FF 3 Factor	**Mkt-Rf**	**SMB**	**HML**						
FF 25P	0.49	0.18	0.37			0.97	0.20	98.38	5.87
	(2.87)	(1.52)	(3.33)					[0.00]	[0.12]
FF 25P + 5 Ind.	0.53	0.15	0.31			0.96	0.16	130.39	10.65
	(3.10)	(1.27)	(2.78)					[0.00]	[0.01]
FF 30 Ind.	0.58	-0.11	-0.02			0.91	0.24	78.18	36.73
	(3.36)	(-0.71)	(-0.17)					[0.00]	[0.00]
FF 5 Factor	**Mkt-Rf**	**SMB**	**HML**	**RMW**	**CMA**				
FF 32P	0.51	0.22	0.36	0.31	0.30	0.97	0.38	98.65	8.88
	(2.97)	(1.86)	(2.99)	(3.59)	(3.75)			[0.00]	[0.11]
FF 32P + 5 Ind.	0.52	0.22	0.23	0.28	0.29	0.96	0.31	133.31	9.08
	(3.04)	(1.84)	(1.91)	(3.25)	(3.66)			[0.00]	[0.11]
FF 30 Ind.	0.50	0.05	-0.12	0.19	0.16	0.94	0.40	87.46	51.71
	(2.90)	(0.34)	(-0.90)	(1.39)	(1.13)			[0.00]	[0.00]

Table 5: Estimation Results, Raw Returns Specification

This table reports second-pass estimation results for the raw returns specification. Only OLS point estimates are reported. Aside from the constant, GLS point estimates (not reported) are identical to the time-series averages reported in Table 3. The GLS constant is reported. T-statistics (in parenthesis) are adjusted for error-in-variable (EIV) using Shanken (1992). Cross-sectional GLS R^2 are calculated using the test-assets covariance matrix. The asset pricing test (AP-Test), adjusted for EIV, is asymptotically distributed $\chi(N+1)$. The Hausman H-test, adjusted for EIV, is asymptotically distributed $\chi(K+1)$. P-values for the AP-Test and the H-Test are reported in brackets.

Model/Assets	Variables							OLS R^2	GLS R^2	AP-Test	H-Test
CAPM	**GLS Const.**	**OLS Const.**	**Mkt-Rf**								
FF 25 P	0.39	0.71	0.33					0.13	0.07	117.47	19.67
	(38.63)	(6.25)	(1.51)							[0.00]	[0.00]
FF 25 P + 5 Ind. P	0.39	0.71	0.32					0.12	0.06	150.23	23.60
	(38.50)	(5.88)	(1.44)							[0.00]	[0.00]
FF 30 Ind. P	0.38	0.71	0.25					0.15	0.17	42.96	10.72
	(37.97)	(6.03)	(1.18)							[0.06]	[0.00]
FF 3 Factor	**GLS Const.**	**OLS Const.**	**Mkt-Rf**	**SMB**	**HML**						
FF 25 P	0.39	0.42	0.46	0.18	0.36			0.74	0.20	98.38	20.71
	(38.11)	(28.07)	(2.66)	(1.46)	(3.24)					[0.00]	[0.00]
FF 25 P + 5 Ind.	0.39	0.49	0.44	0.14	0.29			0.64	0.16	130.39	36.84
	(38.04)	(24.48)	(2.53)	(1.17)	(2.62)					[0.00]	[0.00]
FF 30 Ind.	0.38	0.70	0.30	-0.13	-0.11			0.36	0.24	78.18	40.10
	(37.15)	(10.69)	(1.67)	(-0.86)	(-0.80)					[0.00]	[0.00]
FF 5 Factor	**GLS Const.**	**OLS Const.**	**Mkt-Rf**	**SMB**	**HML**	**RMW**	**CMA**				
FF 32 P	0.38	0.37	0.51	0.22	0.36	0.31	0.30	0.83	0.38	98.65	13.77
	(36.03)	(21.35)	(3.00)	(1.87)	(2.96)	(3.59)	(3.76)			[0.00]	[0.03]
FF 32 P + 5 Ind.	0.38	0.43	0.48	0.21	0.21	0.28	0.28	0.75	0.31	133.31	25.45
	(35.96)	(23.01)	(2.79)	(1.82)	(1.75)	(3.17)	(3.56)			[0.00]	[0.00]
FF 30 Ind.	0.38	0.61	0.32	-0.01	-0.16	0.17	0.04	0.59	0.40	87.46	52.27
	(36.31)	(14.90)	(1.82)	(-0.08)	(-1.20)	(1.23)	(0.25)			[0.00]	[0.00]

Appendix A

This appendix shows, that for the augmented test-assets, the GLS cross-sectional, cross-sectional slopes are equal to the factors time-series averages. The excess return case is discussed in pp. 244-245 in Cochrane (2005). The focus here is on the raw returns case when the test-assets are augmented by the traded factors plus the pseudo risk-free asset. First, the express raw returns time-series dynamics for the pseudo risk-free asset and each factor k as follows:

$$R_{f,t} = a_{R_f} + \beta_{R_f,1} f_{1,t} + \cdots + \beta_{R_f,K} f_{K,t} + \epsilon_{R_f,t} \tag{A.1}$$

$$f_{k,t} + R_{f,t} = a_k + \beta_{k,1} f_{1,t} + \cdots + \beta_{k,K} f_{K,t} + \epsilon_{k,t} \tag{A.2}$$

Subtracting (A.1) from (A.2) yields:

$$f_{k,t} = \left(a_k - a_{R_f}\right) + \left(\beta_{k,1} - \beta_{R_f,1}\right) f_{1,t} + \cdots + \left(\beta_{k,K} - \beta_{R_f,K} f_{K,t}\right) f_{K,t} + \left(\epsilon_{k,t} - \epsilon_{R_f,t}\right) \tag{A.3}$$

Estimating time-series regression (A.3) leads to a perfect fit where $a_k - a_{R_f} = 0$, $\epsilon_{k,t} - \epsilon_{R_f,t} = 0$ and $\hat{\beta}_{k,q} - \hat{\beta}_{R_f,q} = 1$ for $q = k$ and $\hat{\beta}_{k,q} - \hat{\beta}_{R_f,q} = 0$ for $q \neq k$. Observe that $\epsilon_{k,t} = \epsilon_{R_f,t}$ implies that Σ_a, the covariance matrix of the augmented time-series disturbances, is singular. Next, the cross-sectional regressions for the pseudo risk-free and each factor k are given by:

$$E_T\left(R_{f,t}\right) = \gamma_0 + \hat{\beta}_{R_f,1}\gamma_1 + \cdots + \hat{\beta}_{R_f,K}\gamma_K + \alpha_{Rf} \tag{A.4}$$

$$E_T(f_{k,t} + R_{f,t}) = \gamma_0 + \hat{\beta}_{k,1}\gamma_1 + \cdots + \hat{\beta}_{k,K}\gamma_K + \alpha_k \tag{A.5}$$

Subtracting (A.4) from (A.5) yields:

$$E_T(f_{k,t}) = \left(\hat{\beta}_{k,1} - \hat{\beta}_{R_f,1}\right)\gamma_1 + \cdots + \left(\hat{\beta}_{k,K} - \hat{\beta}_{R_f,K}\right)\gamma_K + (\alpha_k - \alpha_{Rf}) \tag{A.6}$$

where $\hat{\beta}_{k,q} - \hat{\beta}_{R_f,q} = 1$ for $q = k$ and $\hat{\beta}_{k,q} - \hat{\beta}_{R_f,q} = 0$ for $q \neq k$. Consequently, from the previously noted observation that $\epsilon_{k,t} = \epsilon_{R_f,t}$ is embedded in the estimated covariance matrix $\hat{\Sigma}_a$, a GLS cross-sectional regression will force a perfect fit such that $\gamma_K = E_T(f_{k,t})$ and $\alpha_k - \alpha_{Rf} = 0$. Finally, observe that (A.4) implies that $\widehat{\gamma}_0$, the pseudo risk-free asset, is not exactly equal to $E_T(R_{f,t})$, the time-series average of the risk-free rate realizations given that the pseudo risk-free asset has both non-zero factor loadings and cross-sectional pricing error α_{Rf}.

Appendix B

This appendix shows a derivation of the variance of the contrast $Var(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS})$ as reported in (5) in the text. From Theorem 3.4 in Jagannathan, Skoulakis, and Wang (2010), the asymptotic covariance $E(\alpha\alpha')$ of second-pass pricing errors is given by:

$$\hat{\Sigma}_{\alpha} = \frac{1}{T}\left[\hat{X}\hat{\Sigma}_f^{*}\hat{X}' + \hat{\Sigma}_a(1+\hat{c})\right]$$

(B.1)

where $\hat{\Sigma}_a$ denotes the estimate of first-pass augmented residual's variance and $\hat{c} = \hat{\gamma}'\hat{\Sigma}_f^{-1}\hat{\gamma}$ is the Shanken (1992) adjustment for error-in-variables. The Shanken adjustment is required because the second-pass regression is carried out with first-pass estimated betas as opposed to true betas as explanatory variables. As previously pointed out, $\hat{\Sigma}_a$ is necessarily singular, while $\hat{\Sigma}_{\alpha}$ is assumed full-rank. Using standard regression results (see e.g. Greene (2012) pp. 258), cross-sectional slope estimates $\hat{\Gamma}_{OLS} = \Gamma + \left(\hat{X}'\hat{X}\right)^{-1}\hat{X}'\alpha$ and $\hat{\Gamma}_{GLS} = \Gamma + \left(\hat{X}'\hat{\Sigma}_a^{-1}\hat{X}\right)^{-1}\hat{X}'\hat{\Sigma}_a^{-1}\alpha$. Hence,

$$Var(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}) = E\left(\left(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}\right)\left(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}\right)'\right) = ME(\alpha\alpha')M' = M\,\hat{\Sigma}_{\alpha}M'$$

(B.2)

with $M = \left(\hat{X}'\hat{X}\right)^{-1}\hat{X}' - \left(\hat{X}'\hat{\Sigma}_a^{-1}\hat{X}\right)^{-1}\hat{X}'\hat{\Sigma}_a^{-1}$. Using (B.1) and matrix algebra yields:

$$Var(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}) = T^{-1}(1+\hat{c})\left[\left(\hat{X}'\hat{X}\right)^{-1}\hat{X}'\,\hat{\Sigma}_a\hat{X}\left(\hat{X}'\hat{X}\right)^{-1} - \left(\hat{X}'\hat{\Sigma}_a^{-1}\hat{X}\right)^{-1}\right]$$

(B.3)

It can be shown that (B.3) is equivalent to Proposition 2 in Shanken and Zhou (2007). Because of the singularity of $\hat{\Sigma}_a$ in this paper, expression (B.3) is computationally challenging. An equivalent expression is as follows:

$$Var(\hat{\Gamma}_{OLS} - \hat{\Gamma}_{GLS}) = T^{-1}\left[\left(\hat{X}'\hat{X}\right)^{-1}\hat{X}'\,\hat{\Sigma}_{\alpha}\hat{X}\left(\hat{X}'\hat{X}\right)^{-1} - \left(\hat{X}'\hat{\Sigma}_{\alpha}^{-1}\hat{X}\right)^{-1}\right]$$

(B.4)

where $\hat{\Sigma}_{\alpha}$ is assumed full-rank. To see the equivalence, first insert (B.1) in the left-hand side of (B.4) to obtain:

$$\left(\hat{X}'\hat{X}\right)^{-1}\hat{X}'\,\hat{\Sigma}_{\alpha}\hat{X}\left(\hat{X}'\hat{X}\right)^{-1} = T^{-1}\left[(1+\hat{c})\left(\hat{X}'\hat{X}\right)^{-1}\hat{X}'\,\hat{\Sigma}_{a}\hat{X}\left(\hat{X}'\hat{X}\right)^{-1} + \hat{\Sigma}_{f}^{*}\right]$$

(B.5)

Next, write:

$$\left(\hat{X}'\hat{\Sigma}_{\alpha}^{-1}\hat{X}\right)^{-1} = \left(T\hat{X}'\left(\hat{X}\hat{\Sigma}_{f}^{*}\hat{X}' + \hat{\Sigma}_{a}(1+\hat{c})\right)^{-1}\hat{X}\right)^{-1} =$$

$$\left(T\hat{X}'\left(\hat{X}'^{-1}\hat{\Sigma}_{f}^{*-1}\hat{X}^{-1} + \hat{\Sigma}_{a}^{-1}(1+\hat{c})^{-1}\right)\hat{X}\right)^{-1} =$$

$$\left(T\left(\hat{\Sigma}_{f}^{*-1} + \hat{X}'\hat{\Sigma}_{a}^{-1}(1+\hat{c})^{-1}\hat{X}\right)\right)^{-1} = T^{-1}\left[(1+\hat{c})\left(\hat{X}'\hat{\Sigma}_{a}^{-1}\hat{X}\right)^{-1} + \hat{\Sigma}_{f}^{*}\right]$$

(B.6)

Subtracting (B.6) from (B.5) delivers (B.3). Hence (B.4) is equivalent to (B.3) and (5) in the text.

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