Market Neutrality and Beta Crashes

Xia Xu*

Abstract

Market neutrality is a key feature of Frazzini and Pedersen (2014)'s betting against beta (BAB) strategy. However, we find that BAB fails to systematically remain market neutral, and the deviation often arrives in the shape of crashes. Such a concern is common to a broad range of market neutral low-beta strategies, which are particularly vulnerable to bull markets. This vulnerability is not explained by liquidity and leverage constraints. We show that beta crashes can be interpreted as negative market timing and negative volatility timing. Managing the crash risk of low-beta strategies promotes significant performance improvement after transaction costs.

JEL Code: G10, G11, G12

Keywords: Betting Against Beta, Conditional CAPM, Market Neutrality, Low-Beta Anomaly

*X. Xu is at ESSCA School of Management. xia.xu@essca.fr. Address: 4, Pont Pasteur, 69007 Lyon, France.

1 Introduction

Low-beta assets often have higher returns than the CAPM predictions, while high-beta assets are with lower returns.[1] The literature notices that investments overweighting low-beta stocks and underweighting high-beta ones produce significant performance. One of the most successful low-beta strategies is Frazzini and Pedersen (2014)'s betting against beta (BAB) factor, which greatly inspires academic explorations and investment practices (Novy-Marx and Velikov, 2022).

If a low-beta strategy's conditional beta covaries with market risk premium and market variance, the conditional CAPM framework is able to explain the low-beta anomaly.[2] However, a key challenge to this prospect is that BAB is designed to be market neutral, which implies insignificant comovement between conditional betas and market variations. Asness et al. (2020, page 644) further point out that the conditional CAPM is irrelevant unless the arrangement of market neutrality "could be sufficiently imperfect."

We find that BAB fails to remain market neutral in a systematic way, and the deviations from market neutrality often arrive in the shape of crashes. Such a concern is shared by a broad range of low-beta strategies covering different weighting schemes, beta estimators, and hedging methods. Market neutral low-beta strategies take negative market timing and negative volatility timing amid volatile markets, promoting beta crashes. The particular vulnerability of low-beta strategies to booming markets is not explained by leverage constraints, and the conditional CAPM contributes to accounting for the low-beta effect.

Our study is motivated by two expressive BAB crashes. The first one exhibits that BAB

[1] Friend and Blume (1970), Black et al. (1972), Miller and Scholes (1972), Haugen and Heins (1975), Reinganum (1981), and Fama and French (1992) find that the security market line is too flat and low-beta stocks earn higher returns than the CAPM implications. Black (1972, 1993) suggests beta arbitrage strategies to benefit from such a flat line. The low-beta anomaly is closely related to a general low-risk anomaly. See Ang et al. (2006), Baker et al. (2011), and Asness et al. (2020), among others.

[2] Market timing refers to the comovement between conditional betas and market risk premiums, while volatility timing refers to the comovement between conditional betas and market variances. See Jagannathan and Wang (1996), Lewellen and Nagel (2006), Boguth et al. (2011), Cederburg and O'Doherty (2016), and Liu et al. (2018), for example.

is negatively exposed to booming markets: during the Internet bubble of 1999, BAB realizes a large loss of 40% while the market gains 20%. By contrast, the second one features BAB's positive exposure to market crashes: during the Great Recession of 2008, the market and BAB plunge similarly by about 40%. A further examination of the 10 worst BAB losses reaffirms that BAB suffers from large market movements, bull markets in particular.

BAB's vulnerability to bull markets is not captured by the rationale of funding liquidity and leverage constraints that underlies BAB. Brunnermeier and Pedersen (2009), Adrian and Shin (2014), and He et al. (2017) show that liquidity and leverage are pro-cyclical, suggesting that funding liquidity constraints are less binding during economic expansions. That is to say, funding liquidity risk and leverage constraint cannot address BAB crashes and the deviation from market neutrality in bull markets.

Novy-Marx and Velikov (2022) specify three concerns for BAB implementation: Frazzini and Pedersen (2014)'s beta (FP beta) raises significant bias; the rank-weighting is a backdoor to equal-weighting stocks; the non-linear hedging for market neutrality is a backdoor to overweighting tiny stocks. We develop low-beta strategies in line with the three critiques. Specifically, we consider six beta estimators: the FP beta, the simple OLS market model beta, Dimson (1979) beta, Vasicek (1973) beta, Welch (2022)'s slope-winsorized beta, and Liu et al. (2018, LSY) beta. Besides, we adopt Jensen et al. (2022)'s capped value-weighting and equal-weighting. The former well balances the effect of tiny stocks and facilitates more tradable strategies, and we take it as the default weighting scheme. Additionally, we apply the non-linear hedging, linear hedging, as well as the conventional dollar hedging.

In this way, we construct 36 beta arbitrate (BA) strategies to characterize investors' attempts to benefit from the low-beta anomaly. Revisiting the two expressive years of 1999 and 2008, we register that crashes are a common concern for these BA strategies. Betting against the OLS and Dimson betas, which Novy-Marx and Velikov (2022) find the least biased and superior to the FP beta, leads to the most significant bankruptcy risk as the strategy minimum return often goes below -100%.

We notice that beta estimator's value-weighted average bias does not necessarily deteriorate performance. Simply buying low-beta stocks and selling high-beta stocks is an elementary reflection of Black (1972)'s insight to exploit a "too flat" security market line, yet the literature also documents that such dollar-neutral beta strategies end up with mediocre returns (see Bali et al., 2017, Bai et al., 2019, and Schneider et al., 2020, for example). The primary goal of non-linear market neutrality is to leverage the low-beta portfolio to take full advantage of the flat security market line. This requires relatively stable beta estimates to facilitate leverage employment. The beta bias concern is secondary as long as the cross-sectional beta ordering does not substantially deviate from the true ordering.

The OLS and Dimson betas, despite their advantage in the value-weighted average bias, promote capricious leverage employment. For instance, the ex ante OLS beta for the low-beta portfolio much fluctuates and often approaches zero, thus the implied leverage can be implausible. By contrast, the FP beta leads to the most robust time series of beta estimates for the low-beta portfolio, outweighing its disadvantage in the value-weighted average bias. As a result, the BA-FP strategies outperform the BA-OLS strategies with considerably higher Sharpe ratios when investors practice non-linear market neutrality.

The beta bias concern prevails once leveraging the low-beta portfolio is no longer a key consideration. Linear market neutrality hedges exposure by buying the market portfolio, so betting against the OLS beta with this method becomes much more successful. Across different hedging methods, the BA-Welch strategies offer persistently favorable performance. This outperformance indicates that the Welch beta achieves the best balance between robust leverage employment and beta bias confinement, consistent with Welch (2022)'s finding that it provides the best beta prediction and the best portfolio hedging.

Simply regressing BA returns on market returns, we find realized betas prevalently significant at the 1% level. The market exposure, nonetheless, is heavily dependent on market conditions. We consider three market conditions: normal, bear, and bull markets. Bear (bull) markets are the months with market risk premium below -5% (above 5%). The normal market, where the market risk premium is between -5% and 5%, is the benchmark condition. Bear (bull) markets account for about 11% (15%) of our sample months, and this distribution is close to Lu and Qin (2021)'s determination of market state.[3]

We identify a strangle market neutrality for the market neutral BA-FP strategies, as market neutrality is systematically maintained in normal markets. For instance, the value-weighted BA-FP strategy with non-linear market neutrality has a trivial beta of -0.08. Bear markets significantly raise the beta estimate by 0.41, and bull markets substantially reduce the beta estimate by 0.69. Consequently, the point estimate of strategy beta is 0.33 in bear markets and -0.77 in bull markets. This helps to explain the two expressive BAB crashes, where the market exposure is positive (negative) in economic crises (booms). That is to say, beta crashes are the result of negative market timing in volatile markets.

For robustness, we futher use a simple threshold of zero to determine upside and downside markets, consistent with Grundy and Martin (2001) and Daniel and Moskowitz (2016). Since the bull-market beta estimate is highly negative, we perform Daniel and Moskowitz (2016)'s optionality analysis to check if BA is exposed to market upswing risk. We confirm significant option-like behavior of the value-weighted BA-FP strategy with non-linear market neutrality, and the size is close to Daniel and Moskowitz (2016)'s registration of momentum optionality. The point estimate of strategy beta is an important -0.62 upon market upswings, as if this market neutral strategy shorts a call option on the market. The other strategies also assume material market exposure when the market rebounds.

Moreover, negative market timing is concurrent with negative volatility timing due to the changes of market risk return tradeoff. The tradeoff is significantly negative in bear markets and marginally negative in normal markets, thus volatility timing improves performance. However, the tradeoff is significantly positive in bull markets, and managing market volatility is disadvantageous in this case. As a result, conditional beta negatively (positively) covaries with market variance in bull (bear) markets, effectuating negative volatility timing.

[3]Specifically, Lu and Qin (2021) refer the bottom 10% of three-year cumulative market returns as low market state, the top 10% as high state, and the 80% in between as middle state.

Beta crashes can be considered as significant volatility shocks that deteriorate BA's risk return tradeoff. As weak risk return tradeoff is key for volatility management (Moreira and Muir, 2017), beta crashes extend a favorable condition to manage volatility. To evaluate the volatility management benefits, we refer to the abnormal return from a 9-factor model as benchmark. This model augments Fama and French (2016)'s six-factor (FF-6) model with Bali et al. (2017)'s FMAX factor and Stambaugh and Yuan (2017)'s two mispricing factors.[4] The 9-factor model remarkably outperforms the FF-6 model in explaining BA performance: only three alphas are significant against the former, while 16 are significant against the latter.

Volatility management considerably improves investment performance. We register 32 (25) significant FF-6 (9-factor) model alphas, much higher than the corresponding number for the unmanaged versions. The benefit is particularly striking for dollar neutrality: the 9-factor model, which fully prices all the dollar-neutral BA strategies, fails to subsume any of the managed strategies. Such observations corroborate Barroso and Maio (2021)'s argument that the success of managed-BAB is puzzling.

The benefits of volatility management are robust to transaction costs. Following Moreira and Muir (2017, Section II.B), we consider three assumptions of trading cost: 1bps, 10bps, and 14bps. We analyze the effect of transaction costs for all the 36 BA strategies and find most of the alphas remaining significantly positive. In addition, we examine a global BAB sample of 24 markets and confirm that the concern of market neutrality and beta crashes is not unique to the US market. Managing international BAB factors also leads to remarkable performance improvements.

The paper proceeds as follows. Section 2 provides the motivation of our study and previews BAB crashes. Section 3 describes the data and develops a wide range of BA strategies. Section 4 presents the main results and discussions, and Section 5 concludes.

[4] Bali et al. (2017), Schneider et al. (2020), and Asness et al. (2020) find skewness preference important to subsume the low-beta anomaly. Stambaugh and Yuan (2017), Liu et al. (2018), and Barroso and Maio (2021) point out that market sentiment, mispricing, and limits to arbitrage give rise to the low-beta effect. Stambaugh and Yuan (2017) highlight that their mispricing factors (MGMT and PERF) are consistent with investor sentiment and related to arbitrage risk.

2 Motivation

2.1 Economic intuition

Following the framework of Lewellen and Nagel (2006), Boguth et al. (2011), and Cederburg and O'Doherty (2016), we express the unconditional alpha of a low-beta strategy as

$$\alpha^U \approx \underbrace{\mathrm{cov}(\beta_t, E_{t-1}[r_{MKT,t}])}_{\text{Market timing}} - \underbrace{\frac{E[r_{MKT,t}]}{\sigma^2_{MKT}}\mathrm{cov}(\beta_t, \sigma^2_{MKT,t})}_{\text{Volatility timing}}, \tag{1}$$

where β_t is the strategy's conditional beta, $E_{t-1}[r_{MKT,t}]$ is the conditional market risk premium, $E[r_{MKT,t}]$ is the unconditional market risk premium, $\sigma^2_{MKT,t}$ is the conditional market variance, and σ^2_{MKT} is the unconditional market variance.

Cederburg and O'Doherty (2016) argue that the low-minus-high beta strategy effectively times market risk premium and volatility, thus time-varying beta explains the low-beta anomaly. However, Asness et al. (2020, Section 5.3.3) point out that the low-minus-high beta strategy is not developed as market neutral as BAB. When market neutrality is systematically maintained, market timing and volatility timing cannot capture the BAB outperformance. Therefore, time-varying beta does not subsume the low-beta effect.

Asness et al. (2020, page 644) summarize that the explanation of time-varying beta matters if "the ex-ante hedge used to construct BAB could be sufficiently imperfect." At the same time, Novy-Marx and Velikov (2022) show that Frazzini and Pedersen (2014)'s beta estimates can be predicted by using market volatility, which is related to volatility timing. Therefore, we are particularly interested in the role of hedging method and beta estimator for the performance of low-beta strategies.

A convenient perspective is from market neutrality and low-beta crashes. If beta crashes are systematically related to market conditions, imperfect market neutrality admits the contribution of time-varying beta in explaining the low-beta anomaly.

2.2 A preview of beta crashes

Figure 1 presents two expressive BAB crashes.[5] In 1999, the market displays an evident uptrend with a cumulative return of 20%, in contrast to the underperforming BAB with a large loss of 40%. The BAB debacle is driven by the Internet bubble, as the successful dot-com stocks have high betas. Here are two simple statistics showing how these technology stocks triumph in 1999: the NASDAQ index soars by 85.6%, and the price of Qualcomm skyrockets by 2,619%.[6] BAB shorts these high-beta stocks, and its performance is heavily affected. Although BAB is intended to be market neutral, the episode of 1999 suggests that bull markets promote significant BAB declines.

BAB crashes can also hit during market crises. In 2008, both the market and BAB suffer a gloomy performance, and their cumulative losses are about 40%. The performance similarity throughout the year indicates that BAB bears a beta close to one rather than around zero as implied by market neutrality. Frazzini and Pedersen (2014, Proposition 3) highlight that a worsening funding liquidity depresses contemporary BAB return and raises future BAB return. Nonetheless, BAB's cumulative performance stably diminishes in 2008, hinting that liquidity shocks do not necessarily elevate future required return.

A relevant question is that if the two episodes are representative or outliers. Table 1 lists the 10 worst BAB returns and the corresponding market returns from 1930 to 2020. The greatest BAB crash is -21.95% in September 1939, while the contemporary market return is a large 16.88%. The second largest crash is in July 1932, as BAB realizes a considerable loss of 19.07% against a huge market increase of 33.84%. The year of 1999 witnesses two large BAB losses, yet the episode of 2008 cannot even get on the list. The top losing months do not come randomly; rather, they are associated with significant market variations such as the Great Depression and its consequences in the 1930s, the market crash of 1987, as well as the Internet bubble in the early 2000s.

[5] Data of the BAB and market factors are from AQR's datasets and Kenneth French's data library.

[6] See the article titled "The Year in the Markets; 1999: Extraordinary Winners and More Losers" at *The New York Times* in January 3, 2000, Section C, Page 17.

BAB's vulnerability to booming markets is evident. The average BAB loss is 14.21%, accompanied by a remarkable market gain of 9.18%. By contrast, we report the 10 best BAB returns and find that BAB is successful amid mild market declines. Specifically, the average BAB profit is 12.62% and the corresponding market risk premium is -1.67% on average. In short, the two expressive BAB crashes and the worst BAB returns exhibit a notable departure from the intended market neutrality.

2.3 Hedging method and beta variation

Consistent with Liu et al. (2018, Equation 9), we decompose BAB into a simple dollar-neutral low-minus-high beta strategy and an enhanced beta bet as follows:[7]

$$\begin{aligned} r_{BAB} &= \frac{r_L - r_f}{\beta_L} - \frac{r_H - r_f}{\beta_H} \\ &= \underbrace{r_L - r_H}_{\text{Dollar hedge}} + \underbrace{(1 - \beta_H^{-1})(r_H - r_f) - (1 - \beta_L^{-1})(r_L - r_f)}_{\text{Enhancement of beta bet}}, \end{aligned} \tag{2}$$

where r_L and β_L (r_H and β_H) denote the return and beta on the low (high)-beta portfolio, and r_f is the risk-free return.

The dollar-neutral low-minus-high beta strategy directly reflects Black (1972)'s insight to benefit from a "too flat" security market line. However, a flat security market line does not necessarily guarantee that r_L is significantly superior to r_H, so the low-minus-high beta strategy can be unprofitable. Examining CAPM beta decile portfolios, Bali et al. (2017, Table 1) register an insignificant return difference between the highest and lowest beta deciles. Bai et al. (2019, Table 3) and Schneider et al. (2020, Table 3) document similar results.

Therefore, it is necessary to overweight the low-beta portfolio and underweight the high-beta portfolio to take full advantage of the weak security market line. Market neutrality fulfills this mission by enhancing the beta bet with $(1 - \beta_H^{-1})(r_H - r_f) - (1 - \beta_L^{-1})(r_L - r_f)$.

[7] Han (2022) decomposes BAB returns into three parts with specified weighting schemes. Comparatively, Equation 2 does not identify weighting scheme or beta estimator, thus the implication is more general.

Since the market average beta is one, we have $1-\beta_H^{-1}>0$ and $1-\beta_L^{-1}<0$. As long as r_H and r_L are greater than r_f, the enhanced beta bet raises BAB returns.

The dollar-neutral low-minus-high beta strategy and the enhanced beta bet are, individually or jointly, exposed to crash risk. First, the dollar-neutral low-minus-high beta strategy is vulnerable to booming markets, and this vulnerability can be inferred from coskewness and downside beta.[8] Bali et al. (2017) report that the coskewness (downside beta) is -4.75 (0.09) for the lowest beta decile and -1.96 (2.10) for the highest beta decile. This suggests that low-beta stocks are sensitive to large market movements but robust to large market declines. In other words, they suffer from large market gains.

Second, the enhanced beta bet implicitly assumes that the ex ante beta estimates reliably predict future betas. If the beta estimates are poor forecasts, the enhanced beta bet backfires and distances BAB from its intention of market neutrality (see Novy-Marx and Velikov, 2022, page 85). Cederburg and O'Doherty (2016, Figure 1) show that the 5th percentile of stock betas can approach -1, so β_L can be too small and the implied leverage too large. For instance, if β_L is estimated to be 0.04, the leverage for the low-beta portfolio is 24 (which is 1/0.04-1). Investors find it implausible because the accessible leverage is only up to 20 even for equity hedge funds (Ang et al., 2011).

Novy-Marx and Velikov (2022, Figure 6) further exhibit that the value-weighted average beta across the market dips below 0.8 for several beta estimators. Such a general downward bias makes it possible that $\beta_H<1$ and $(1-\beta_H^{-1})(r_H-r_f)<0$, depressing BAB performance. Given the strong comovement between the 5th percentile of stock betas and the median stock beta (Cederburg and O'Doherty, 2016, Figure 1), one may expect that $\beta_H<1$ is concurrent with $\beta_L<0$ and the enhanced beta bet amplifies losses. The beta estimation concern is aggravated in bull markets (Hollstein et al., 2020), which supports our intuition that BAB is particularly vulnerable to booming markets.

[8] Harvey and Siddique (2000) add square market excess returns to the CAPM regression and refer to the slope coefficient as coskewness. Downside beta is the slope coefficient by regressing stock excess returns on market excess returns upon downside markets (see Bawa and Lindenberg, 1977 and Ang et al., 2006).

3 Data and strategy devlopment

3.1 Data description

We obtain daily and monthly stock data from the Center for Research in Security Prices (CRSP), including all common stocks listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11. Our sample is from July 1926 to December 2020. Consistently, we collect data for the market (MKT), size (SMB), and value (HML) factors from Fama and French (1993), the momentum (MOM) factor from Carhart (1997), the profitability (RMW) and investment (CMA) factors from Fama and French (2015). Factor data and monthly NYSE market equity breakpoints are from Kenneth French's data library.

3.2 Development of beta arbitrage strategies

Novy-Marx and Velikov (2022) identify three concerns about BAB's non-standard procedures: Frazzini and Pedersen (2014)'s novel beta (FP beta) raises significant bias; the rank-weighting scheme is a backdoor to equal-weighting stocks; the non-linear hedging for market neutrality is a backdoor to overweighting tiny stocks. We next develop low-beta strategies corresponding to the three concerns.

Since value-weighting stock betas across the market should be one, Novy-Marx and Velikov (2022) calculate the value-weighted average for different beta estimators to quantify beta bias. They find the FP beta inferior to the simple OLS market model beta (OLS beta) and Dimson (1979) beta (Dimson beta). Besides, we also consider Vasicek (1973) beta (Vasicek beta), Welch (2022)'s slope-winsorized beta (Welch beta), and Liu et al. (2018) beta (LSY beta). The six beta estimators employ different estimation windows and return frequencies, constituting a representative set of betas that investors can readily bet against. We provide more details for these beta estimators in the Internet Appendix.

As for weighting method, Novy-Marx and Velikov (2022) argue that the rank-weighting assigns disproportionately high weights to micro and nano stocks (market equity below the NYSE 20th percentile), which are illiquid and difficult to trade. To address this problem, we follow Jensen et al. (2022, Section 2)'s capped value-weighting: beta terciles are determined by non-micro and non-nano stocks, then micro and nano stocks are assigned based on the same terciles. Compared to the plain value-weighting, the capped version balances the effect of tiny stocks across tercile portfolios and forms more tradable strategies. This advantage is particularly relevant to the low-beta context. We set the capped value-weighting as primary option and also include the conventional equal-weighting.[9]

In terms of hedging method, Frazzini and Pedersen (2014) use non-linear market neutrality as represented by Equation 2. We further consider the following linear market neutrality, which is also denoted as direct hedging by Novy-Marx and Velikov (2022):

$$r_{BAB} = \underbrace{r_L - r_H}_{\text{Dollar hedge}} + \underbrace{(\beta_H - \beta_L) \cdot r_{MKT}}_{\text{Bet on the market}}. \tag{3}$$

Generally, market neutrality is organized as dollar neutrality plus an enhanced beta bet. Non-linear market neutrality enhances the beta bet by tilting towards the low-beta portfolio, while linear market neutrality takes the bet on the whole market.

The market bet can take on the equal-weighted or value-weighted market factor. Novy-Marx and Velikov (2022, Figure 3) compare the two versions and find the former more profitable. However, stock betas are conventionally estimated relative to the value-weighted market factor. The ex ante beta of the equal-weighted market factor is not always one, thus investors have to adjust the market exposure $\beta_H - \beta_L$ accordingly. The exact adjustment is also dependent on beta estimation window, which raises a problem when the interested beta estimator (like the FP beta) employs multiple estimation windows or when multiple beta estimators are involved. Comparatively, the value-weighted market factor is free from such a problem, and we choose it to implement linear market neutrality.

[9]Novy-Marx and Velikov (2022, Section 2.1) accentuate that the value-weighted BAB performance is a more accurate and practical reflection of investment gains.

We recapitulate our strategy design as follows:

- 6 beta estimators: FP, OLS, Dimson, Vasicek, Welch, and LSY betas;
- 2 weighting schemes: capped value-weighting and equal-weighting;
- 3 hedging methods: non-linear and linear market neutrality, as well as dollar neutrality.

Therefore, we develop 36 ($6 \times 2 \times 3$) strategies, and we refer to them as beta arbitrage (BA) strategies in case of confusion with Frazzini and Pedersen (2014)'s original BAB. We believe such diverse strategies account for the concerns in BAB's construction, and a comprehensive evaluation of these strategies sheds light on the low-beta effect.

4 Main results and discussions

4.1 Revisiting beta crashes

We first revisit the two expressive years of 1999 and 2008 for the six value-weighted BA strategies with non-linear market neutrality in Figure 2. Large declines are prevalent across strategies, and investors hit greater crashes when they bet against more standard betas such as the OLS beta. In 1999, the maximum drawdown is about 30% for the BA-FP strategy and about 50% for the others. Besides, the year of 2008 witnesses a depreciation of 20% for the BA-FP strategy and about 60% for the rest. Such huge and persistent losses reaffirm that beta crashes are a general concern for the low-beta effect.

Inspecting key indicators for the 36 BA strategies, we notice similar performance for the BA-FP and LSY, for the BA-OLS and Dimson, as well as for the BA-Vasicek and Welch strategies. To facilitate exposition, we present the BA-FP, OLS, and Welch results in Table 2, with the complete summary reported in the Internet Appendix. Frazzini and Pedersen (2014, Table 3) document that BAB has a standard deviation of 11% and a Sharpe ratio of 0.8, close to that of the equal-weighted BA-FP strategy with non-linear market neutrality (11.91% and 0.81, respectively). This resemblance corroborates Novy-Marx and Velikov (2022)'s argument that the rank-weighting is a backdoor to equal-weighting stocks.

Dollar neutrality renders poor Sharpe ratios, while market neutrality remarkably improves performance. For instance, the dollar-neutral equal-weighted BA-FP strategy has a Sharpe ratio of -0.03. Linear market neutrality raises the Sharpe ratio to 0.44 and non-linear market neutrality brings a notable 0.81. This pattern holds across beta estimators, reaffirming our argument in Section 2.3 that the dollar-neutral low-minus-high beta strategy can be unfavorable and the enhanced beta bet makes the best of the flat security market line.

Investors may expect that market neutrality promotes risk mitigation, but we find this intuitive expectation not coming true. Non-linear market neutrality leads to a significant bankruptcy risk as minimum returns are often below -100%. For instance, it renders the equal-weighted BA-Welch strategy a minimum return of -116.55%, a straight bankruptcy once it occurs. The two BA-OLS strategies even have a minimum return below -2,000%. In practice, such strategies are hardly implementable.

To illustrate the impact of beta crashes on investment experience, we show the cumulative performance of BA strategies across the sample period in the Internet Appendix. In December 1989, the BA-OLS and Dimson strategies with non-linear market neutrality hit direct bankruptcy, wiping out all cumulative profits. The value-weighted BA-FP strategy with linear market neutrality also suffers from precipitous declines: its cumulative return falls from \$0.67 at the beginning of August 1932 to \$0.13 at the end of the month. A full recovery to the former level takes up almost three decades until February 1960. The month of August 1932 registers the bankruptcy of the value-weighted BA-FP strategy with dollar neutrality, too.

The primary examination of BA strategies confirms that crash risk is not specific to BAB but rather a common concern for low-beta strategies. Beta crashes are not mitigated when investors adopt more standard procedures for strategy construction, such as using conventional beta estimators like the OLS beta or using linear market neutrality. We are further interested in if beta variations play a role in precipitating beta crashes, and we investigate this question in the next subsection.

4.2 Time-varying beta and leverage

Novy-Marx and Velikov (2022, Section 4.2) argue that the FP beta is inferior to the OLS and Dimson betas because of its substantial bias in the value-weighted average. Specifically, they find the mean bias of the FP beta six times that of the OLS and Dimson betas. We extend their analysis back from 1926 with the inclusion of the Vasicek, Welch, and LSY betas. Figure 3 plots the time series of value-weighted average for different beta estimators. Novy-Marx and Velikov (2022, Figure 6) register that the FP beta bias reaches a peak of 30% around 1994, while we note the maximum bias of 41% in December 1964. The mean bias of the FP beta is the highest 5.4% and the standard deviation is also the highest 9.4%.

Surprisingly, the LSY beta has the second worst mean bias and standard deviation. Estimated with monthly returns, the LSY beta is broadly considered robust to microstructure noises (see, Liu et al., 2018 and Barroso and Maio, 2021, for instance). Nonetheless, the value-weighted average of LSY beta estimates is persistently below one, reaching a nadir of 0.8 in 1973. The time-series fluctuation is much smaller for the rest beta estimators: the mean bias is only 0.2% for the OLS beta, 0.1% for the Dimson beta, 1.4% for the Vasicek beta, and 2.3% for the Welch beta, with similar standard deviations around 2.8%.

As the OLS and Dimson betas are the least biased, one may expect that betting against them with non-linear market neutrality produces the best performance. Contrarily, such strategies significantly underperform. Table 2 Panel A reminds that the BA-OLS strategies have a poor Sharpe ratio of 0.17, while the average Sharpe ratio is 0.72 for the BA-FP strategies and 0.71 for the BA-Welch strategies. It is still preferred to bet against the FP beta. Comparatively, betting against the OLS beta with linear market neutrality is substantially favorable, as the average Sharpe ratio rockets to 0.79. By contrast, the BA-FP strategies have a dwarfed Sharpe ratio of 0.44 on average.

The OLS beta badly matches non-linear market neutrality because it promotes mistaken leverage in the low-beta portfolio. Figure 4 displays the time series of β_L estimates. When betting against the OLS and Dimson betas, investors often find β_L too small and the implied leverage implausible. For instance, the ex ante OLS beta of the low-beta portfolio is -0.0015 in May 1989 and 0.0018 in June. That is to say, investors have to radically change leverage from -658 to 552 over one month. Such variations are persistent for the OLS and Dimson betas from the late 1980s to the mid-1990s. The other betas are less affected.[10]

Besides, stable β_L fosters stable leverage. The OLS and Dimson betas have the highest standard deviations of β_L estimates while the FP and Welch betas have the lowest ones.[11] Correspondingly, the BA-FP and Welch strategies have the best Sharpe ratios while the BA-OLS and Dimson strategies have the worst ones. The disadvantage of the OLS and Dimson betas is much mitigated by linear market neutrality, which leverages the whole market rather than the low-beta portfolio. In such a case, the edge of the two betas in confining the value-weighted average bias prevails, and betting against them obtains a large Sharpe ratio.

In short, the FP beta better matches non-linear market neutrality than the OLS beta, indicating that the value-weighted average bias does not necessarily undermine performance. The primary goal of non-linear market neutrality seems leveraging the low-beta portfolio than hedging. This requires relatively stable beta estimates to facilitate leverage employment. The beta bias concern is secondary as long as the cross-sectional beta ordering does not substantially deviate from the true ordering (see also Novy-Marx and Velikov, 2022, footnote 16). The bias concern prevails when leveraging the low-beta portfolio is no longer a consideration, as in the case of linear market neutrality.

It is worth noting that the BA-Welch strategies have the most consistent performance with a favorable Sharpe ratio of 0.75 on average, regardless of market neutrality versions. This suggests that the Welch beta achieves the best balance between robust leverage employment and beta bias confinement. Our results support Welch (2022)'s finding that the Welch beta provides the best prediction for future beta and the best portfolio hedging.

[10]The minimum β_L estimate is 0.46 for the FP beta, 0.38 for the LSY beta, 0.18 for the Welch beta, and 0.14 for the Vasicek beta. The implied leverage is more reasonable for the FP and LSY betas.

[11]The standard deviation is 11.5% for the FP beta, 12.8% for the Welch beta, 13.2% for the LSY beta, 14.3% for the Vasicek beta, 16.5% for the OLS beta, and 18.8% for the Dimson beta.

4.3 Market exposure

If market neutrality is maintained, the strategy's realized beta should be statistically insignificant. For the value-weighted BA strategies, we calculate their realized betas and plot the absolute value of the associated t-statistics in Figure 5. All betas are significant, most of them even at the 1% level. As expected, the dollar-neutral BA strategies are greatly exposed to the market. However, the large t-statistics of the market neutral BA strategies come as a surprise, signaling notable deviations from the intended market neutrality. Comparatively, the BA-FP strategies have the lowest t-statistics around 2, while betting against the other betas assumes a much stronger market exposure.

Following Novy-Marx and Velikov (2022, Section 5), we examine BA's exposure to market risk premium and market volatility ratio. Specifically, we consider the following regression:

$$r_{BA,t} = \alpha + \beta_1 \mathcal{V}_t + \beta_2 r_{MKT,t} + \beta_3 r_{MKT,t-1} + \beta_4 r_{MKT,t-2} + \varepsilon_t, \tag{4}$$

where $\mathcal{V} = r_{MKT} \cdot \ln(\sigma_{MKT}^{-1Y}/\sigma_{MKT}^{-5Y})$ is the interactive term between market risk premium and the log ratio of the prior one-year market volatility σ_{MKT}^{-1Y} to the prior five-year market volatility σ_{MKT}^{-5Y}. Novy-Marx and Velikov (2022) argue that BAB's significantly negative loading on the log market volatility ratio alludes to volatility timing.

Table 3 shows that the BA-FP strategies always have negative $\hat{\beta}_1$, suggesting higher market tilt when recent market volatility is low. The significance of $\hat{\beta}_1$, however, seems primarily driven by market neutrality rather than the FP beta. Betting against the FP beta with dollar neutrality no long exhibits significant $\hat{\beta}_1$. Similarly, $\hat{\beta}_1$ of the value-weighted BA-OLS strategies is a significant -0.21 upon linear market neutrality but an insignificant 0.13 upon dollar neutrality. The complete analysis for all the 36 strategies in the Internet Appendix reaffirms that no $\hat{\beta}_1$ attains statistical significance upon dollar neutrality, while linear market neutrality promotes more negative $\hat{\beta}_1$ than non-linear market neutrality.

Meanwhile, the market factor coefficients are often significant, attesting BA's imperfect market neutrality. The equal-weighted BA-FP strategy with non-linear market neutrality has a zero $\hat{\beta}_2$, yet its $\hat{\beta}_3$ is a highly significant 0.20 (t-statistic is 8.56). The result conforms to Novy-Marx and Velikov (2022, Table 5, Specification 3)'s argument that BAB's long position in tiny and illiquid stocks facilitates greater beta significance in longer horizon. Comparatively, the capped value-weighting balances tiny stocks across beta terciles, pacifying the nonsynchronous trading effect. Thus, we see that the value-weighted BA-FP strategy with non-linear market neutrality has a trivial $\hat{\beta}_3$ of 0.05.

In addition, the BA-FP strategies are more robust to the nonsynchronous trading concern than the BA-OLS and Welch strategies. Most of the BA-FP strategies have insignificant $\hat{\beta}_3$, which is not the case for the other strategies. In the Internet Appendix, we further show that the Dimson beta does not cure the nonsynchronous trading problem, as half of the BA-Dimson strategies have highly significant $\hat{\beta}_3$. This finding is different from Novy-Marx and Velikov (2022, Table 5), and the main reason is that they use five-year daily data to estimate Dimson beta while we use one-year window to be consistent with other beta estimators (see Cederburg and O'Doherty, 2016 and Welch, 2022, for instance).[12]

We also inspect all the BA strategies relative to Carhart (1997)'s four-factor model in the Internet Appendix. The estimated market exposure remains prevalently significant, and the lowest significance goes for the BA-FP strategies. Compared to traditional betas, the FP beta better suits the objective of leveraging the low-beta portfolio and containing market exposure. At the same time, the BA-FP strategies are persistently exposed to momentum risk. Such momentum exposure corroborates the deviation from market neutrality, as momentum is strongly dependent on market risk (Daniel and Moskowitz, 2016).

Imperfect market neutrality admits the role of time-varying beta in explaining the low-beta anomaly. Section 2 indicates that BAB crashes are triggered by large market variations. We next examine how market timing accounts for beta crashes.

[12] Another reason can be about the differences in strategy construction and sample period. Novy-Marx and Velikov (2022) adopt the plain value-weighting and hedge the exposure with the equal-weighted market factor, while we take the capped value-weighting and hedge with the value-weighted market factor. Their sample period is from 1968 to 2019, while ours from 1926 to 2020.

4.4 Strangle market neutrality

Both economic crises and booms promote beta crashes, yet they entail different market exposure. To distinguish such effects, we plot BA returns against market returns for the value-weighted BA-FP strategy with non-linear market neutrality. Figure 6 displays a negative relationship over the whole sample period, with a slope estimate of -0.18 (t-statistic is -2.10). However, a strongly positive relationship emerges when market return goes below -5%, as the slope estimate is 0.33 (t-statistic is 2.96).

The negative relationship is restored once market return is above 5%. The slope estimate is an important -0.77 (t-statistic is -5.52), corroborating that large market gains foster greater beta crashes. We notice a flat relationship when market return is between -5% and 5%. The slope estimate is a trivial -0.08 (t-statistic is -1.45), meaning that market neutrality is systematically maintained as long as market changes are not dramatic. The piecewise relationship between BA and market returns indicates conditional market timing.

Correspondingly, we consider three market conditions: normal, bear, and bull markets. We define a bear (bull) market dummy variable D_t (U_t) taking the value of one if the market risk premium in month t is below -5% (above 5%) and zero otherwise. The normal market, where the market risk premium is between -5% and 5%, is the benchmark condition. Bear markets account for about 11% of our sample months and bull markets take up about 15%. The distribution is close to Lu and Qin (2021), who refer the bottom 10% as low market state, the top 10% as high state, and the 80% in between as middle state.

We examine the following conditional CAPM regression:

$$r_{BA,t} = \alpha + \alpha_D D_t + \alpha_U U_t + (\beta + \beta_D D_t + \beta_U U_t) r_{MKT,t} + \varepsilon_t. \tag{5}$$

Table 4 summarizes the results for the market neutral BA-FP strategies. Based on the simple CAPM regression, the equal-weighted BA-FP strategy with non-linear market neutrality well achieves its goal of zero market beta. However, the other BA-FP strategies have a significantly negative beta, indicating their particular vulnerability to booming markets.

Introducing market conditions, we find market neutrality systematically maintained in normal markets. For instance, the value-weighted BA-FP strategy with non-linear market neutrality has an insignificant $\hat{\beta}$ of -0.08,[13] while bear markets significantly raise the beta estimate by 0.41 (t-statistic is 2.77). On the contrary, bull markets substantially lower the beta estimate by 0.69 (t-statistic is -3.01). Therefore, the point estimate of beta is 0.33 (-0.08+0.41) in bear markets and a remarkable -0.77 (-0.08-0.69) in bull markets.

The material exposure to bear and bull markets is not unique to market neutral strategies. In the Internet Appendix, we register that dollar-neutral strategies have more significant $\hat{\beta}_D$ and $\hat{\beta}_U$. One may concern that the threshold specification of $\pm 5\%$ can affect the diagnostics of BA's sensitivity to market conditions. For robustness, we next use a simple threshold of zero to determine upside and downside markets, consistent with Grundy and Martin (2001) and Daniel and Moskowitz (2016). We conduct Daniel and Moskowitz (2016)'s optionality analysis to check if BA is exposed to market upswing risk.

We run the following regression:

$$r_{BA,t} = \gamma_0 + \gamma_1 D^*_{t-1} + \big(\gamma_2 + D^*_{t-1}(\gamma_3 + \gamma_4 CU^*_t)\big) r_{MKT,t} + \varepsilon_t, \tag{6}$$

where D^*_t is a cumulative downside market indicator taking the value of one if the prior one-year cumulative market return is negative and zero otherwise, and CU^*_t is a contemporary upside market indicator taking the value of one if market risk premium is positive and zero otherwise.[14] The two indicators help to capture the effect of market upswings.

Table 5 provides the key results. In Panel A, we see highly significant $\hat{\gamma}_4$ for the BA-FP strategies. Specifically, the value-weighted BA-FP strategy with non-linear market neutrality

[13]The t-statistic is slightly different from Figure 6 Panel C because we use the full sample to estimate the conditional CAPM regression.

[14]Daniel and Moskowitz (2016) adopt a lookback window of two years to determine downside market. Since beta estimates predominantly rely on return information over the past year, the one-year window better conforms to the methodology of BA strategies. However, as we show in the Internet Appendix, using the two-year window or the NBER-based recession indicator renders similar results.

has a $\hat{\gamma}_4$ of -0.69 (t-statistic is -3.11), which is close to the momentum optionality (Daniel and Moskowitz, 2016, Table 3). The point estimate of strategy beta is an important -0.62 (-0.02+0.09-0.69) when the market rebounds, suggesting that market upswings precipitate large declines. For example, the strategy incurs a heavy loss around 10% if the market regains 20% from recession, as if it shorts a call option on the market.

Panels B and C confirm this optionality for the BA-FP strategies with different hedging. The other BA strategies often have insignificant $\hat{\gamma}_4$, but their market timing behaviors upon market upswings are still persistent. Figure 8 exhibits the point estimate of strategy beta when the market rebounds from recession ($\hat{\gamma}_2 + \hat{\gamma}_3 + \hat{\gamma}_4$). Interestingly, the dollar-neutral BA strategies take the least deviation from zero beta, while the market neutral BA strategies assume a much stronger market exposure.

Frazzini and Pedersen (2014) emphasize that funding shocks impair market neutrality. Following their choice of TED-spread volatility to proxy for funding difficulties,[15] we find that funding liquidity risk cannot fully accommodate the deviation from market neutrality. The average TED-spread volatility is about 0.06% in bull and normal markets, while it doubles to 0.12% in bear markets. Since liquidity difficulties asymmetrically fall in bear markets (Brunnermeier and Pedersen, 2009, Adrian and Shin, 2014, and He et al., 2017), the prevalent market timing behaviors in bull markets are left unexplained.

Besides, beta dispersion is negatively related to TED-spread volatility (Novy-Marx and Velikov, 2022, Table 4) and it neither accounts for the beta crashes in bull markets. In the Internet Appendix, we regress the beta spread between the high-beta and low-beta portfolios $\beta_H - \beta_L$ on market risk premium with and without market condition indicators. We do not find evidence that bull markets promote large losses by compressing beta. The slope coefficients are economically small and statistically indistinguishable from zero. Moreover, market conditions neither help to predict beta spread. These observations reaffirm that beta compression is unlikely responsible for the bull-market beta crashes.

[15] Daily data of the TED spread (TEDRATE) are from Federal Reserve Bank of St. Louis, and the earliest date is January 2, 1986. See Novy-Marx and Velikov (2022, Section 4.4) for more discussions.

4.5 Market variance risk

We have seen that BA performance is related to market risk premium, and next we show it is also related to market variance. Consistently, we run the following regression:

$$r_{BA,t} = \kappa + \kappa_D D_t + \kappa_U U_t + (\lambda + \lambda_D D_t + \lambda_U U_t)\hat{\sigma}^2_{MKT,t} + \varepsilon_t, \tag{7}$$

where $\hat{\sigma}^2_{MKT,t}$ is the variance of daily market factor returns in month t.

Table 6 shows the results for the market neutral BA-FP strategies. Simply regressing BA returns on market variances, we register highly negative slope coefficients. For example, the equal-weighted BA-FP strategy with non-linear market neutrality has a $\hat{\lambda}$ of -1.65 (t-statistic is -3.79). As the value of a call option on the market is positively related to market variance, writing a call loses in volatile markets. The prevalently negative $\hat{\lambda}$ supports our finding of BA's optionality. Comparatively, the Internet Appendix reports that the dollar-neutral BA-FP strategies have insignificant $\hat{\lambda}$, consistent with Figure 8 that dollar neutrality realizes smaller deviation from zero beta than market neutrality upon market upswings.

Adding market condition indicators further reveals that BA returns are particularly relevant to bull-market variance risk. Take the value-weighted BA-FP strategy with non-linear market neutrality for example. It is trivially exposed to market variance risk in normal markets given an insignificant $\hat{\lambda}$ of 0.43 (t-statistic is 0.71). Bull markets bring greater changes in the sensitivity of market variance risk, as the strategy's $\hat{\lambda}_U$ (-6.51 with a t-statistic of -2.08) is more significant than its $\hat{\lambda}_D$ (-1.23 with a t-statistic of -1.89). For the other strategies, the absolute value of $\hat{\lambda}_U$ is always larger than that of $\hat{\lambda}_D$, corroborating that the variance risk in bull markets is more detrimental to BA performance.

The negative impact of bull-market variance raises a question: BA's reduced market exposure in bull markets should bring volatility timing benefits upon increased bull-market variance. We find this bull-market volatility timing ineffective because the market risk return tradeoff changes. Figure 7 illustrates that the tradeoff is negative when market risk premium is below 5%. Specifically, the negative tradeoff is highly significant in bear markets (the slope coefficient's t-statistic is -3.55) and marginally significant in normal markets (t-statistic is -1.69). In both cases, less market exposure is profitable.

However, the bull-market slope coefficient is a highly significant 5.91 (t-statistic is 3.47), dominating its peers in size (-1.74 in bear markets and -1.19 in normal markets). Managing market volatility is disadvantageous in the presence of a strong risk return tradeoff. Moreover, BA's market exposure and market variance significantly go up in bear markets, effectuating negative volatility timing. As a result, volatility timing in bull and bear markets fails to improve the performance of BA strategies.

Both bull and bear markets have high variance. Comparatively, funding liquidity risk predominantly falls in bear markets, and this asymmetric feature facilitates the separation of market variance effects on BA performance. We consider the following regression:

$$r_{BA,t} = \theta + \theta_1 \hat{\sigma}^2_{MKT,t} + \theta_2 \hat{\sigma}_{TED,t} + \theta_3 TED_t + \theta_4 \Delta TED_t + \varepsilon_t, \tag{8}$$

where $\hat{\sigma}_{TED,t}$ is the volatility of daily TED spreads in month t, TED_t is the TED spread at the end of month t, and ΔTED_t is the TED spread difference between month t and month $t-1$. The definition of the three TED-spread variables conforms to Frazzini and Pedersen (2014) and Novy-Marx and Velikov (2022).

Figure 9 exhibits the t-statistics for the slope estimates. Market variance risk remains the most important in explaining BA performance, indicating that the bull-market volatility timing is a greater concern than the bear-market volatility timing. Market variance risk is much more significant than funding liquidity risk, particularly for linear market neutrality. Consistent with Novy-Marx and Velikov (2022), we note that the TED-spread volatility plays a marginal role when the level of TED spread is present. We also consider lagged terms as explanatory variables, and we find the lagged market variance insignificant in predicting BA performance. This is not surprising due to the ex ante market neutrality.

4.6 Managing beta crashes

Moreira and Muir (2017) and Cederburg et al. (2020) register that volatility management is particularly successful for BAB. Beta crashes deteriorate the risk return tradeoff by depressing return and elevating risk, extending a favorable condition to manage volatility. We next explore the volatility-managed BA performance with the following strategy:

$$r^{\sigma}_{BA,t} = \frac{c}{\hat{\sigma}_{BA,t-1}} r_{BA,t}, \tag{9}$$

where c corresponds to an annualized volatility level of 12% following Barroso and Santa-Clara (2015) and Barroso and Maio (2021), and $\hat{\sigma}_{BA,t}$ is the realized volatility calculated from daily BA returns in month t.[16]

Barroso and Maio (2021) argue that the FF-6 model subsumes BAB performance. We expand the analysis for the 36 BA strategies and report the results in the Internet Appendix. Significant FF-6 alphas go to half of the market neutral BA strategies and one-third of the dollar-neutral BA strategies. Non-linear market neutrality is remarkably advantageous in promoting abnormal performance. For example, Table 7 shows that the FF-6 alphas of the value-weighted BA-FP strategies are persistent, and non-linear market neutrality brings the most significant 0.55% (t-statistic is 3.34).

We consider extending the FF-6 model for better explanatory power. Bali et al. (2017), Schneider et al. (2020), Asness et al. (2020), Stambaugh and Yuan (2017), Liu et al. (2018), and Barroso and Maio (2021) highlight that skewness preference, market sentiment, mispricing, and limits to arbitrage contribute to explaining the low-beta anomaly. In particular, Bali et al. (2017) develop FMAX factor to characterize skewness preference; Stambaugh and Yuan (2017) propose two mispricing factors (MGMT and PERF), which capture investor sentiment and arbitrage risk. Therefore, we augment the FF-6 model with the three factors

[16] In the literature, realized variance is also used for risk scaling, yet the performance difference is not material (see Moreira and Muir, 2017, Table 5). We choose realized volatility for its advantage in mitigating transaction costs, as noted by Moreira and Muir (2017, page 1625) and Barroso and Detzel (2021, page 751).

to arrive at an extended 9-factor model.[17]

The 9-factor model well subsumes the BA abnormal performance, as only three alphas are left significantly positive. Linear market neutrality and dollar neutrality fail to sustain significant alphas. Novy-Marx and Velikov (2022, Tables C.8 and C.9) note that the value-weighted BAB has lower FF-6 alphas than the equal-weighted BAB. We do not observe such a pattern for the 9-factor model, given that the value-weighted BA-FP strategies always have higher alphas. A plausible reason is that small stocks suffer from strong mispricing error (Stambaugh et al., 2015 and Stambaugh and Yuan, 2017), which is captured by the mispricing factors. Therefore, the equal-weighted strategies are no longer rewarded for their tilt to small stocks in the 9-factor model.

Managing BA risk strongly improves performance. In particular, the economical and statistical significance of alphas is considerably enhanced for the BA-FP strategies. For example, the value-weighted BA-FP strategy with linear market neutrality has an insignificant 9-factor model alpha of 0.23%, while volatility management doubles it to 0.47% (t-statistic is 2.73). Volatility management is also beneficial for the other strategies. The equal-weighted BA-Welch strategy with linear market neutrality has a trivial 9-factor model alpha of 0.11% (t-statistic is 0.68), while its managed version obtains 0.97% (t-statistic is 3.52).

In addition, volatility management is especially profitable for the dollar-neutral BA strategies. The 9-factor model, which fully prices all the dollar-neutral BA strategies, fails to subsume any of the managed strategies. The dollar-neutral equal-weighted BA-FP strategy is a case in point: volatility management favorably raises the 9-factor model alpha from -0.06% (t-statistic is -0.45) to 0.63% (t-statistic is 4.30). In general, we register 32 (25) significant FF-6 (9-factor) model alphas in the Internet Appendix, corroborating Barroso and Maio (2021)'s argument that the success of managed-BAB is puzzling.

A practical concern of volatility management profits is transaction cost. Following Mor-

[17] The 9-factor model fully incorporates Stambaugh and Yuan (2017)'s M-4 factor model to accommodate the low-beta anomaly, as a parsimonious model is limited to explain expected returns in the presence of mispricing (Stambaugh and Yuan, 2017, page 1037).

eira and Muir (2017, Section II.B), we consider three assumptions of trading cost: 1bps, 10bps, and 14bps.[18] We examine if the significant benefits of BA risk management survive transaction costs. Table 8 summarizes the results for 10bps and 14bps trading costs, as the results for 1bp are highly close to Table 7. All alphas remain positive, except for the 9-factor model alphas of the value-weighted BA-OLS strategy with non-linear market neutrality. Furthermore, most of the positive alphas keep statistical significance.

Take the value-weighted BA-FP strategies for example. With non-linear market neutrality, the 9-factor model alpha is 0.63% (t-statistic is 3.57) when trading cost is 10bps and 0.62% (t-statistic is 3.50) when trading cost is 14bps. With linear market neutrality, the alpha remains a large 0.44% (t-statistic is 2.56) upon 10bps trading cost and 0.42% (t-statistic is 2.48) upon 14bps trading cost. In the Internet Appendix, we analyze the effect of transaction costs for all the 36 BA strategies. We observe prevalent performance improvement with significance, and the profitability of BA risk management is robust to transaction costs.

4.7 International evidence

To check if the concern of imperfect market neutrality and beta crashes is unique to the US market or not, we investigate a global BAB sample covering 24 markets such as France, Germany, and UK. Maintained by AQR, the international BAB factors are composed with FP beta, rank-weighting, and non-linear market neutrality. We provide the sample description as well as the detailed results in the Internet Appendix.

Briefly, the international BAB factors similarly encounter crashes. The average drawdown of the global sample is 52%, slightly lower than the US drawdown of 55%. For example, the BAB drawdown is 69% in the UK market, suggesting a greater impact of BAB crashes than in the US market. Calculating realized betas for the international sample, we find that most factors have an economically large and statistically significant market beta, a clear evidence

[18] Barroso and Detzel (2021, Appendix A) point out that ETFs provide a cost-effective way to manage portfolios, and we assume that investors track BA strategies with ETFs. Barroso and Detzel (2021) comprehensively examine the effect of transaction cost at stock level when common factors are under management.

that market neutrality is not systematically maintained across markets. Compared to the US level of 0.08, the average absolute market beta is a higher 0.12. The concern of market neutrality deviations seems more acute in the international markets.

As BAB crashes deteriorate the risk return tradeoff, managing them helps to ameliorate performance. Volatility management brings 17 Sharpe ratio increases, and most of them are statistically significant. As the international risk factors such as the mispricing factors are not available, we directly regress each managed BAB factor on its original version. We register 19 positive alphas, and most of them are significant. The average annualized alpha is about 2%, and this size is even greater than that of the managed HML factor in the US market (Moreira and Muir, 2017, Table 1).

5 Conclusion

Frazzini and Pedersen (2014) intend their BAB strategy to be market neutral, and this feature challenges the conditional CAPM in explaining the low-beta anomaly. Nonetheless, we find that BAB takes a systematic deviation from market neutrality, and BAB crashes often come in volatile markets. Such a concern is common to a broad range of 36 low-beta strategies, covering six different beta estimators, three hedging methods, and two weighting schemes. We document that betting against more standard betas such as the OLS beta brings worse crash risk.

Our paper contributes to understanding BAB risk. Frazzini and Pedersen (2014) point out that BAB may depart from market neutrality upon funding liquidity shocks. We notice that BAB's deviation from market neutrality in bull markets is more significant than in bear markets. However, liquidity tends to be pro-cyclical and leverage constraints are less binding in economic expansion. Thus, funding liquidity risk cannot address BAB's particular vulnerability to bull markets. The crashes of market neutral low-beta strategies can be interpreted as negative market timing and negative volatility timing amid large market variations, suggesting that the conditional CAPM sheds light on the low-beta effect.

In addition, managing beta crashes leads to significant performance improvements. Leading explanations such as missing risk factors, skewness preference, mispricing, and limits to arbitrage cannot subsume the abnormal performance of managed low-beta strategies. The benefits of volatility management are also robust to transaction costs.

References

Adrian, T. and H. S. Shin (2014). Procyclical leverage and Value-at-Risk. *Review of Financial Studies 27*(2), 373–403.

Ang, A., J. Chen, and Y. Xing (2006). Downside risk. *Review of Financial Studies 19*(4), 1191–1239.

Ang, A., S. Gorovyy, and G. B. Van Inwegen (2011). Hedge fund leverage. *Journal of Financial Economics 102*(1), 102–126.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *Journal of Finance 61*(1), 259–299.

Asness, C., A. Frazzini, N. J. Gormsen, and L. H. Pedersen (2020). Betting against correlation: Testing theories of the low-risk effect. *Journal of Financial Economics 135*(3), 629–652.

Bai, H., K. Hou, H. Kung, E. X. Li, and L. Zhang (2019). The CAPM strikes back? An equilibrium model with disasters. *Journal of Financial Economics 131*(2), 269–298.

Baker, M., B. Bradley, and J. Wurgler (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. *Financial Analysts Journal 67*(1), 40–54.

Bali, T. G., S. J. Brown, S. Murray, and Y. Tang (2017). A lottery-demand-based explanation of the beta anomaly. *Journal of Financial and Quantitative Analysis 52*(6), 2369–2397.

Barroso, P. and A. Detzel (2021). Do limits to arbitrage explain the benefits of volatility-managed portfolios? *Journal of Financial Economics 140*(3), 744–767.

Barroso, P. and P. Maio (2021). Managing the risk of the beta anomaly. *Working paper*.

Barroso, P. and P. Santa-Clara (2015). Momentum has its moments. *Journal of Financial Economics 116*(1), 111–120.

Bawa, V. S. and E. B. Lindenberg (1977). Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics 5*(2), 189–200.

Black, F. (1972). Capital market equilibrium with restricted borrowing. *Journal of Business 45*(3), 444–455.

Black, F. (1993). Beta and return. *Journal of Portfolio Management 20*(1), 8–18.

Black, F., M. C. Jensen, and M. S. Scholes (1972). The capital asset pricing model: Some empirical tests. In M. C. Jensen (Ed.), *Studies in the Theory of Capital Markets*, pp. 79–121. New York: Praeger.

Boguth, O., M. Carlson, A. Fisher, and M. Simutin (2011). Conditional risk and performance evaluation: Volatility timing, overconditioning, and new estimates of momentum alphas. *Journal of Financial Economics 102*(2), 363–389.

Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *Review of Financial Studies 22*(6), 2201–2238.

Carhart, M. M. (1997). On persistence in mutual fund performance. *Journal of Finance 52*(1), 57–82.

Cederburg, S. and M. S. O'Doherty (2016). Does it pay to bet against beta? On the conditional performance of the beta anomaly. *Journal of Finance 71*(2), 737–774.

Cederburg, S., M. S. O'Doherty, F. Wang, and X. S. Yan (2020). On the performance of volatility-managed portfolios. *Journal of Financial Economics 138*(1), 95–117.

Daniel, K. and T. J. Moskowitz (2016). Momentum crashes. *Journal of Financial Economics 122*(2), 221–247.

Dimson, E. (1979). Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics 7*(2), 197–226.

Fama, E. and K. French (1992). The cross-section of expected stock returns. *Journal of Finance 47*(2), 427–465.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Fama, E. F. and K. R. French (2015). A five-factor asset pricing model. *Journal of Financial Economics 116*(1), 1–22.

Fama, E. F. and K. R. French (2016). Dissecting anomalies with a five-factor model. *Review of Financial Studies 29*(1), 69–103.

Frazzini, A. and L. H. Pedersen (2014). Betting against beta. *Journal of Financial Economics 111*(1), 1–25.

Friend, I. and M. Blume (1970). Measurement of portfolio performance under uncertainty. *American Economic Review 60*(4), 561–575.

Grundy, B. D. and J. S. M. Martin (2001). Understanding the nature of the risks and the source of the rewards to momentum investing. *Review of Financial Studies 14*(1), 29–78.

Han, X. (2022). Understanding the performance of components in betting against beta. *Critical Finance Review 11*(1), 1–36.

Harvey, C. R. and A. Siddique (2000). Conditional skewness in asset pricing tests. *Journal of Finance 55*(3), 1263–1295.

Haugen, R. A. and A. J. Heins (1975). Risk and the rate of return on financial assets: Some old wine in new bottles. *Journal of Financial and Quantitative Analysis*, 775–784.

He, Z., B. Kelly, and A. Manela (2017). Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics 126*(1), 1–35.

Hollstein, F., M. Prokopczuk, and C. W. Simen (2020). Beta uncertainty. *Journal of Banking & Finance 116*, 105834.

Jagannathan, R. and Z. Wang (1996). The conditional CAPM and the cross-section of expected returns. *Journal of Finance 51*(1), 3–53.

Jensen, T. I., B. T. Kelly, and L. H. Pedersen (2022). Is there a replication crisis in finance? *Journal of Finance, forthcoming*.

Lewellen, J. and S. Nagel (2006). The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics 82*(2), 289–314.

Liu, J., R. F. Stambaugh, and Y. Yuan (2018). Absolving beta of volatility's effects. *Journal of Financial Economics 128*(1), 1–15.

Lu, Z. and Z. Qin (2021). Leveraged funds and the shadow cost of leverage constraints. *Journal of Finance 76*(3), 1295–1338.

Miller, M. H. and M. Scholes (1972). Rates of return in relation to risk: A reexamination of some recent findings. In M. C. Jensen (Ed.), *Studies in the Theory of Capital Markets*, pp. 47–48. New York: Praeger.

Moreira, A. and T. Muir (2017). Volatility-managed portfolios. *Journal of Finance 72*(4), 1611–1644.

Newey, W. K. and K. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–708.

Novy-Marx, R. and M. Velikov (2022). Betting against betting against beta. *Journal of Financial Economics 143*(1), 80–106.

Reinganum, M. R. (1981). A new empirical perspective on the CAPM. *Journal of Financial and Quantitative Analysis*, 439–462.

Schneider, P., C. Wagner, and J. Zechner (2020). Low-risk anomalies? *Journal of Finance 75*(5), 2673–2718.

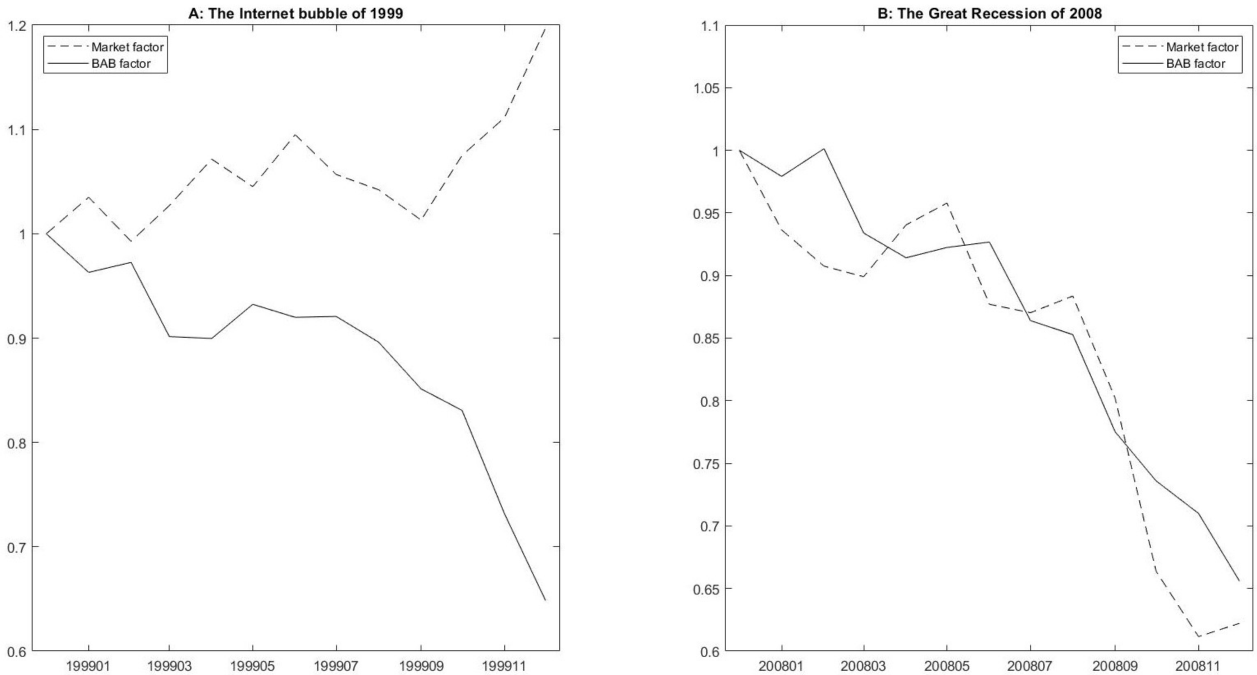
Stambaugh, R. F., J. Yu, and Y. Yuan (2015). Arbitrage asymmetry and the idiosyncratic volatility puzzle. *Journal of Finance 70*(5), 1903–1948.

Stambaugh, R. F. and Y. Yuan (2017). Mispricing factors. *Review of Financial Studies 30*(4), 1270–1315.

Vasicek, O. A. (1973). A note on using cross-sectional information in bayesian estimation of security betas. *Journal of Finance 28*(5), 1233–1239.

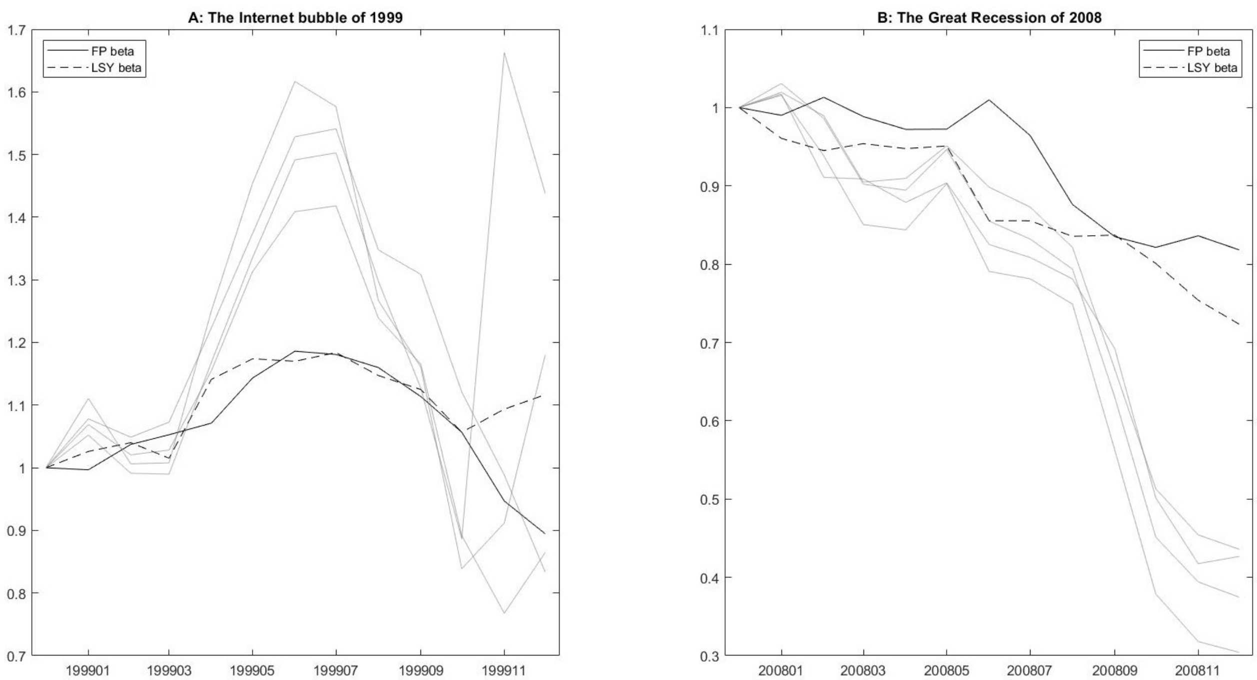
Welch, I. (2022). Simply better market betas. *Critical Finance Review 11*(1), 37–64.

Figures



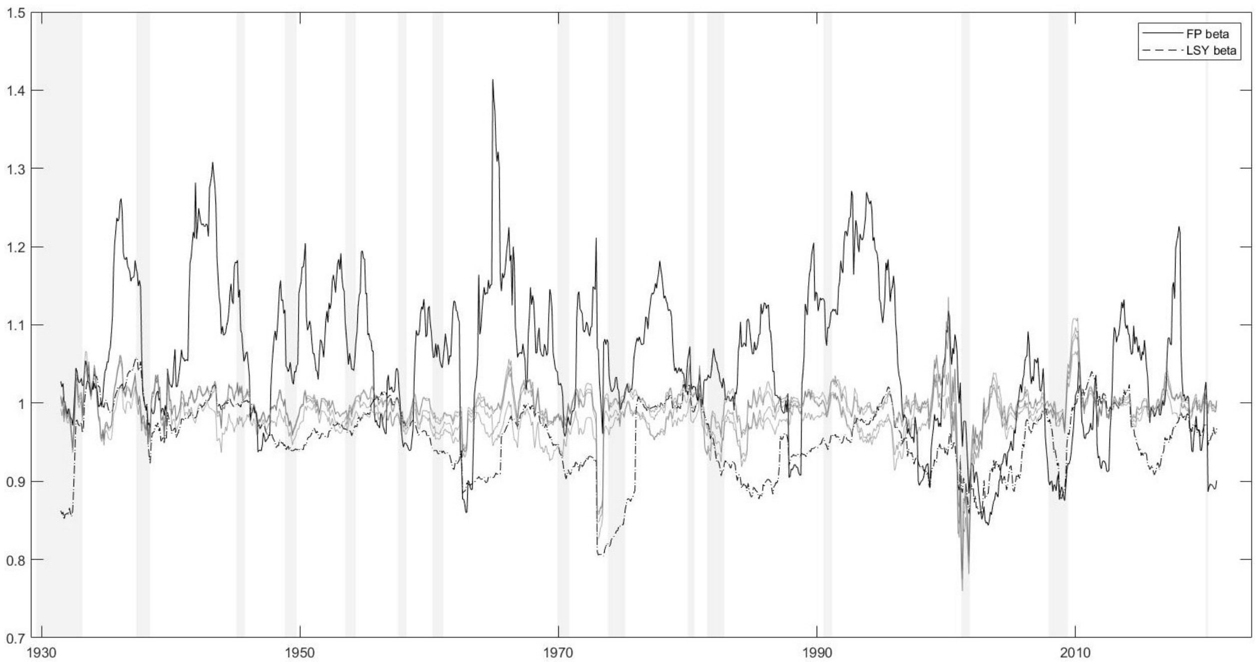
This figure shows two expressive crashes for the betting against beta (BAB) factor. Panel A is during the Internet bubble of 1999, and Panel B is during the Great Depression of 2008. Assuming an initial investment of $1 at the beginning of the year, we plot the cumulative return to the market (by dashed line) and BAB (by solid line) factors. Data of the BAB and market factors are from AQR's datasets and Kenneth French's data library, respectively.

Figure 1: Two expressive BAB crashes



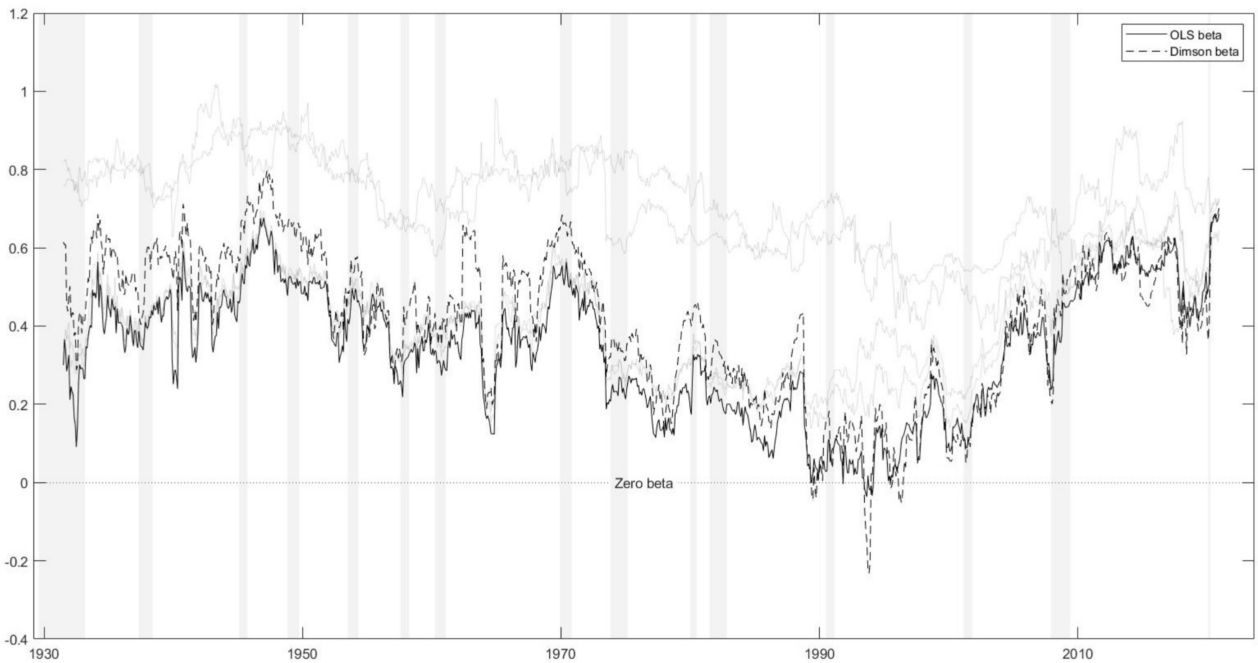
This figure revisits the two expressive episodes for the six value-weighted beta arbitrage (BA) strategies with non-linear market neutrality. Panel A is during the Internet bubble of 1999, and Panel B is during the Great Depression of 2008. We use six beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, Dimson (1979) beta, Vasicek (1973) beta, Welch (2022) beta, and Liu et al. (2018) (LSY) beta. We use Jensen et al. (2022)'s capped value-weighting scheme and Frazzini and Pedersen (2014)'s non-linear market neutrality to compose BA strategies. Individual stock data are from the CRSP, and the sample period is from July 1926 to December 2020.

Figure 2: Crashes revisited



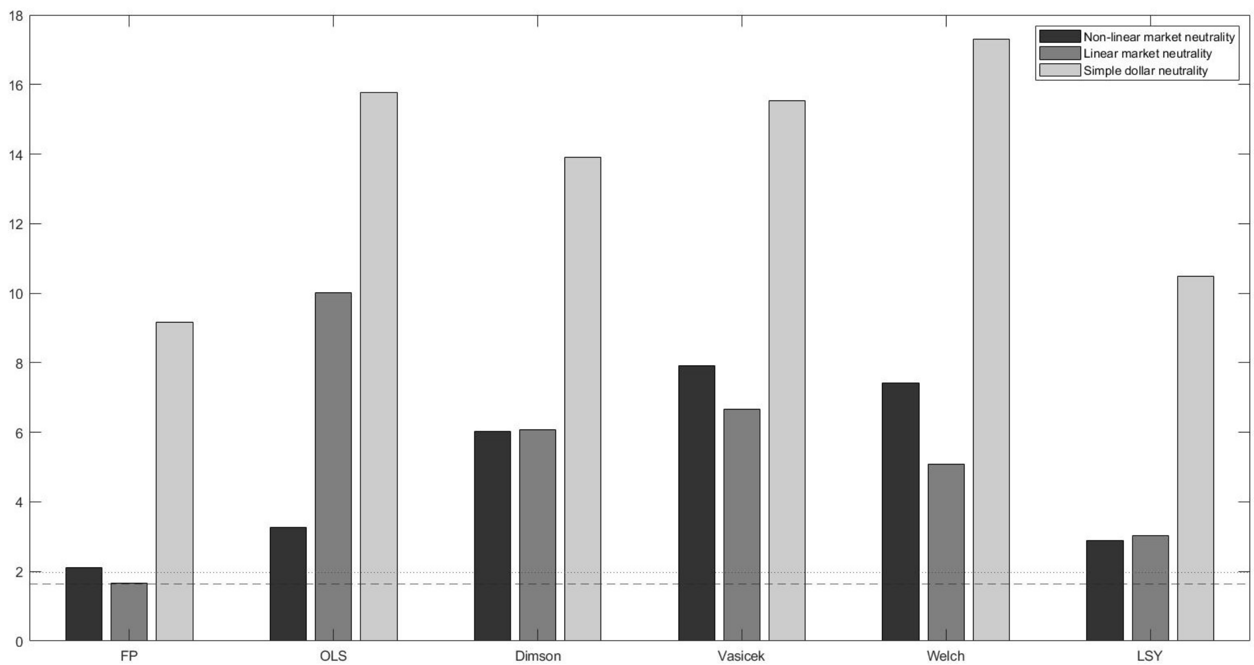
This figure shows the value-weighted average of beta estimates across the sample period. We use six beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, Dimson (1979) beta, Vasicek (1973) beta, Welch (2022) beta, and Liu et al. (2018) (LSY) beta. The series is plotted by black solid line for the FP beta, by black dashed line for the LSY beta, and by grey solid lines for the other four betas. Individual stock data are from the CRSP, and the sample period is from July 1926 to December 2020. Shaded bars indicate NBER economic recessions.

Figure 3: Time series of the value-weighted average of beta estimates



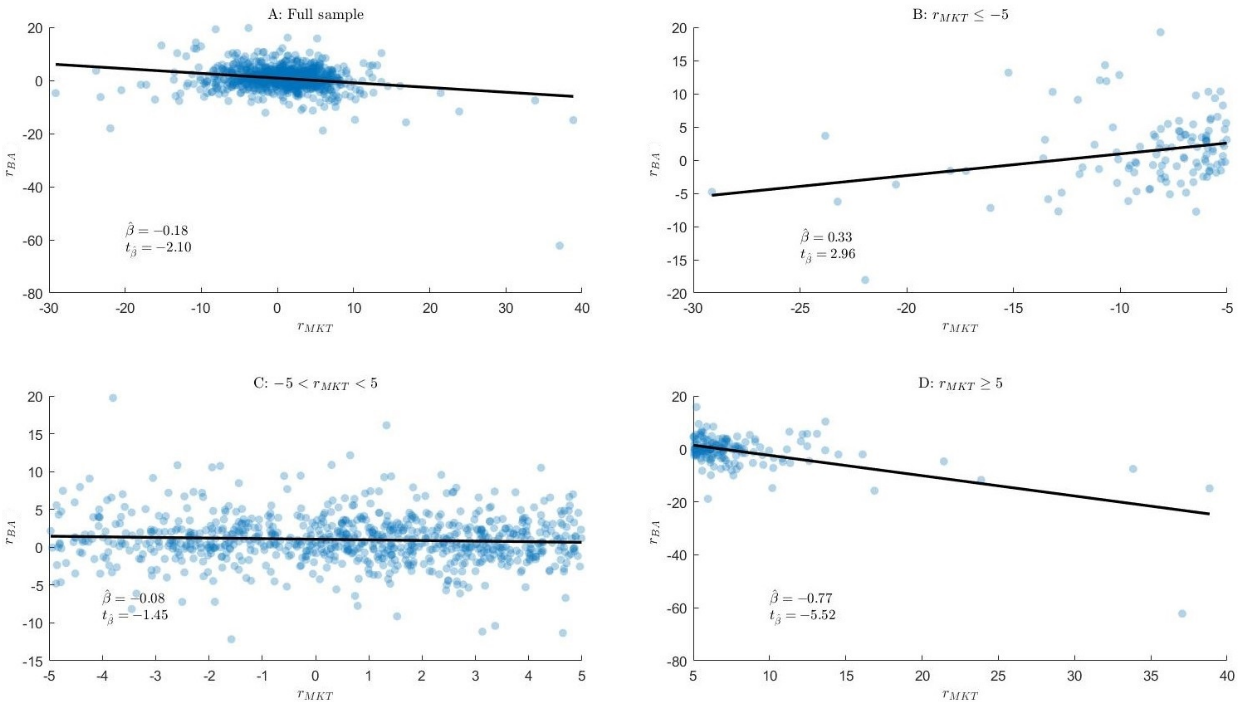
This figure shows the estimated beta of the low-beta portfolio across the sample period. We use six beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, Dimson (1979) beta, Vasicek (1973) beta, Welch (2022) beta, and Liu et al. (2018) (LSY) beta. The low-beta portfolio is developed following Jensen et al. (2022)'s capped value-weighting scheme. The series is plotted by black solid line for the OLS beta, by black dashed line for the Dimson beta, and by grey solid lines for the other four betas. Individual stock data are from the CRSP, and the sample period is from July 1926 to December 2020. Shaded bars indicate NBER economic recessions.

Figure 4: Time series of the estimated beta of the low-beta portfolio



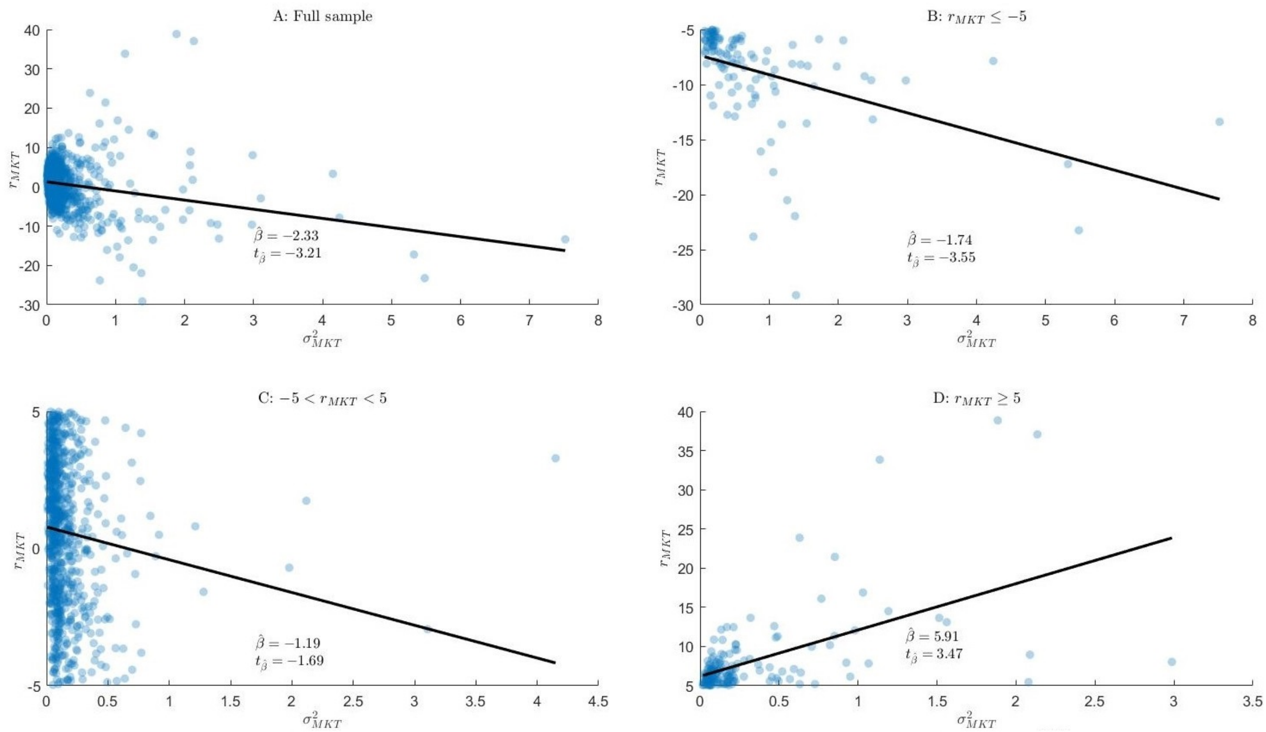
This figure shows the absolute value of t-statistics associated with realized betas for the 18 value-weighted beta arbitrage (BA) strategies. We use six beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, Dimson (1979) beta, Vasicek (1973) beta, Welch (2022) beta, and Liu et al. (2018) (LSY) beta. We use Jensen et al. (2022)'s capped value-weighting scheme to compose BA strategies. For hedging methods, we use non-linear market neutrality as explained by Frazzini and Pedersen (2014), linear market neutrality as explained by Novy-Marx and Velikov (2022), and simple dollar neutrality. The t-statistics are computed from the CAPM regressions using Newey and West (1987) standard errors with 12 monthly lags. The dashed line denotes a cut-off t-statistic of 1.645 and the dotted line denotes a cut-off t-statistic of 1.96.

Figure 5: Market exposure and realized betas



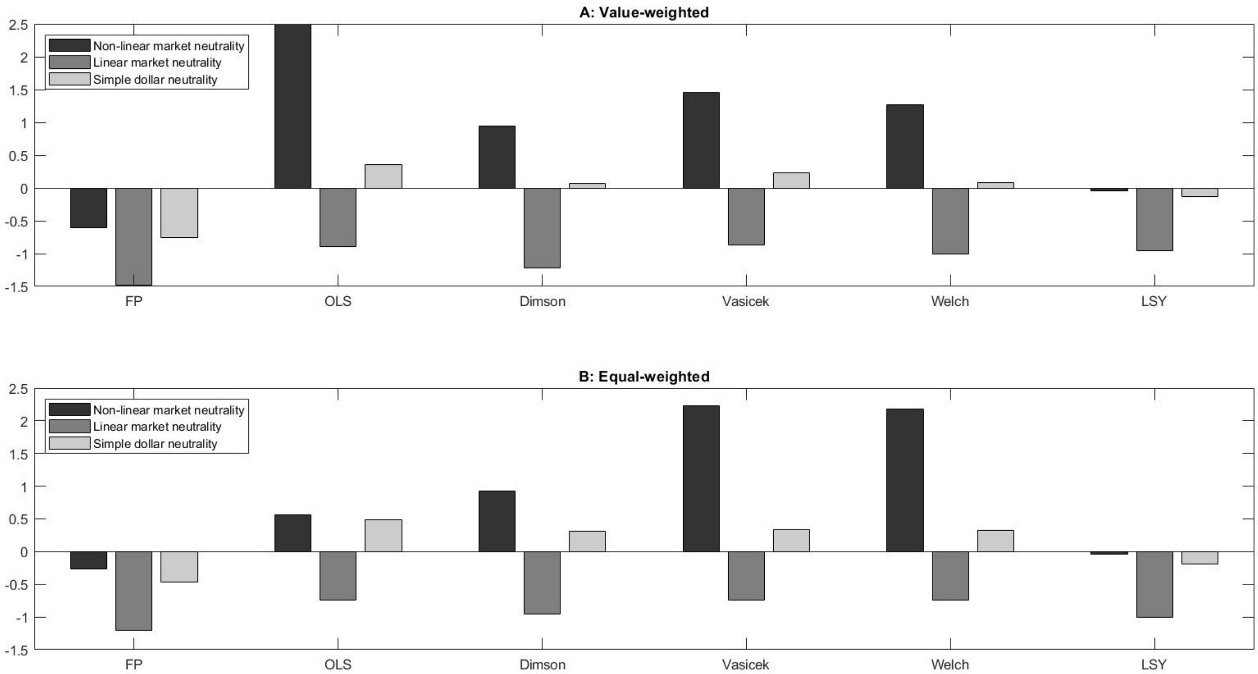
This figure shows the relationship between the market factor and the value-weighted BA-FP strategy in different market states. To construct the value-weighted BA-FP strategy, we use Frazzini and Pedersen (2014) (FP) beta, Jensen et al. (2022)'s capped value-weighting, and non-linear market neutrality as explained by Frazzini and Pedersen (2014). Panel A plots the scatter points of market returns and BA-FP returns, as well as the fitted line, over the full sample; Panel B refers to the subsample when the return of the market factor is below -5%; Panel C refers to the subsample when the return of the market factor is between -5% and 5%; Panel D refers to the subsample when the return of the market factor is above 5%. We also report the beta estimates and t-statistics, using Newey and West (1987) standard errors with 12 monthly lags. Individual stock data are from the CRSP, and the sample period is from July 1926 to December 2020.

Figure 6: Market neutrality and market states



This figure shows the risk return relationship of the market factor in different market states. Panel A plots the scatter points of market returns and market variances, as well as the fitted line, over the full sample; Panel B refers to the subsample when the return of the market factor is below -5%; Panel C refers to the subsample when the return of the market factor is between -5% and 5%; Panel D refers to the subsample when the return of the market factor is above 5%. We also report the beta estimates and t-statistics, using Newey and West (1987) standard errors with 12 monthly lags. The data and sample period are consistent with Figure 6.

Figure 7: Market variance and market states

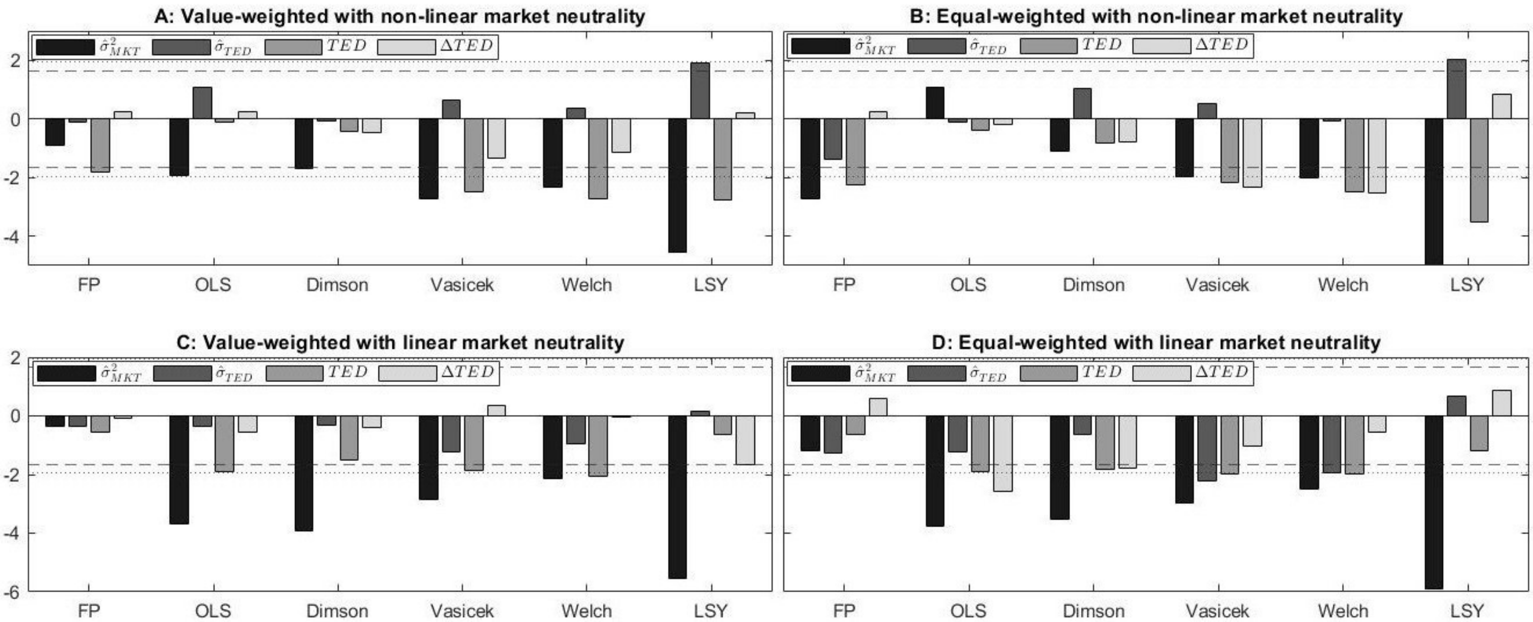


This figure shows the realized beta of the 36 beta arbitrage (BA) strategies in market upswings. We use six beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, Dimson (1979) beta, Vasicek (1973) beta, Welch (2022) beta, and Liu et al. (2018) (LSY) beta. We use Jensen et al. (2022)'s capped value-weighting scheme and simple equal-weighting scheme to compose BA strategies. For hedging methods, we use non-linear market neutrality as explained by Frazzini and Pedersen (2014), linear market neutrality as explained by Novy-Marx and Velikov (2022), and simple dollar neutrality. We run the following regression:

$$r_{BA,t} = \gamma_0 + \gamma_1 D^*_{t-1} + \big(\gamma_2 + D^*_{t-1}(\gamma_3 + \gamma_4 CU^*_t)\big) r_{MKT,t} + \varepsilon_t,$$

where r_{BA} is the BA return, r_{MKT} is the market factor return, D^*_t is a cumulative downside market indicator taking the value of one if the past one-year cumulative market excess return is negative and zero otherwise, and CU^*_t is a contemporary upside market indicator taking the value of one if the market risk premium is positive and zero otherwise. The point estimate of strategy beta is $\hat{\gamma}_2 + \hat{\gamma}_3 + \hat{\gamma}_4$ when the market rebounds from recessions.

Figure 8: Realized beta in market upswings



This figure shows the effects of market variance risk and funding liquidity risk on the 24 market neutral beta arbitrage (BA) strategies. We use six beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, Dimson (1979) beta, Vasicek (1973) beta, Welch (2022) beta, and Liu et al. (2018) (LSY) beta. We use Jensen et al. (2022)'s capped value-weighting scheme and simple equal-weighting scheme to compose BA strategies. For hedging methods, we use non-linear market neutrality as explained by Frazzini and Pedersen (2014) and linear market neutrality as explained by Novy-Marx and Velikov (2022). We run the following regression:

$$r_{BA,t} = \theta + \theta_1 \hat{\sigma}^2_{MKT,t} + \theta_2 \hat{\sigma}_{TED,t} + \theta_3 TED_t + \theta_4 \Delta TED_t + \varepsilon_t,$$

where r_{BA} is the BA return, $\hat{\sigma}^2_{MKT,t}$ is the variance of daily market factor returns in month t, $\hat{\sigma}_{TED,t}$ is the volatility of daily TED spreads in month t, TED_t is the TED spread at the end of month t, and ΔTED_t is the TED spread difference between month t and month $t-1$. The t-statistics are computed by using Newey and West (1987) standard errors with 12 monthly lags. We plot the t-statistics associated with the coefficients of $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_4$. The dashed lines denote cut-off t-statistics of ± 1.645 and the dotted lines denote cut-off t-statistics of ± 1.96.

Figure 9: Market variance and TED spread

Tables

Table 1: Worst and best BAB returns

This table lists the 10 worst and best monthly returns to the betting against beta (BAB) factor from December 1930 to December 2020. The contemporary returns to the market factor are also reported. Data of the BAB and market factors are from AQR's datasets and Kenneth French's data library, respectively.

	Top losing months			Top winning months		
	Month	BAB	Market	Month	BAB	Market
1	193909	-21.95	16.88	193305	18.65	21.43
2	193207	-19.07	33.84	200102	15.39	-10.05
3	200101	-15.68	3.13	200011	13.50	-10.72
4	200211	-14.25	5.96	200204	12.92	-5.20
5	200002	-13.46	2.45	200212	12.15	-5.76
6	198710	-12.69	-23.24	193306	11.96	13.11
7	199911	-11.94	3.37	200901	11.90	-8.12
8	193208	-11.54	37.06	200103	10.52	-7.26
9	199912	-11.33	7.72	200202	9.70	-2.29
10	200006	-10.15	4.64	193404	9.56	-1.79
Average		-14.21	9.18		12.62	-1.67

Table 2: Performance of BA strategies

This table presents the performance of the 18 beta arbitrage (BA) strategies. We use three beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, and Welch (2022) beta. We use Jensen et al. (2022)'s capped value-weighting (VW) scheme and simple equal-weighting (EQ) scheme to compose BA strategies. Panel A refers to the strategies with non-linear market neutrality as explained by Frazzini and Pedersen (2014), Panel B refers to the strategies with linear market neutrality as explained by Novy-Marx and Velikov (2022), and Panel C refers to the strategies with simple dollar neutrality. Performance measures include mean return, standard deviation, Sharpe ratio, minimum return, maximum return, skewness, and kurtosis. Individual stock data are from the CRSP, and the sample period is from July 1926 to December 2020. The extended table for all the 36 BA strategies is presented in the Internet Appendix.

	FP		OLS		Welch	
	VW	EQ	VW	EQ	VW	EQ
Panel A: Non-linear market neutrality						
Mean	9.10	9.68	48.63	114.34	25.51	36.73
St. Dev	14.58	11.91	289.81	663.80	34.67	52.99
SR	0.62	0.81	0.17	0.17	0.74	0.69
Min	-62.20	-22.29	-2218.97	-2544.19	-52.25	-116.55
Max	19.73	21.58	821.77	3378.04	134.47	212.97
Skewness	-3.10	-0.52	-16.01	6.63	3.57	4.10
Kurtosis	51.08	9.72	478.32	192.35	48.71	60.52
Panel B: Linear market neutrality						
Mean	7.41	6.07	12.88	13.09	11.12	11.17
Std	17.16	13.86	17.59	15.51	15.29	13.82
SR	0.43	0.44	0.73	0.84	0.73	0.81
Min	-81.44	-39.00	-40.92	-37.57	-29.40	-28.67
Max	14.97	13.89	38.49	46.55	29.86	47.10
Skewness	-5.00	-2.41	-0.61	0.69	-0.50	1.30
Kurtosis	75.52	19.68	15.09	27.27	10.32	34.27
Panel C: Dollar neutrality						
Mean	0.47	-0.71	2.22	2.40	2.44	2.57
Std	24.06	20.95	21.26	17.38	21.42	17.83
SR	0.02	-0.03	0.10	0.14	0.11	0.14
Min	-107.87	-53.85	-51.98	-32.55	-54.60	-35.48
Max	22.51	22.05	24.47	22.00	24.70	24.37
Skewness	-4.39	-2.06	-1.12	-0.51	-1.00	-0.58
Kurtosis	61.93	16.92	13.28	8.01	12.65	9.20

Table 3: Market exposure and the log market volatility ratio

This table presents the results of estimating the following time-series regression:

$$r_{BA,t} = \alpha + \beta_1 \mathcal{V}_t + \beta_2 r_{MKT,t} + \beta_3 r_{MKT,t-1} + \beta_4 r_{MKT,t-2} + \varepsilon_t,$$

where r_{BA} is the beta arbitrage (BA) return, r_{MKT} is the market factor return, and $\mathcal{V} = r_{MKT} \cdot \ln(\sigma^{-1Y}_{MKT}/\sigma^{-5Y}_{MKT})$ is the interactive term between the market factor return and the log ratio of the prior one-year market volatility to the prior five-year market volatility. We use three beta estimators: Frazzini and Pedersen (2014) (FP) beta, the simple OLS market model beta, and Welch (2022) beta. We use Jensen et al. (2022)'s capped value-weighting (VW) scheme and simple equal-weighting (EQ) scheme to compose BA strategies. Below the estimated coefficients in square brackets are robust Newey and West (1987) t-statistics. The extended table for all the 36 BA strategies is presented in the Internet Appendix.

	FP		OLS		Welch	
	VW	EQ	VW	EQ	VW	EQ
Panel A: Non-linear market neutrality						
$\hat{\beta}_1$	-0.48	-0.23	2.25	7.91	0.36	1.02
	[-2.41]	[-2.39]	[1.23]	[1.64]	[1.23]	[2.16]
$\hat{\beta}_2$	-0.14	0.00	1.77	-0.21	0.89	1.51
	[-2.50]	[0.03]	[3.61]	[-0.12]	[9.83]	[10.01]
$\hat{\beta}_3$	0.05	0.20	0.89	0.59	0.47	0.88
	[0.95]	[8.56]	[3.26]	[0.86]	[4.52]	[5.95]
Panel B: Linear market neutrality						
$\hat{\beta}_1$	-0.76	-0.48	-0.21	-0.11	-0.09	0.04
	[-3.31]	[-4.14]	[-1.80]	[-0.95]	[-0.80]	[0.32]
$\hat{\beta}_2$	-0.11	-0.07	0.48	0.56	0.23	0.32
	[-1.67]	[-1.46]	[10.22]	[13.33]	[5.09]	[7.25]
$\hat{\beta}_3$	-0.06	0.05	0.08	0.16	0.07	0.17
	[-0.98]	[1.70]	[2.58]	[3.98]	[2.04]	[3.57]
Panel C: Dollar neutrality						
$\hat{\beta}_1$	-0.37	-0.11	0.13	0.13	0.09	0.11
	[-1.43]	[-0.88]	[1.21]	[1.28]	[0.78]	[0.99]
$\hat{\beta}_2$	-0.91	-0.87	-0.83	-0.74	-0.86	-0.74
	[-12.82]	[-17.79]	[-17.14]	[-19.72]	[-18.27]	[-18.78]
$\hat{\beta}_3$	-0.07	0.05	0.09	0.17	0.07	0.17
	[-1.04]	[1.40]	[2.64]	[4.09]	[2.19]	[3.74]

Table 4: Market states and market factor loadings

This table presents the results of estimating two specifications of the following time-series regression:

$$r_{BA,t} = \alpha + \alpha_D D_t + \alpha_U U_t + (\beta + \beta_D D_t + \beta_U U_t) r_{MKT,t} + \varepsilon_t,$$

where r_{BA} is the beta arbitrage (BA) return, r_{MKT} is the market factor return, D_t is a bear market dummy variable taking the value of one if the market factor return is below -5% and zero otherwise, and U_t is a bull market dummy variable taking the value of one if the market factor return is above 5% and zero otherwise. Specification 1 is the simple market model by setting $\alpha_D = \alpha_U = \beta_D = \beta_U = 0$, and Specification 2 is the full form. We use Frazzini and Pedersen (2014)'s beta estimator, Jensen et al. (2022)'s capped value-weighting (VW) scheme and simple equal-weighting (EQ) scheme to compose BA-FP strategies. We use non-linear market neutrality as explained by Frazzini and Pedersen (2014), as well as linear market neutrality as explained by Novy-Marx and Velikov (2022). Below the estimated coefficients in square brackets are robust Newey and West (1987) t-statistics. The extended table for all the three hedging methods is presented in the Internet Appendix.

	Non-linear market neutrality				Linear market neutrality			
	VW		EQ		VW		EQ	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
$\hat{\alpha}$	0.88	1.05	0.81	1.11	0.75	0.85	0.58	0.78
	[6.08]	[7.69]	[5.28]	[8.21]	[4.89]	[5.80]	[4.07]	[5.91]
$\hat{\alpha}_D$		3.15		3.62		2.74		3.34
		[2.81]		[3.16]		[2.08]		[3.18]
$\hat{\alpha}_U$		4.16		-0.08		6.70		2.61
		[2.55]		[-0.08]		[3.50]		[2.35]
$\hat{\beta}$	-0.18	-0.08	0.00	-0.04	-0.18	-0.01	-0.11	-0.08
	[-2.10]	[-1.56]	[0.01]	[-0.73]	[-1.67]	[-0.16]	[-1.97]	[-1.39]
$\hat{\beta}_D$		0.41		0.56		0.31		0.48
		[2.77]		[4.16]		[1.83]		[3.79]
$\hat{\beta}_U$		-0.69		-0.11		-1.03		-0.44
		[-3.01]		[-0.71]		[-3.86]		[-2.68]

Table 5: Market exposure in market upswings

This table presents the results of estimating the following time-series regression:

$$r_{BA,t} = \gamma_0 + \gamma_1 D^*_{t-1} + \big(\gamma_2 + D^*_{t-1}(\gamma_3 + \gamma_4 CU^*_t)\big) r_{MKT,t} + \varepsilon_t,$$

where r_{BA} is the beta arbitrage (BA) return, r_{MKT} is the market factor return, D^*_t is a cumulative downside market dummy variable taking the value of one if the past one-year cumulative market excess return is negative and zero otherwise, and CU^*_t is a contemporary upside market dummy variable taking the value of one if the market risk premium is positive and zero otherwise. We develop BA strategies as described in Tables 2. Below the estimated coefficients in square brackets are robust Newey and West (1987) t-statistics. The extended table for all the 36 BA strategies is presented in the Internet Appendix.

	FP		OLS		Welch	
	VW	EQ	VW	EQ	VW	EQ
Panel A: Non-linear market neutrality						
$\hat{\gamma}_0$	0.94	0.97	3.55	1.29	1.45	2.03
	[5.98]	[6.42]	[1.27]	[0.25]	[4.89]	[3.74]
$\hat{\gamma}_1$	1.27	0.32	-9.42	34.56	-1.51	-2.34
	[2.13]	[0.93]	[-1.48]	[1.07]	[-0.98]	[-1.19]
$\hat{\gamma}_2$	-0.02	0.10	1.17	-0.62	0.93	1.52
	[-0.33]	[1.55]	[2.06]	[-0.26]	[4.82]	[4.85]
$\hat{\gamma}_3$	0.09	0.04	-0.12	3.64	-0.29	-0.17
	[0.98]	[0.41]	[-0.12]	[1.35]	[-0.91]	[-0.44]
$\hat{\gamma}_4$	-0.69	-0.41	3.06	-2.46	0.63	0.83
	[-3.11]	[-3.63]	[1.16]	[-0.52]	[0.96]	[1.14]
Panel B: Linear market neutrality						
$\hat{\gamma}_4$	-0.90	-0.58	-0.13	-0.10	-0.24	-0.05
	[-3.24]	[-3.82]	[-0.54]	[-0.60]	[-1.46]	[-0.29]
Panel C: Dollar neutrality						
$\hat{\gamma}_4$	-0.89	-0.56	-0.11	-0.03	-0.23	-0.02
	[-3.25]	[-3.71]	[-0.51]	[-0.15]	[-1.36]	[-0.11]

Table 6: Market states, market variances, and BA performance

This table presents the results of estimating two specifications of the following time-series regression:

$$r_{BA,t} = \kappa + \kappa_D D_t + \kappa_U U_t + (\lambda + \lambda_D D_t + \lambda_U U_t)\hat{\sigma}^2_{MKT,t} + \varepsilon_t,$$

where r_{BA} is the beta arbitrage (BA) return, $\hat{\sigma}^2_{MKT,t}$ is the market variance calculated from daily market factor returns in month t, D_t is a bear market dummy variable taking the value of one if the market factor return is below -5% and zero otherwise, and U_t is a bull market dummy variable taking the value of one if the market factor return is above 5% and zero otherwise. Specification 1 sets $\kappa_D = \kappa_U = \lambda_D = \lambda_U = 0$, and Specification 2 is the full form. We use Frazzini and Pedersen (2014)'s beta estimator, Jensen et al. (2022)'s capped value-weighting (VW) scheme and simple equal-weighting (EQ) scheme to compose BA-FP strategies. We use non-linear market neutrality as explained by Frazzini and Pedersen (2014), as well as linear market neutrality as explained by Novy-Marx and Velikov (2022). Below the estimated coefficients in square brackets are robust Newey and West (1987) t-statistics. The extended table for all the three hedging methods is presented in the Internet Appendix.

	Non-linear market neutrality				Linear market neutrality			
	VW		EQ		VW		EQ	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
$\hat{\kappa}$	1.03	0.94	1.21	1.18	0.90	0.88	0.80	0.89
	[7.64]	[6.43]	[8.32]	[8.18]	[5.66]	[4.87]	[5.79]	[6.11]
$\hat{\kappa}_D$		1.05		0.40		0.25		0.32
		[1.54]		[0.51]		[0.36]		[0.46]
$\hat{\kappa}_U$		0.00		-0.63		0.79		-0.25
		[0.00]		[-1.82]		[0.88]		[-0.52]
$\hat{\lambda}$	-1.10	0.43	-1.65	-0.65	-1.18	-0.23	-1.22	-1.09
	[-3.00]	[0.71]	[-3.79]	[-1.01]	[-2.07]	[-0.26]	[-3.38]	[-1.52]
$\hat{\lambda}_D$		-1.23		-1.19		0.03		0.38
		[-1.89]		[-1.53]		[0.03]		[0.51]
$\hat{\lambda}_U$		-6.51		-1.53		-7.77		-3.41
		[-2.08]		[-1.38]		[-1.89]		[-1.96]

Table 7: Volatility-managed BA performance

This table presents the abnormal returns relative to Fama and French (2016)'s 6-factor model ($\hat{\alpha}^6$) and the 9-factor model ($\hat{\alpha}^9$) for the 18 beta arbitrage (BA) strategies. See Table 2 for BA development and Equation 9 for volatility management strategy. Below the estimated coefficients in square brackets are robust Newey and West (1987) t-statistics. The extended table for all the 36 BA strategies is presented in the Internet Appendix.

	FP		OLS		Welch	
	VW	EQ	VW	EQ	VW	EQ
Panel A: Non-linear market neutrality						
$\hat{\alpha}^6(r_{BA})$	0.55	0.38	2.65	14.37	0.72	1.45
	[3.34]	[2.19]	[0.83]	[1.01]	[2.13]	[2.39]
$\hat{\alpha}^9(r_{BA})$	0.43	0.20	2.03	12.82	0.41	1.00
	[2.44]	[1.23]	[0.45]	[0.87]	[1.18]	[1.56]
$\hat{\alpha}^6(r^\sigma_{BA})$	0.76	1.28	0.29	1.65	0.46	1.43
	[4.35]	[3.74]	[0.69]	[2.23]	[2.35]	[2.83]
$\hat{\alpha}^9(r^\sigma_{BA})$	0.66	1.16	-0.03	1.62	0.30	1.27
	[3.74]	[3.55]	[-0.04]	[1.86]	[1.45]	[2.47]
Panel B: Linear market neutrality						
$\hat{\alpha}^6(r_{BA})$	0.38	0.14	0.20	0.37	0.24	0.30
	[2.30]	[0.88]	[0.96]	[2.54]	[1.44]	[1.81]
$\hat{\alpha}^9(r_{BA})$	0.23	-0.06	-0.02	0.20	0.02	0.11
	[1.45]	[-0.42]	[-0.12]	[1.34]	[0.11]	[0.68]
$\hat{\alpha}^6(r^\sigma_{BA})$	0.63	1.05	0.41	0.98	0.42	1.24
	[3.53]	[3.74]	[2.41]	[4.60]	[2.39]	[4.31]
$\hat{\alpha}^9(r^\sigma_{BA})$	0.47	0.81	0.19	0.72	0.21	0.97
	[2.73]	[3.16]	[1.15]	[3.42]	[1.18]	[3.52]
Panel C: Dollar neutrality						
$\hat{\alpha}^6(r_{BA})$	0.39	0.15	0.19	0.35	0.25	0.28
	[2.48]	[0.98]	[0.90]	[2.30]	[1.46]	[1.67]
$\hat{\alpha}^9(r_{BA})$	0.22	-0.06	-0.08	0.12	-0.01	0.05
	[1.50]	[-0.45]	[-0.42]	[0.74]	[-0.07]	[0.32]
$\hat{\alpha}^6(r^\sigma_{BA})$	0.60	0.70	0.43	0.77	0.39	0.71
	[4.47]	[4.57]	[3.23]	[5.72]	[3.19]	[4.89]
$\hat{\alpha}^9(r^\sigma_{BA})$	0.53	0.63	0.30	0.71	0.30	0.66
	[4.05]	[4.30]	[2.43]	[5.39]	[2.52]	[4.70]

Table 8: Volatility-managed BA performance and trading costs

This table presents the abnormal returns relative to Fama and French (2016)'s 6-factor model ($\hat{\alpha}^6$) and the 9-factor model ($\hat{\alpha}^9$) for the 18 beta arbitrage (BA) strategies in the presence of transaction costs. See Table 2 for BA development and Equation 9 for volatility management strategy. Following Moreira and Muir (2017, Section II.B), we consider two assumptions of trading costs: 10bps and 14bps. The extended table for all the 36 BA strategies is presented in the Internet Appendix.

	FP		OLS		Welch	
	VW	EQ	VW	EQ	VW	EQ
Panel A: Non-linear market neutrality						
$\hat{\alpha}^6$(10bps)	0.73	1.23	0.27	1.63	0.44	1.41
	[4.18]	[3.59]	[0.66]	[2.21]	[2.27]	[2.79]
$\hat{\alpha}^9$(10bps)	0.63	1.11	-0.04	1.59	0.28	1.24
	[3.57]	[3.39]	[-0.07]	[1.84]	[1.35]	[2.40]
$\hat{\alpha}^6$(14bps)	0.72	1.20	0.27	1.62	0.44	1.40
	[4.12]	[3.53]	[0.65]	[2.19]	[2.23]	[2.76]
$\hat{\alpha}^9$(14bps)	0.62	1.09	-0.05	1.58	0.27	1.22
	[3.50]	[3.32]	[-0.08]	[1.83]	[1.31]	[2.37]
Panel B: Linear market neutrality						
$\hat{\alpha}^6$(10bps)	0.60	0.99	0.38	0.94	0.39	1.18
	[3.34]	[3.53]	[2.25]	[4.41]	[2.22]	[4.12]
$\hat{\alpha}^9$(10bps)	0.44	0.75	0.16	0.68	0.18	0.92
	[2.56]	[2.94]	[0.99]	[3.22]	[1.02]	[3.32]
$\hat{\alpha}^6$(14bps)	0.59	0.96	0.37	0.93	0.38	1.16
	[3.28]	[3.46]	[2.18]	[4.33]	[2.16]	[4.04]
$\hat{\alpha}^9$(14bps)	0.42	0.73	0.15	0.66	0.17	0.89
	[2.48]	[2.85]	[0.92]	[3.14]	[0.96]	[3.24]
Panel C: Dollar neutrality						
$\hat{\alpha}^6$(10bps)	0.57	0.66	0.40	0.73	0.37	0.67
	[4.26]	[4.32]	[3.04]	[5.47]	[3.00]	[4.65]
$\hat{\alpha}^9$(10bps)	0.50	0.60	0.28	0.68	0.28	0.63
	[3.87]	[4.07]	[2.26]	[5.16]	[2.34]	[4.48]
$\hat{\alpha}^6$(14bps)	0.56	0.65	0.39	0.72	0.36	0.66
	[4.19]	[4.23]	[2.98]	[5.38]	[2.93]	[4.57]
$\hat{\alpha}^9$(14bps)	0.50	0.59	0.27	0.67	0.27	0.62
	[3.80]	[3.97]	[2.19]	[5.07]	[2.27]	[4.39]