Accounting Transparency and the Implied Volatility Skew

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Abstract

Stock price jump risk is known to be important for explaining the option-implied volatility skew generated by the Black-Scholes model. Financial leverage (distress) has an important impact on the shape of the implied volatility skew, however, we find that the impact of leverage on the implied volatility skew depends on the quality of the firm's accounting transparency. In this paper, we propose a model where incomplete accounting information and the risk of financial distress together act as important drivers of jump rates and sizes for individual stocks. Consistent with our model, empirical tests using individual stock option data indicate that the impact of leverage on the skew is weaker for firms with lower accounting transparency and stronger for firms with higher accounting transparency.

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I. Introduction

The volatility smile in the options market refers to the stylized fact that the Black and Scholes [1973] implied volatility as a function of the strike price is skewed, not flat as it should be if that model were correct. This model has since been extended in many ways to better fit various aspects of the time-series and cross-sectional properties of option prices.[6] Through these studies we have learned that stock price jumps are integral to fitting the time-series and cross-sections of option prices.[7] The economic sources of jump risk, however, are less well understood.

In this paper, we study an economically intuitive explanation for the jump risk embedded in the implied volatility skew in individual stock options. We model the equity option price for a firm financed with debt and equity, for which investors do not know the precise value of the firm, but periodically receive noisy accounting reports. In contrast to the perfect information firm structural models of Merton [1974] and Leland [1994] models, imperfect corporate disclosure introduces the possibility that the firm can default unexpectedly, with equity then jumping to zero. We study the effect of this jump risk on the implied volatility skew by incorporating the imperfect corporate disclosure of the Duffie and Lando [2001] (DL) model into the pricing of stock options with risky debt of Toft and Pryck [1997].

Since equity is essentially a call option on the value of the firm, one cannot consistently model both firm value and stock price as geometric Brownian motions. This led to the Geske [1979] (Geske, Subrahmanyam, and Zhou [2016]) treatment of stock options as compound options (options on an underlying Merton [1974] firm structure). Since equity value is equal to zero when bankruptcy occurs, the stock return attains a heavier left tail than in a Lognor-

[6]Beginning with stock price jumps (Merton [1976]), then deterministic local volatility models of Derman and Kani [1994] and Dupire [1994], then stochastic (Heston [1993]), stochastic jumps in volatility (Bates [1996]), and stochastic jumps in volatility and the price process (Duffie, Pan, and Singleton [2000]).

[7]Our paper specifically focuses on the endogenous size and arrival rates of jumps associated with a firm's default through the channel of quality of accounting information. In contrast, the literature has largely focused on studying the magnitude of jump risk premia required by the data to price stock market index options and individual firms options in: Eraker, Johannes, and Polson [2003], Broadie, Chernov, and Johannes [2009], Christoffersen, Jacobs, and Ornthanalai [2012], Andersen, Fusari, and Todorov [2015], and others.

mal distribution, implying a downward-sloping volatility skew—the more levered the firm is, the more skewed is the implied volatility function. The Toft and Pryck [1997] (TP) option pricing model improves upon the underlying firm structure by using a Leland [1994] firm structure allowing the firm to default at any time between inception and debt maturity or option maturity.

In DL, the firm's asset value is assumed to follow a Geometric Brownian Motion. However, investors do not observe the precise value. Instead, the firm periodically issues noisy accounting reports. DL show that there is a positive probability per unit time that the firm can go bankrupt within the next instant even if the firm is reported to be *safe*, because the true firm value may lie somewhere closer to the default threshold than expected. This introduces jump risk into the model, as equity value can suddenly go to zero from any positive level. The crucial parameter in the DL model, the variance of the accounting noise, governs the intensity of the jump to default.

Large jumps in stock prices are closely tied to news about future cash flows and discount rates.[8] It therefore seems plausible that the volatility smile of stock options may be related to the quality of corporate disclosures. Intuitively, a firm with timely, clear, and detailed disclosures will seldom impose a *surprise* on the market. In contrast, a firm that makes only infrequent and less informative disclosures, is more likely to catch investors off guard. In this respect, the quality of corporate disclosures may proxy for the overall level of jump risk (both likelihood and magnitude of jumps) in stock returns.

Toft and Pryck [1997] show that financial leverage and thus distress risk influence the implied volatility smile empirically.[9] Our empirical analysis suggests that the leverage ratio and the quality of corporate disclosure jointly explain the cross-sectional variation of the implied volatility skew. Specifically, we highlight an important interaction between the two effects. When a firm is perceived to be opaque, whether its leverage ratio is high or low has

[8] See for instance Maheu and McCurdy [2004].

[9] Dennis and Mayhew [2002] find less of an impact of leverage on the implied volatility skew as well as a different sign using a different data set of options data in a time period (1986-1996) that pre-dates our analysis (1997-2017).

less impact on the price of a stock option. Conversely, when the firm is subject to greater accounting transparency, the higher the leverage ratio, the more skewed the implied volatility smile, linking a lower distance to default with more expensive out-of the money options. Our theory is able to provide option prices in closed form up to Bivariate Cumulative Standard Normal Distributions. Our empirical tests confirm our model's theoretical prediction and are robust to a variety of controls, measures of skewness, and measures of accounting transparency.

To the best of our knowledge, our paper is the first to suggest and document a relationship between the implied volatility skew and accounting transparency. As such it offers interesting support for the DL insight and also furthers our understanding of the economic determinants of the volatility skew. However, our approach is not without its limitations. For example, to emphasize the role of firm-level accounting noise, we have omitted to model economy-wide jump risk. This risk is linked to the index option skew and is also partly responsible for the skew in individual stock options through its effect on the pricing kernel (Bakshi, Kapadia, and Madan [2003]). Therefore, our model cannot be expected to fully account for the magnitude of the skew.

Our empirical results should not be confused with empirical tests of option models of heterogeneous beliefs (Buraschi and Jiltsov [2006]). From a Bayesian point of view, the reason why different investors have different beliefs about the fundamentals of a firm is that they are updating with different information. In the absence of truthful reporting, any news about a company is necessarily noise, and the difference in beliefs will be high. However, tests of heterogeneous beliefs also jointly estimate the empirical measures of the consistency of investor trading behaviour (open interest and open volume) and measures of beliefs or disagreement of firm value quantities. Whereas our empirical tests measure the differential impact of firm leverage across measures of accounting quality (beliefs) of firm value, we make no statement about the trading behaviour of investors.

More broadly, we contribute to the research agenda that focuses on the economic reasons for empirical departures from Black-Scholes. In this sense, our framework complements existing option models that use structural models such as Toft and Pryck [1997], Chen and Kou [2009], Geske et al. [2016], and Morellec and Zhdanov [2019].[10] Chen and Kou [2009] extend the TP framework to explicitly include jumps in the underlying asset price process framework in order to generate a variety of predictions for implied volatilities and credit spreads. In our framework, jumps arise endogenously as a result of imperfect information.[11] Additionally we contribute to a recent literature that studies the effect of information (accounting and other) disclosure on option prices.[12]

The rest of this paper is organized as follows. In Section II we present our theoretical model and in Section III use numerical examples to illustrate the relationship between accounting transparency and the volatility smile. In Section IV we document the construction of the volatility skew, the accounting transparency measures, and other aspects of the data. In Section V we conduct regression tests of the main predictions of the model with robustness tests in Section VI. We conclude with Section VII.

II. The Model

Our goal is to examine how equity option prices relate to a firm's capital structure as well as the fact that public information about the firm provides only a noisy estimate of its true value. The natural framework for this is the DL structural credit risk model with incomplete accounting information.

Our derivation of the stock option pricing formula takes three steps. Step 1 takes into account endogenous bankruptcy by shareholders and expresses equity value as the solution to an optimal stopping problem. Step 2 assumes perfectly observed firm value and prices

[10] Morellec and Zhdanov [2019] constructs a production economy option model where firm product competition is an important determinant in explaining the cross-sectional variation in implied volatility skew.

[11] Xiao and Vasquez [2020] study the impact of measures of default and credit risk on future option returns with a structural option pricing model with embedded jumps in the firm asset value process.

[12] Dubinsky, Johannes, Kaeck, and Seeger [2019] quantify the impact of earnings announcements on option prices using deterministic jump components and Smith [2018] option pricing model incorporates the impact of an accounting *disclosure event date* on the price of an option (not the *quality* of the information)

stock options using the compound options approach. Step 3 incorporates noisy accounting information into the pricing formula. The results of the first two steps are directly from DL and TP, and the final step is a combination of the two.

A. Equities

To begin, assume that we have a firm whose asset level is described by V_t, a Geometric Brownian Motion with drift μ and volatility σ. The firm generates cash flow at a rate δV_t, which is distributed to shareholders as well as being used to service a par consol bond with a total coupon rate C. The tax rate is θ, generating tax benefits for the bond at a rate of θC. All agents are risk-neutral and discount cash flows at a constant rate of r.

In this setting, one can conjecture that shareholders will choose to liquidate the firm if the asset level V_t becomes sufficiently low. Namely, the stopping policy that maximizes the discounted present value from operating the firm, including tax benefits,

$$E\left(\int_t^{\tau} e^{-r(s-t)}\left(\delta V_s - C + \theta C\right) ds | V_t\right), \tag{1}$$

takes the form $\tau\left(V_B\right) = \inf\left\{t : V_t \leq V_B\right\}$. Indeed, DL verify the optimality of such a stopping rule and show the corresponding equity value as

$$w\left(v\right) = \frac{\delta v}{r-\mu} - \frac{v_B\left(C\right)\delta}{r-\mu}\left(\frac{v}{v_B\left(C\right)}\right)^{-\gamma} - \left(1-\theta\right)\frac{C}{r}\left(1-\left(\frac{v}{v_B\left(C\right)}\right)^{-\gamma}\right) \tag{2}$$

when $v > v_B\left(C\right)$, and $w\left(v\right) = 0$ for $v \leq v_B\left(C\right)$. The terms represent, respectively, the present value of future cash flows generated by the assets, the present value of cash flows lost to bankruptcy, and the cost of debt service minus the tax benefit. Here, $v_B\left(C\right)$ is the default threshold

$$v_B\left(C\right) = \frac{\left(1-\theta\right)C\gamma\left(r-\mu\right)}{r\left(1+\gamma\right)\delta}, \tag{3}$$

where $\gamma = \frac{m+\sqrt{m^2+2r\sigma^2}}{\sigma^2}$ and $m = \mu - \frac{1}{2}\sigma^2$.

Notice that the total coupon rate C can be determined by maximizing the initial value of equity plus the market value of the consol bond. This is the notion of optimal capital structure pursued in Leland [1994] and Leland and Toft [1996]. It is not essential to our subsequent results and is therefore omitted.

B. *Stock Options*

Given shareholders' optimal decision to liquidate the firm once the firm asset level drops below $V_B = v_B(C)$, we can price stock options as compound options on the firm's assets.

To facilitate the derivations, define a standard Brownian motion $Z_t \equiv \log V_t$ and let $\underline{v} = \log V_B$. The price at time t of a stock option with maturity T and strike price K is

$$h_t(u) = e^{-r(T-t)} E\left(\left(w\left(e^{Z_T}\right) - K\right)^+ 1_{\{\tau > T\}} | Z_t = u\right). \tag{4}$$

The indicator function means that the option payoff is zero if bankruptcy is declared prior to its maturity.

Using Bayes' rule, this can be rewritten as

$$\begin{aligned} & e^{-r(T-t)} \int_{\underline{v}}^{\infty} (w(e^x) - K)^+ P(Z_T \in dx, \tau > T | Z_t = u) \\ = \; & e^{-r(T-t)} \int_{\underline{v}}^{\infty} (w(e^x) - K)^+ P(\tau > T | Z_t = u, Z_T \in dx) P(Z_T \in dx | Z_t = u). \end{aligned} \tag{5}$$

This expression involves two probabilities. The first is the probability of survival through T given that the standard Brownian motion Z is *pinned* at the two end points, and can be written as $\psi\left(u - \underline{v}, x - \underline{v}, \sigma\sqrt{T-t}\right) = 1 - \exp\left(-\frac{2(u-\underline{v})(x-\underline{v})}{\sigma^2(T-t)}\right)$.[13] The second is simply the density of a normal random variable with mean $u + m(T-t)$ and variance $\sigma^2(T-t)$. We

[13] This can be interpreted as the probability that the first time the diffusive log-firm value (which began at time t with value u) reaches the absorbing default boundary ($Z_t \equiv \log V_t$ and let $\underline{v} = \log V_B$) exceeds time T. The result relies on the density of the First Passage Time, which is the time it takes a Brownian motion to hit an absorbing boundary.

therefore obtain

$$
\begin{aligned}
h_t(u) &= \frac{e^{-r(T-t)}}{\sqrt{2\pi\sigma^2(T-t)}} \int_{\underline{v}}^{\infty} (w(e^x) - K)^+ \exp\left(-\frac{(x-u-m(T-t))^2}{2\sigma^2(T-t)}\right) \\
&\quad \left(1 - \exp\left(-\frac{2(u-\underline{v})(x-\underline{v})}{\sigma^2(T-t)}\right)\right) dx. \qquad (6)
\end{aligned}
$$

After substituting in the value of equity in equation (2) and some further manipulations, we arrive at the final expression:

$$
\begin{aligned}
h_t(u) &= Fe^{u+\left(m+\frac{1}{2}\sigma^2-r\right)l} \left(N\left(y^* + \sigma\sqrt{l}\right) - e^{\left(2m\sigma^{-2}+2\right)(\underline{v}-u)} N\left(y^* + \sigma\sqrt{l} + \frac{2(\underline{v}-u)}{\sigma\sqrt{l}}\right)\right) \\
&\quad + Be^{\gamma(\underline{v}-u)} \left(N\left(y^* - \gamma\sigma\sqrt{l}\right) - e^{\left(2m\sigma^{-2}-2\gamma\right)(\underline{v}-u)} N\left(y^* - \gamma\sigma\sqrt{l} + \frac{2(\underline{v}-u)}{\sigma\sqrt{l}}\right)\right) \\
&\quad - (A+K)e^{-rl} \left(N(y^*) - e^{2m\sigma^{-2}(\underline{v}-u)} N\left(y^* + \frac{2(\underline{v}-u)}{\sigma\sqrt{l}}\right)\right), \qquad (7)
\end{aligned}
$$

where

$$
\begin{aligned}
l &= T - t, \\
F &= \frac{\delta}{r-\mu}, \\
A &= \frac{(1-\theta)C}{r}, \\
B &= \frac{(1-\theta)C}{r(1+\gamma)}, \\
y^* &= -\frac{x^* - u - m(T-t)}{\sigma\sqrt{T-t}},
\end{aligned}
$$

and e^{x^*} is the firm asset level that corresponds to an equity value of K:

$$
Fe^x - A + Be^{-\gamma(x-\underline{v})} = K. \qquad (8)
$$

With some differences in notation, this pricing formula is first derived by TP. They show that it leads to a downward-sloping volatility skew. Furthermore, the skewness of the pattern increases with firm leverage. Our comparative statics in Section III will re-visit these findings.

C. Incomplete Accounting Information

The pricing formula (7) assumes that the firm asset level V_t is known to investors, which may seem like a harmless assumption. However, recent accounting scandals suggest that this can be quite far from reality. DL assume that firm assets are observed only periodically, and with noise. Therefore, at time t, the value of the firm's assets is not known with perfect precision. Instead, it is governed by a conditional distribution that depends on the reported firm assets as well as the absence of bankruptcy. DL use this framework to investigate the pricing of defaultable bonds, producing strong predictions for short-maturity credit spreads. We intend to study its impact on the pricing of stock options.

First, we follow DL in assuming that Z_0 is observed without noise. Then, at time t, the firm reports the value of $Y_t = Z_t + U_t$, where U_t is independent of Z_t and normally distributed with mean $-\frac{a^2}{2}$ and variance a^2, so that the reported firm asset level $\widehat{V}_t = V_t e^{U_t}$ is a noisy but unbiased version of V_t.

Next, we show that the stock option value can be expressed as an integral of equation (7) over the distribution of Z_t given information available at t. This can be seen by writing the option price H_t as

$$\begin{aligned} H_t &= e^{-r(T-t)} E\left(\left(w\left(e^{Z_T}\right) - K\right)^+ 1_{\{\tau > T\}} | Y_t = y, Z_0 = z_0, \tau > t\right) \\ &= e^{-r(T-t)} \int_{\underline{v}}^{\infty} \left(w\left(e^x\right) - K\right)^+ P\left(Z_T \in dx, \tau > T | Y_t = y, Z_0 = z_0, \tau > t\right). \end{aligned} \quad (9)$$

The joint density of Z_T and no default until T given the current and lagged asset values, as well as survival to t, can be decomposed by repeated applications of Bayes' rule:

$$\begin{aligned} & P\left(Z_T \in dx, \tau > T | Y_t = y, Z_0 = z_0, \tau > t\right) \\ = & \int_{u=\underline{v}}^{\infty} P\left(Z_T \in dx, Z_t \in du, \tau > T | Y_t = y, Z_0 = z_0, \tau > t\right) \\ = & \int_{u=\underline{v}}^{\infty} P\left(\tau > T | Z_t = u, Z_T = x\right) P\left(Z_T \in dx, Z_t \in du | Y_t = y, Z_0 = z_0, \tau > t\right) \\ = & \int_{u=\underline{v}}^{\infty} P\left(\tau > T | Z_t = u, Z_T = x\right) P\left(Z_T \in dx | Z_t = u\right) P\left(Z_t \in du | Y_t = y, Z_0 = z_0, \tau > t\right). \end{aligned} \tag{10}$$

Of the three probabilities above, the first is simply the survival probability of the *pinned* Brownian motion, previously referred to as $\psi\left(u - \underline{v}, x - \underline{v}, \sigma\sqrt{T-t}\right)$. The second is the density of a normal random variable with mean $u + m\left(T-t\right)$ and variance $\sigma^2\left(T-t\right)$. The last is the density of Z_t given current and lagged asset reports and survival. We denote it by $g\left(u|y, z_0, t\right) du$.

Combining these two equations, we obtain

$$\begin{aligned} H_t = & \frac{e^{-r(T-t)}}{\sqrt{2\pi\sigma^2\left(T-t\right)}} \int_{\underline{v}}^{\infty} \left(w\left(e^x\right) - K\right)^+ \int_{\underline{v}}^{\infty} \exp\left(-\frac{\left(x - u - m\left(T-t\right)\right)^2}{2\sigma^2\left(T-t\right)}\right) \\ & \left(1 - \exp\left(-\frac{2\left(u-\underline{v}\right)\left(x-\underline{v}\right)}{\sigma^2\left(T-t\right)}\right)\right) g\left(u|Y_t, Z_0, t\right) du dx. \end{aligned} \tag{11}$$

Switching the order of integration, it is easy to see that

$$H_t = \int_{\underline{v}}^{\infty} h_t\left(u\right) g\left(u|y, z_0, t\right) du, \tag{12}$$

which is in some sense obvious given the assumed risk-neutrality.

Following DL, the last unknown quantity g can be written as

$$\begin{aligned} g\left(u|y,z_0,t\right)du &= P\left(Z_t\in du|Y_t=y,Z_0=z_0,\tau>t\right) \\ &= \frac{P\left(Z_t\in du,\tau>t|Y_t=y,Z_0=z_0\right)}{P\left(\tau>t|Y_t=y,Z_0=z_0\right)} \\ &= \frac{P\left(Z_t\in du,\tau>t|Y_t=y,Z_0=z_0\right)}{\int_{\underline{v}}^{\infty}P\left(Z_t\in du,\tau>t|Y_t=y,Z_0=z_0\right)}. \end{aligned} \tag{13}$$

The numerator, denoted as $b\left(u|y,z_0,t\right)du$, is equal to

$$\begin{aligned} b\left(u|y,z_0,t\right)du &= \frac{P\left(\tau>t|Z_t=u,Z_0=z_0\right)P\left(Z_t\in du,Y_t\in dy\right)}{P\left(Y_t\in dy\right)} \\ &= \frac{\psi\left(z_0-\underline{v},z-\underline{v},\sigma\sqrt{t}\right)\phi_U\left(y-u\right)\phi_Z\left(u\right)du}{\phi_Y\left(y\right)}, \end{aligned} \tag{14}$$

where ϕ_U, ϕ_Z and ϕ_Y are respectively the densities of U_t, Z_t and Y_t. We note that $U_t\sim N\left(-a^2/2,a^2\right)$, $Z_t\sim N\left(z_0+mt,\sigma^2t\right)$ and $Y_t\sim N\left(-a^2/2+z_0+mt,a^2+\sigma^2t\right)$. The last step above uses the independence between Z_t and U_t.

Putting everything together, we obtain

$$g\left(u|y,z_0,t\right)=\frac{\sqrt{\frac{\alpha}{2\pi}}\left(1-\exp\left(-\frac{2}{\sigma^2t}\widetilde{z}_0\widetilde{u}\right)\right)\exp\left(-\frac{1}{2a^2}\left(\widetilde{y}-\widetilde{u}\right)^2\right)\exp\left(-\frac{1}{2\sigma^2t}\left(\widetilde{u}-\widetilde{z}_0-mt\right)^2\right)}{N\left(\sqrt{\alpha}\beta\right)-N\left(\sqrt{\alpha}\left(\beta-\frac{2}{\sigma^2t\alpha}\widetilde{z}_0\right)\right)\exp\left(-\frac{\alpha\eta}{2}+\frac{2}{\sigma^4t^2\alpha}\widetilde{z}_0\left(\widetilde{z}_0-\alpha\beta\sigma^2t\right)\right)}, \tag{15}$$

where $\widetilde{z}_0=z_0-\underline{v}$, $\widetilde{u}=u-\underline{v}$, $\widetilde{y}=y-\underline{v}+a^2/2$, and

$$\begin{aligned} \alpha &= \frac{\sigma^2t+a^2}{a^2\sigma^2t}, \\ \beta &= \frac{\sigma^2t\widetilde{y}+a^2\left(\widetilde{z}_0+mt\right)}{\sigma^2t+a^2}, \\ \eta &= \frac{\sigma^2ta^2\left(\widetilde{y}-\left(\widetilde{z}_0+mt\right)\right)^2}{\left(\sigma^2t+a^2\right)^2}. \end{aligned}$$

This expression is slightly different in form from, but equivalent to, the one in DL.

In order to solve for our option pricing model with incomplete accounting information, given by H_t, in closed form, we first note that the function $g\left(u|y,z_0,t\right)$ can be re-written as a difference of Normal distribution probability density functions with different means and variances in the general form (see appendix for details of derivation and notation):

$$g\left(u|z_0, y\right) = \frac{L_1}{L_0} \times e^{\frac{-(u-M_1)^2}{2\psi}} - \frac{L_2}{L_0} \times e^{\frac{-(u-M_2)^2}{2\psi}}. \tag{16}$$

Our formula for H_t involves the integral of the product of the TP option pricing formula (denoted $h_t\left(u\right)$) and the density function $g\left(u|y, z_0, t\right)$. Expanding the product results in computing twelve integrals of the form in equation 17 (see Owen [1980]):

$$\int_R^\infty \Phi(A + Bx)\phi(x)dx = \Phi\left(\frac{A}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, R; \frac{-B}{\sqrt{1+B^2}}\right). \tag{17}$$

The resulting sum of the twelve integrals results in a closed form solution, up to Standard Bivariate Normal Cumulative Probability Distribution Function, of the expression H_t. The resulting expression and sketch of the proof for H_t is available in Section VIII of this paper. A full step by step derivation is available in Section VIII.B of Section VIII.B of the online mathematical appendix.

III. Numerical Examples

In this section we use numerical examples to illustrate the theoretical model outlined in the preceding section. We focus on the interaction between accounting transparency and leverage, asset volatility, as well as the maturity of the equity option in the determination of the implied volatility skew.

The model parameters are chosen to be as close as possible to those used by TP, so that comparisons can be made where appropriate. The basic parameters are $m = 0.01$, $\sigma = 0.2$, $r = 0.08$, and $\theta = 0.35$. The starting point of the firm value is set to $V_0 = 100$, and the reported firm value at $t = 1$ is $Y_1 = 100$. The leverage ratio of the firm is controlled by the coupon rate of the consol bond, C. Given the payout rate δ, the dividend yield on the stock is set to $\frac{\delta Y_1-(1-\theta)C}{E_1}$ where E_1 is the equity value corresponding to an asset level of Y_1. The dividend yield and the equity value are useful when inverting the Black-Scholes formula for the implied volatility.

In the examples below, the case of low leverage assumes $C = 1.72$ and $\delta = 3.70\%$; medium leverage, $C = 3.77$ and $\delta = 4.54\%$; high leverage, $C = 6.42$ and $\delta = 5.66\%$. These parameter values are taken directly from TP's Table I. We also assume that the accounting precision parameter a can take on three values: 0.01, 0.05, and 0.1. Lastly, we take the maturity of the stock option to be 1 month.

Our primary interest is the role of leverage where the accounting reports are assumed to be precise. We do this by setting $a = 0$, which reduces the model to that of TP. To isolate the shape of the volatility skew pattern, we normalize the strike price by the current value of equity, and divide all of the implied volatilities by the at-the-money implied volatility, resulting in a *standardized* plot.

Figure 1 presents the volatility smile for cases with low, medium, or high leverage ratios. Panel 1 is where the accounting precision is set to zero, representing precisely observed asset values. It shows that the volatility skew increases with leverage, something that TP document both in theory and empirically. Notably, the magnitude of the skew is very close to what TP present in their Table II.

The remaining three panels then illustrate what happens when we have imprecise accounting reports. Notice that for the case with $a = 0.01$ (Panel 2) we still have an implied volatility skew that increases with leverage, but this relationship reverses itself under the presence of somewhat higher accounting noise (Panels 3 and 4). This observation suggests that there is a level of accounting precision under which the skewness of the smile is insensitive to leverage. Consequently, one has to be careful in designing regression tests for the relationship between skewness and leverage. It is possible, for example, that simply regressing the skewness on leverage would not be able to uncover any relation at all.

To explain this puzzling behavior, we examine the relationship between the volatility smile and accounting precision under different assumptions of firm leverage. First, Figure 2 shows that this relationship is clearly monotonic, with higher skewness associated with higher accounting noise. Since option value is a convex function of equity value in TP's model, the accounting noise boosts option value due to Jensen's effect. With deep-in-the-money call options (with low strike prices) that are very close to the no-arbitrage lower bound, a small increase in the option value requires a large increase in equity volatility.[14] This results in the elevated volatility skew.

In Figure 2, the impact of accounting imprecision can be ostensibly much larger than the effect of leverage, but one has to keep in mind that we do not yet know what a *reasonable* level of a is. A value of $a = 0.1$, for example, implies an accounting report that can regularly be 10 percent off target.

We also see from Figure 2 that accounting noise disproportionately affects the left side of the volatility skew. The intuition is of course that the imprecise observation of firm assets introduces downward jump risk due to the presence of the default boundary.

The most important observation from Figure 2 is perhaps the decreasing effect of accounting noise with leverage—the variation of the skew is noticeably smaller in Panel 3 than in Panel 1. This observation, which helps to explain the relationship between skewness and leverage in Figure 1, can be understood intuitively as follows:

In our model, the volatility skew is generated by default through two channels. First, the firm asset level can diffuse down to the default boundary. Second, the firm value can suddenly *jump* to default due to incomplete accounting information. As the firm moves closer to the default boundary (represented by higher leverage ratios), it becomes easier to default through the normal diffusion channel, and the relative importance of the jump risk decreases.

[14]Another way to understand it is that these options have very low vega according to the Black-Schoes model.

IV. Data

The data used for our empirical tests is merged from the following standard sources: quarterly balance sheet data is from COMPUSTAT, daily and monthly stock data is from CRSP, equity options data is obtained from the OptionMetrics Database, and earnings information is from the I/B/E/S data. The sample period is from January 1997 to December 2017. OptionMetrics equity options data is first available as of January 1996, however, the merge between OptionMetrics and the earnings information is sparse during 1996 hence we begin our sample at January 1, 1997.

Our equity options data consists of closing end of day call and put option best bid and best offer quotes from the OptionMetrics Database which collects closing option quotes data from all U.S. equity option exchanges. Equity options for individual firms are American in nature in the sense that they can be exercised at any time. For each quoted option contract price the corresponding contract open interest, daily trade volume, and Black-Scholes delta and implied volatility are reported. The Black-Scholes delta and implied volatility are computed using the Cox, Ross, and Rubinstein [1979] binomial lattice model in order to incorporate the early exercise features of American options. We apply filters to our options data set, specifically we remove contracts with missing implied volatility, open interest, trade volume, and delta. We remove contracts that have less than 10 days remaining to maturity in order to account for the rollover of option contracts. We remove option contracts with option best bid or best offer prices that are less than or equal to zero, and cases where the best offer is less than or equal to the best bid. We also require the absolute value of the option contract delta to be between 0.02 and 0.98 to avoid using very deep in-the-money and out-of-the-money contracts that are mis-priced and have low liquidity. Additionally we remove options that have zero open interest as they are unlikely to contain any important information.

As a metric for the volatility skew we take the scaled difference between an at-the-money option and an out-of-the-money option. Our logic is that the at-the-money option will have a lower implied volatility (relatively less expensive with a lower implied volatility) than an out-of-the-money option (more expensive with a higher implied volatility. We then scale by the at-the-money implied volatility to control for the level of the implied volatility and the difference in moneyness between the at-the-money option and out-of-the-money option. We take as at-the-money option the closest option contract that as long as the moneyness ((K/S_0)) is between 0.99 and 1.03, with corresponding moneyness ($(K/S_0)_t^{High}$) and implied volatility ($IVOL_t^{High}$) for every firm for each day trading day. For the out-of-the-money option, we choose the closest option contract that as long as the moneyness below 0.97 but above 0.92 (if not then choose that option with moneyness closest to 0.92), with corresponding moneyness ($(K/S_0)_t^{Low}$) and implied volatility ($IVOL_t^{Low}$). We choose the at-the-money and out-of-the-money options having the same date of expiration each day for the same firm. As a metric for the volatility skew we define the Slope Skew the following variable:

$$SLOPE_t = \frac{\frac{IVOL_t^{High} - IVOL_t^{Low}}{IVOL_t^{High}}}{(K/S_0)_t^{High} - (K/S_0)_t^{Low}} \tag{18}$$

So for a typical skew, this variable is negative and the more negative the more skewed the smirk/smile is. Note that this variable only measures the left hand side of the skew, which is where our theory's predictions are the strongest. Our measure in equation 18 is computed each day for every firm and averaged across all options with less than a year of remaining time to maturity and then averaged over the quarter for each firm. For each stock trading day we estimate the model free option implied skewness ($Skew^Q$) of Bakshi et al. [2003]. We detail the construction of the model free option implied skewness ($Skew^Q$) of Bakshi et al. [2003] to Appendix Section VIII.B.[15]

Our measure of leverage is computed using quarterly COMPUSTAT data as the ratio of book value of debt divided by sum of debt and market value of equity using quarterly data from COMPUSTAT. We assume that book values of debt and preferred stock are adequate

[15] We remove large outliers of equation 18 and $Skew^Q$ that are above (below) the 99th (1st) percentile.

proxies for the corresponding market values. In order to account for heterogeneity of firm characteristics, we use several control variables known to impact the implied volatility skew in the most recent empirical tests in Morellec and Zhdanov [2019]. Specifically we control for the: market-to-book ratio (M/B), market capitalization of the firm (Size), stock momentum (Momentum), stock beta (Beta), idiosyncratic stock return skewness (Idio Skew), and at-the-money option implied volatility (Atm Ivol). The market-to-book ratio (M/B) is the ratio of quarterly market equity divided by book equity using quarterly data from COMPUSTAT. We also control for the market capitalization of the firm (size) which is the log of the product of the stock price and shares outstanding from CRSP monthly stock files (firms with share codes 10 and 11 common shares). Momentum is the past 6 month cumulative monthly stock returns from CRSP which controls for the stock momentum over the previous 6 months of returns. Beta is the stock beta with the market estimated from 36 months rolling regressions adjusted by 3 months of lags for asynchronous trading as per the Dimson [1979] adjustment. Idio Skew is the idiosyncratic skewness of daily returns estimated quarterly using daily CRSP stock returns. Atm Ivol is average of call and put contract implied volatility with $|\Delta| = 0.5$ and 30 days to maturity, and using OptionMetrics Volatility surface computed daily on a firm level and then averaged over the quarter. We remove firms that have two digit SIC codes between and including 60 and 69 in order to remove financial firms from our analysis.

Our measures of accounting quality are based on earnings information from I/B/E/S database. We compute the dispersion in analyst forecast (Disp) and number of analyst covering stock (Nanalyst) from the data set of firm characteristics in Green, Hand, and Zhang [2017] (GHZ, henceforth).[16] The list of firm characteristics in GHZ is constructed using all firms with common shares that are listed on the AMEX, NYSE, or NASDAQ, that have end of month value on CRSP, quarterly and annual balance sheet reporting on COMPUSTAT and earnings information reported to the I/B//E/S data.

As the number of analysts (Nanalyst) increases, the quality of the accounting trans-

[16]We thank Jeremiah Green for making the SAS code to construct the data set freely available on his website.

parency of the firm increases since more analysts are paying attention to the earnings statement of the firm and will yield a more precise estimate of firm value. As the dispersion in analyst forecast (Disp), decreases from a high dispersion quantity to a low dispersion quantity the quality of the accounting transparency of the firm increases since more is associated with a more precise statement of the quality of accounting information.

INSERT TABLE I **HERE**

In Section X, Table I presents quarterly summary statistics for main variables from January 1997 to December 2017. Our implied volatility skewness measure equation 18 is negative on average (-0.86) and up to the 99-th percentile. A negative value indicates that the OTM implied volatility ($IVOL_t^{Low}$) is higher than the ATM implied volatility ($IVOL_t^{High}$), the more expensive OTM options are indicated by a higher implied volatility. Similarly the $Skew^Q$ measure is also negative on average (-0.45) and up to the 99-th percentile.The average firm leverage is 0.2 (or 20%) The Nanalyst ranges from zero to over 50 with a mean of five and a median number of analysts of three.

INSERT TABLE II **HERE**

Correlations between variables are presented in Table II. The correlation between the skewness measure equation 18 and the $Skew^Q$ measure is very high 0.7. Our implied volatility skewness measure equation 18 shows only minor correlation with leverage, Nanalyst, and Disp of -0.13, -0.05 and 0.08 respectively. Similarly the $Skew^Q$ measure shows only minor correlation with leverage, Nanalyst, and Disp of -0.16, -0.29 and 0.1 respectively. There is very high correlation between Nanalyst and size of 0.76, however, this is not a concern in our empirical analysis since we do not directly use the level of Nanalyst and our analysis is robust to the exclusion of size. Additionally all of our measures of accounting transparency exhibit low correlation between one another (the highest is between Nanalyst and Disp of -0.09).

V. Empirical Tests

We will focus on the empirical implication of the impact of leverage on the implied volatility skew based on the model prediction in III. We noted that as the quality of accounting data declined, one should expect that the reported leverage becomes less influential on the pricing of options. More precisely, we will test the following hypothesis for different accounting quality measures and the impact of leverage on the volatility smile.

Main Hypothesis: a higher value of number of Nanalysts (lower value of Disp) is associated with a higher level of transparency (lower a in our model) which our model predicts would mean a more negative impact of leverage on skewness.[17]

Recall that TP predict that firms with higher leverage should exhibit more steeply negatively sloped skews. Hence when regressing our skew metric on leverage, the coefficient should be negative as long as the accounting transparency is not too low. Under our main empirical hypothesis, we expect to see that the coefficient is larger in absolute value for firms with greater transparency.

To measure the differential impact of leverage, based on the quality of accounting transparency, on the implied volatility smile, we separate the cross-section of firms based on the 50th percentile of each accounting transparency measure each quarter of each year. We define those firms that have *low accounting transparency* as those with the number of Nanalysts below the 50th percentile (Disp above the 50th percentile) each quarterly cross section. Correspondingly, we define those firms that have *high accounting transparency* as those with the number of Nanalysts above the 50th percentile (and Disp below the 50th percentile) each quarterly cross section.[18]

We then estimate the impact of leverage on the volatility smile using quarterly Fama and Macbeth [1973] regressions within in each subset. The resulting estimates of the impact of leverage on skewness measure equation 18, where the measure of accounting quality is

[17] A more negative number is a negative number further from zero.

[18] In section VI we will show that all of our results are robust to using different percentile levels.

Nanalyst (Disp), is reported in Table III (Table IV). Panel A (B) is estimated using the firms with *low accounting transparency* (*high accounting transparency*).

INSERT TABLES III and IV **HERE**

The variable (and sign) of interest is leverage. In each of the tables III and IV, for panels A and B, column 1 reports the univariate quarterly Fama and Macbeth [1973] regressions within in each subset regressions using leverage. In columns 2 to 7, we add individual control variables (Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol) one at a time to the univariate regression for leverage. Additionally in each of the columns, in each panel A and B, in the Tables III and IV we report the R-squared (R^2) and average number of firms in each quarterly cross-sectional regressions (N obs). R^2, within each subset of data in Panels A(B), range from 1.6% up to 11.01% (when including leverage and all control variables). The average number of firms in each quarterly cross-sectional regressions total between both subset of data in panels A(B) is roughly 1500 in column 1 to roughly 1200 in column 7.

We first look at Table III column 1 in panel A (low transparency firms) and find that the coefficient of leverage is negative (-0.43) and statistically significant at the 1% level (t-stat of -7.6). In the corresponding column 1 in panel B (high transparency firms), the leverage coefficient is again negative (-0.55) and significant at the 1% level (t-stat of -13.23) and more negative (or larger in absolute magnitude) than the coefficient of leverage in panel A. This provides preliminary evidence that low accounting transparency implying a less marked impact of leverage than high accounting transparency which is the main prediction of our model.

In Table III, as we add control variables individually in columns 2 to 7 in panel A (low transparency firms) find that the leverage coefficient remains negative and statistically significant at the 1% level in each case. In the corresponding columns 2 to 7 in panel B (high transparency firms), in each column the leverage coefficient remains negative and significant at the 1% level. In column 7 in panel A (B), the leverage coefficient is -0.23 (-0.38) and both significant at the 1% level with a t-stat of -3.84 (t-stat of -12.34) and the leverage coefficient is more negative than the coefficient of leverage in panel A (controlling for firm characteristics). In each of the regression specifications, as controls are added, we continue to see that the leverage coefficient is more negative for firms with a higher transparency firms (Panel B) than for a lower transparency (Panel A) implying a less marked impact of leverage for firms with a lower accounting transparency and confirms our main prediction of our model in empirical tests.

So far we have shown that our theoretical model predictions are true using the Nanalyst, next in Table IV (Disp) we estimate our regression specification from Table III where our measure of accounting transparency is Disp. For each of columns 1 to 7 in panel A, where low transparency firms are measured by high Disp, in each case find that the leverage coefficient is negative and statistically significant at the 1% level. In the corresponding columns 1 to 7 in panel B, where high transparency firms are measured by low Disp, the leverage coefficient is not only negative and statistically significant at the 1% level but is also more negative than the corresponding coefficient of leverage in panel A (controlling for firm characteristics). Taken together, Table III and Table IV provide strong evidence towards our model's theoretical predictions and shows that our conclusion is robust to different measures of accounting transparency.

So our findings are consistent with low disclosure quality implying a less marked impact of leverage on the skew. This is consistent with the situation depicted in Figure 2 and confirms our main empirical hypothesis.

VI. Robustness Tests

In this section, we report several robustness tests to evaluate our main empirical findings in Section V. We present our results for both measures of accounting quality using (*i*) an alternative measure of implied volatility skewness, and (*ii*) alternative percentile levels of the cross section of accounting transparency levels (for both measures of implied volatility skewness).

First, we redo our empirical tests from Tables III and IV where instead of using the skewness measure of equation 18 as the dependent variable we use the model free option implied skewness ($Skew^Q$) of Bakshi et al. [2003].

INSERT TABLES V and VI **HERE**

Tables V and VI present our results using the $Skew^Q$ measure where the level of accounting transparency is measured using Nanalyst and Disp respectively. In Table V for each of columns 1 to 7 in panel A and corresponding columns 1 to 7 of panel B we find that the leverage coefficient is negative and significant at the 1% level and the leverage coefficient in panel B is more negative than that in panel A in each column. Results in Table VI find the same conclusion for Disp as an accounting transparency measure. These results confirm our main empirical hypothesis using a different measure of implied volatility skewness as well as different measures of accounting transparency.

As a final robustness test, we use alternative percentile levels of the cross section of accounting transparency levels in our empirical tests in order to show that are results are not affected by choosing the sample median as a percentile level each quarter.

INSERT TABLES VII and VIII **HERE**

Panel A (B) of Tables VII and VIII, show each of the seven regression models estimated using the subset of data each quarter that is below (above) the 40th (60th) percentile level of accounting transparency in the cross section where the level of accounting transparency is measured using Nanalyst and Disp respectively and the dependent variable is the slope skewness measure of equation 18. Similarly, Tables IX and X are presented with the regressions estimated using the 40th (60th) percentile level of accounting transparency in the cross section where the level of accounting transparency is measured using Nanalyst and Disp respectively with the dependent variable being the $Skew^Q$ measure. When using Disp as the measure of accounting transparency in Tables VIII and X, for all seven regression models in panels A and B leverage is significant at the 1% level. When using Nanalyst, in Tables VII and IX, in panel A and B of columns 1 to 3 the coefficient of leverage is significant at the 1% level as well as in panel B of columns 4 to 7, whereas panel A of columns 4 to 7, the leverage coefficient is negative and only statistically significant at the 5% level and in the case of column 7 of Table IX significant at the 10% level.

INSERT TABLES IX and X **HERE**

In each of the four tables, in each of the seven regression frameworks, the leverage coefficient is more negative for firms with higher transparency (Panel B) than those with lower transparency (Panel A), after controlling for firm characteristics. So our findings confirm our main empirical hypothesis implied by our theoretical model and show that the main conclusion is robust across different measures of accounting transparency, a variety of firm characteristics, different specifications of implied volatility skewness, and different cross-sectional splits of accounting transparency.

VII. Conclusion

We have developed a model of the interaction between financial leverage, the quality of corporate disclosure and the implied volatility skew for individual firm stock options. We show theoretically that the impact of leverage on the skew is stronger for firms with greater transparency and are able to provide empirical support for this prediction.

Our model provides an economically intuitive and empirically testable explanation for differences in deviations from the Black-Scholes option pricing framework across firms. Taken together with findings of a relationship between accounting transparency and the valuation of corporate debt, this provides support for the modeling framework of Duffie and Lando [2001] who link the precision of accounting information to the valuation of corporate securities.

Our model, like the Duffie and Lando [2001], is based on the Leland [1994] model. However, interesting extensions to evaluate the role of debt maturity and / or variance risk when taking into account accounting transparency, one could extend the underlying model by using the Leland and Toft [1996] model or in the case of stochastic asset variance, the Du, Elkhami, and Ericsson [2019] setup. We leave such extensions to future work.

VIII. Appendix

A. Appendix A: Proofs

We re-write equation $g(u|z,y)$ in the following form.

$$g(u|z,y) = \frac{L_1}{L_0} \times e^{\frac{-(u-M_1)^2}{2\psi}} - \frac{L_2}{L_0} \times e^{\frac{-(u-M_2)^2}{2\psi}} \tag{19}$$

where $\tilde{u} = u - \underline{\nu}$,

$$M_1 = \underline{\nu} + \frac{\left(\tilde{y}\sigma^2 t + a^2(\tilde{z}_0 + mt)\right)}{(a^2+\sigma^2 t)} \qquad M_2 = \underline{\nu} + \frac{\left(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0 - mt)\right)}{(a^2+\sigma^2 t)} \qquad \psi = \frac{\sigma^2 a^2 t}{(a^2+\sigma^2 t)} \tag{20}$$

$$L_0 = \left(1/\sqrt{\alpha/2\pi}\right)\left[\Phi\left(\sqrt{\alpha}\beta\right) - \Phi\left(\sqrt{\alpha}\left(\beta - \frac{2\tilde{z}_0}{\sigma^2 t\alpha}\right)\right) e^{\left(-\frac{\alpha\eta}{2} + \frac{2\tilde{z}_0\left(\tilde{z}_0 - \alpha\beta\sigma^2 t\right)}{\sigma^4 t^2 \alpha}\right)}\right] \tag{21}$$

$$L_1 = e^{\frac{-\left(a^2+\sigma^2 t\right)}{2\sigma^2 a^2 t}\left[\frac{-\left(\tilde{y}\sigma^2 t + a^2(\tilde{z}_0+mt)\right)^2}{(a^2+\sigma^2 t)^2} + \frac{\left(\tilde{y}^2\sigma^2 t + a^2(\tilde{z}_0+mt)^2\right)}{(a^2+\sigma^2 t)}\right]} \qquad L_2 = e^{\frac{-\left(a^2+\sigma^2 t\right)}{2\sigma^2 a^2 t}\left[\frac{-\left(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0-mt)\right)^2}{(a^2+\sigma^2 t)^2} + \frac{\left(\tilde{y}^2\sigma^2 t + a^2(\tilde{z}_0+mt)^2\right)}{(a^2+\sigma^2 t)}\right]} \tag{22}$$

We then compute a closed form expression for an option pricing model of Toft and Pryck [1997] in the spirit of Duffie and Lando [2001] where the firm value is imperfectly observed. We derive the call option pricing model as:

$$\begin{aligned}
DL_{CALL} &= \int_{\underline{\nu}}^{\infty} TP_{CALL}\left(e^u, V_B, C, r, \delta, \tau, \sigma_A, T, K, t\right) g(u|z,y)\, du \\
&= \int_{\underline{\nu}}^{\infty} e^{u+(-r+m+\sigma^2/2)(T-t)} \left[\Phi(-z_1) - e^{(2m/\sigma^2+2)(\underline{\nu}-u)}\Phi(-z_2)\right] g(u|z,y)\, du \\
&+ \int_{\underline{\nu}}^{\infty} B e^{-\gamma(u-\underline{\nu})} \left[\Phi(-z_3) - e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi(-z_4)\right] g(u|z,y)\, du \\
&- \int_{\underline{\nu}}^{\infty} e^{-r(T-t)}(A+K) \left[\Phi(-z_5) - e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi(-z_6)\right] g(u|z,y)\, du
\end{aligned} \tag{23}$$

where

$$\begin{array}{ll} -z_1 = \frac{-y^*+\left(m(T-t)+u+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & -z_2 = \frac{-y^*+\left(m(T-t)-u+2\underline{\nu}+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\ -z_3 = \frac{-y^*+\left(m(T-t)+u-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & -z_4 = \frac{-y^*+\left(m(T-t)-u+2\underline{\nu}-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\ -z_5 = \frac{-y^*+(m(T-t)+u)}{\sigma\sqrt{T-t}} & -z_6 = \frac{-y^*+(m(T-t)-u+2\underline{\nu})}{\sigma\sqrt{T-t}} \end{array} \tag{24}$$

To compute the integral in equation 23, note that equation 19 is essentially a difference in the Normally distributed random variable probability distribution functions (PDF) being integrated over the TP option pricing formula, which is a function of Normal distributed cumulative distribution functions (CDFS). As such the end product will be an integral, over the range of either $(-\infty, \underline{\nu})$ for a put or $(\underline{\nu}, \infty)$ for a call, of the product of a Normally distributed CDF and the Normally PDF. To compute this integral we make use of the equations $10,010.1$ and $10,010.4$ in Owen [1980] (stated in Lemmas VIII.B and VIII.B respectively) which allows for a closed form expression of the integral. The result itself is then just a function of the Standard Bivariate Normal Cumulative Probability Distribution Function.

Let A and B are real valued constants and $Z \sim \mathcal{N}(0,1)$, with probability (cumulative) distribution function denoted $\phi(z)$ $(\Phi(z))$ respectively then

$$\begin{aligned} \int_h^k \Phi(A+Bx)\phi(x)dx &= \int_{-\infty}^{A/\sqrt{1+B^2}} \phi(x)\Phi\left(k\sqrt{B^2+1}+Bx\right)dx \\ &\quad - \int_{-\infty}^{A/\sqrt{1+B^2}} \phi(x)\Phi\left(h\sqrt{B^2+1}+Bx\right)dx \\ &= \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, k; \frac{-B}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \end{aligned} \tag{25}$$

Taking $k \longrightarrow \infty$ in equation 54 yields:

$$\begin{aligned} \int_h^\infty \Phi(A+Bx)\phi(x)dx &= \lim_{k\to\infty} \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, k; \frac{-B}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \\ &= \Phi\left(\frac{A}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \end{aligned} \tag{26}$$

B. Appendix B: Risk-Neutral Skewness

For each stock trading day we estimate the model free option implied skewness ($Skew^Q$) of Bakshi et al. [2003] in equation 27. We denote S_t the underlying stock price at time t and $C(t,T,K)$ ($P(t,T,K)$) as the respective call (put) option prices with maturity date T and exercise strike price of time K.

$$Skew^Q(t,T) = \frac{e^{r(T-t)} - 3\mu(t,T)\, e^{r(T-t)} V(t,T) + 2(\mu(t,T))^3}{\left(e^{r(T-t)} V(t,T) - (\mu(t,T))^2\right)^{3/2}} \tag{27}$$

where denoted below, $\mu(t,T)$ is the risk neutral log return of the stock price from time t to T,$V(t,T)$, $W(t,T)$, and $X(t,T)$ are respectively the price of volatility, cubic, and quadratic contracts.

$$\begin{aligned}
\mu(t,T) &= e^{r(T-t)} - 1 - \frac{e^{r(T-t)}}{2} V(t,T) - \frac{e^{r(T-t)}}{6} W(t,T) - \frac{e^{r(T-t)}}{24} X(t,T) \\
V(t,T) &= \int_{S_t}^{\infty} \frac{2\left(1 - \ln\left(\frac{K}{S_t}\right)\right)}{K^2} C(t,T,K)\, dK + \int_0^{S_t} \frac{2\left(1 + \ln\left(\frac{S_t}{K}\right)\right)}{K^2} P(t,T,K)\, dK \\
W(t,T) &= \int_{S_t}^{\infty} \frac{6\ln\left(\frac{K}{S_t}\right) - 3\left(\ln\left(\frac{K}{S_t}\right)\right)^2}{K^2} C(t,T,K)\, dK - \int_0^{S_t} \frac{6\ln\left(\frac{S_t}{K}\right) + 3\left(\ln\left(\frac{S_t}{K}\right)\right)^2}{K^2} P(t,T,K)\, dK \\
X(t,T) &= \int_{S_t}^{\infty} \frac{12\left(\ln\left(\frac{K}{S_t}\right)\right)^2 - 4\left(\ln\left(\frac{K}{S_t}\right)\right)^3}{K^2} C(t,T,K)\, dK \\
&\quad + \int_0^{S_t} \frac{12\left(\ln\left(\frac{S_t}{K}\right)\right)^2 + 4\left(\ln\left(\frac{S_t}{K}\right)\right)^3}{K^2} P(t,T,K)\, dK
\end{aligned} \tag{28}$$

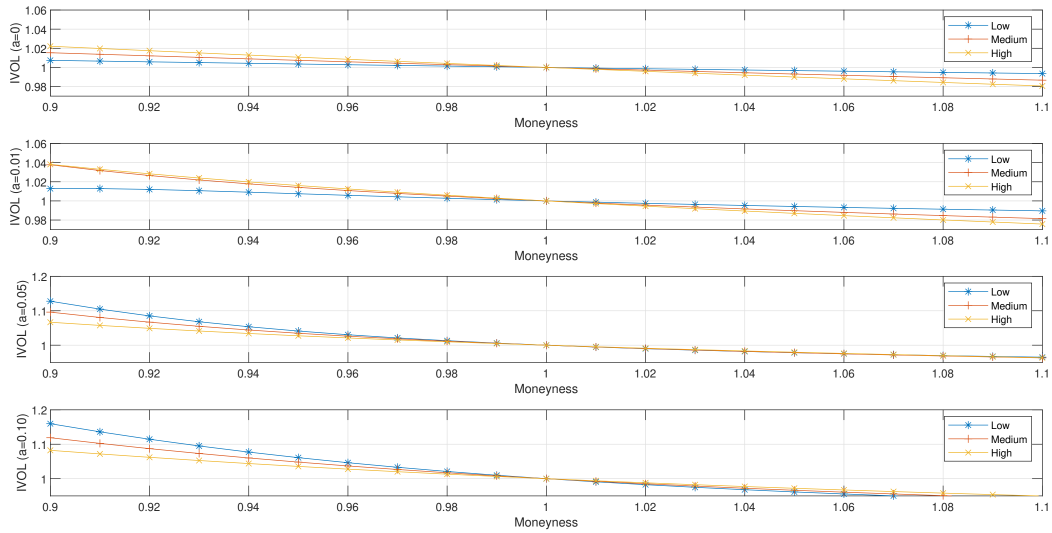
We compute the skew in equation 27 for every single stock for every trading day using all remaining option contracts for a stock on that trading day subject to the following data filters. We remove observations with option bid quotes that are zero, observations where the bid quote exceeds the ask quote. We remove all option contracts with zero open interest as they are unlikely to contain any important information. We remove options who have maturities longer than 365 since they are likely to have less trading activity. Option prices are then computed as the midpoint of the option bid and ask prices where the midpoints are required to be at least 0.5.

Additionally, we make sure that the no-arbitrage condition holds for both calls and puts. Specifically we remove observations where: (1.) the ask quotes for call options is less than the difference between the underlying stock price and the strike price as well as (2.) the ask quotes from put options is less than the difference between the strike price and the underlying stock price. Subsequently we only keep out-of-the-money puts and calls. After applying all filters and conditions we require that in order to estimate the skew we require that there are a minimum of two out-of-the-money puts and calls each trading data.

The resulting skew measure is then computed the average of the skew measure across option maturities each trading day and then averaged across all trading dates with available data within a calendar quarter which results in our final quarterly skew measure. We use trapezoidal approach when numerically approximating integrals.

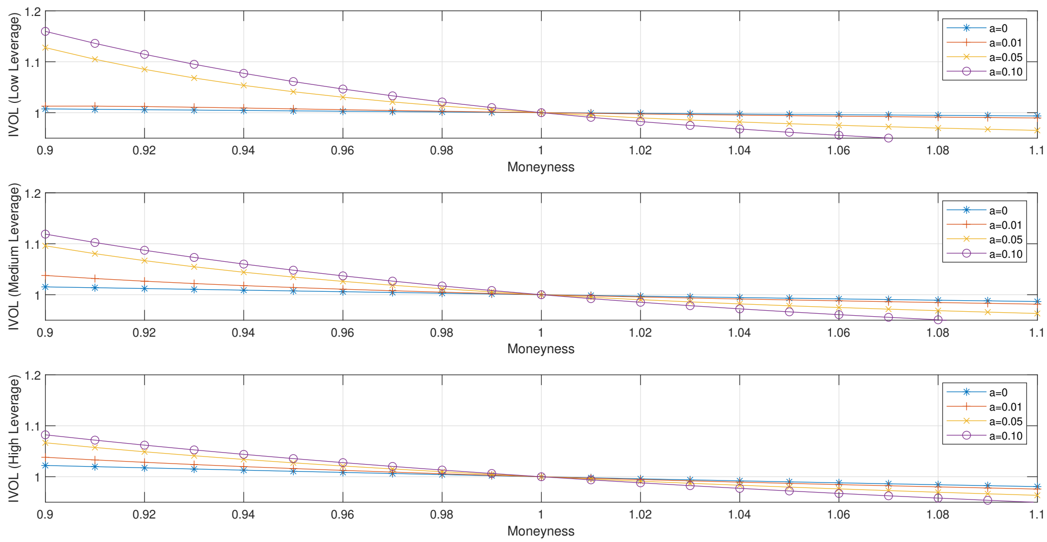
IX. Figures

Figure 1. Impact of Leverage Ratio on Implied Volatility Smile For Different Accounting Transparency



Notes: Volatility smile for low (blue), medium (red), or high (yellow) leverage ratios. The panel closest to the top of the page is where the accounting precision is set to zero, (observed asset values). Normalized volatility smile as a function of moneyness.

Figure 2. Impact of Accounting Transparency on Implied Volatility Smile For Different Leverage Ratios



Notes: Volatility smile for $a = 0.00$ (blue), $a = 0.01$ (red), $a = 0.05$ (yellow), and $a = 0.10$ (purple) accounting noise. Each of Panels represent low, medium, and high leverage ratio.

X. Tables

Table I: Summary Statistics

Variable Name	N	Mean	Std. Dev.	Percentiles 25th	Median	75th
$Skew^Q$	85,742	−0.45	0.32	−0.62	−0.4	−0.22
Slope Skew	139,123	−0.86	0.81	−1.13	−0.68	−0.37
Disp	181,540	0.16	0.41	0.02	0.04	0.11
Nanalyst	290,063	5.54	6.79	0.00	3.00	8.00
Leverage	317,728	0.2	0.22	0.01	0.12	0.32
Momentum	307,111	0.06	0.43	−0.18	0.02	0.22
M/B	319,606	3.69	5.68	1.22	2.06	3.77
Beta	272,691	1.52	1.69	0.49	1.26	2.28
Size	311,655	19.48	2.17	17.9	19.42	20.94
Idio Skew	312,269	0.4	1.32	−0.17	0.33	0.92
Atm Ivol	133,414	0.49	0.24	0.32	0.43	0.61

Notes: The table presents quarterly summary statistics for main variables. We compute the dispersion in analyst forecast (Disp) and number of analyst covering stock (Nanalyst) from I/B/E/S quarterly earnings data. M/B is the ratio of market and book equity computed using quarterly data from COMPUSTAT. Leverage is the book value of debt divided by sum of debt and market value of equity using quarterly data from COMPUSTAT. M/B and Leverage are lagged by one quarter in order to account for the timing of the release of accounting statements. Size is the log of the product of the stock price and shares outstanding (times 1000) from CRSP monthly stock files (of firms with share codes 10 and 11 common shares). Momentum is the past 6 month cumulative monthly stock returns from CRSP. Beta is the stock beta with the market estimated from 36 months rolling regressions adjusted by 3 months of lags for asynchronous trading as per the Dimson [1979] adjustment. Idio Skew is the idiosyncratic skewness of daily returns estimated quarterly using daily CRSP stock returns. Atm Ivol is average of call and put contract implied volatility with $|\Delta| = 0.5$ and 30 days to maturity, and using OptionMetrics Volatility surface. $Skew^Q$, Slope Skew and Atm Ivol are computed daily on a firm level and then averaged over the quarter. The sample period is quarterly observations from January 1997 to December 2017.

Table II: Correlations

	Correlations										
Variable Names	$Skew^Q$	Slope Skew	Disp	Nanalyst	Leverage	Momentum	M/B	Beta	Size	Idio Skew	Atm Ivol
$Skew^Q$	1.00	0.69	0.10	−0.29	−0.16	0.10	0.08	0.21	−0.59	0.03	0.66
Slope Skew	0.69	1.00	0.08	−0.05	−0.13	0.03	0.08	0.17	−0.24	$5e-3$	0.43
Disp	0.10	0.08	1.00	−0.09	0.07	−0.05	−0.02	0.09	−0.18	0.02	0.20
Nanalyst	−0.29	−0.05	−0.09	1.00	−0.05	0.01	0.09	−0.10	0.76	−0.11	−0.30
Leverage	−0.16	−0.13	0.07	−0.05	1.00	−0.05	−0.18	−0.06	−0.08	0.02	−0.07
Momentum	0.10	0.03	−0.05	0.01	−0.05	1.00	0.08	0.01	0.16	0.05	−0.13
M/B	0.08	0.08	−0.02	0.09	−0.18	0.08	1.00	0.08	0.12	$2e-3$	0.09
Beta	0.21	0.17	0.09	−0.10	−0.06	0.01	0.08	1.00	−0.16	0.05	0.30
Size	−0.59	−0.24	−0.18	0.76	−0.08	0.16	0.12	−0.16	1.00	−0.12	−0.59
Idio Skew	0.03	$5e-3$	0.02	−0.11	0.02	0.05	$2e-3$	0.05	−0.12	1.00	0.03
Atm Ivol	0.66	0.43	0.20	−0.30	−0.07	−0.13	0.09	0.30	−0.59	0.03	1.00

Notes: Table contains pooled correlations between all control and accounting quality measures from Table I. The sample period is quarterly observations from January 1997 to December 2017.

Table III: FM Regression with Nanalyst as Accounting Transparency

	Less Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.43	−0.43	−0.4	−0.34	−0.3	−0.3	−0.23
	(−7.6)	(−7.84)	(−6.91)	(−5.76)	(−5.22)	(−5.15)	(−3.84)
Momentum		0.04	0.06	0.12	0.1	0.11	0.07
		(1.29)	(2.49)	(4.53)	(4.14)	(4.51)	(2.46)
Beta			0.07	0.06	0.06	0.06	0.03
			(10.32)	(8.71)	(8.73)	(8.77)	(3.54)
Size				−0.11	−0.11	−0.11	0.01
				(−8.87)	(−8.83)	(−8.85)	(0.67)
M/B					0.01	0.01	$2.3e-3$
					(8.33)	(8.43)	(1.46)
Idio Skew						−0.01	−0.01
						(−1.86)	(−2.35)
Atm Ivol							1.19
							(16.05)
R^2	1.61	2.54	4.07	6.13	6.43	6.69	11.01
N obs	349.00	349.00	329.00	329.00	329.00	329.00	281.00

	More Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.55	−0.56	−0.55	−0.5	−0.47	−0.47	−0.38
	(−13.23)	(−14.32)	(−11.43)	(−11.82)	(−12.03)	(−12.08)	(−12.34)
Momentum		−0.04	−0.04	0.05	0.04	0.04	0.11
		(−0.76)	(−0.85)	(1.63)	(1.28)	(1.31)	(4.6)
Beta			0.12	0.1	0.09	0.09	0.03
			(7.39)	(7.07)	(7.01)	(7.04)	(3.57)
Size				−0.1	−0.1	−0.1	$-0.5e-3$
				(−9.79)	(−9.99)	(−9.99)	(−0.07)
M/B					0.01	0.01	$-0.4e-3$
					(5.31)	(5.31)	(−0.66)
Idio Skew						$-2.6e-3$	$-3.7e-3$
						(−1.48)	(−1.97)
Atm Ivol							1.89
							(11.4)
R^2	3.26	5.2	9.55	15.53	15.76	15.8	25.49
N obs	1232.00	1232.00	1172.00	1172.00	1172.00	1172.00	988.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 50th percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the Slope Skew.

Table IV: FM Regression with Disp as Accounting Transparency

	Less Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.43	−0.43	−0.4	−0.38	−0.35	−0.35	−0.28
	(−10.47)	(−10.83)	(−10.4)	(−10.51)	(−9.68)	(−9.65)	(−11.18)
Momentum		0.03	0.04	0.05	0.04	0.04	0.07
		(1.04)	(1.8)	(2.38)	(1.64)	(1.58)	(3.12)
Beta			0.06	0.05	0.05	0.05	0.02
			(10.22)	(9.38)	(9.15)	(9.07)	(4.21)
Size				−0.02	−0.02	−0.02	0.05
				(−4.1)	(−4.42)	(−4.43)	(6)
M/B					0.01	0.01	$2.3e-3$
					(7.79)	(7.78)	(3.09)
Idio Skew						$1.3e-3$	$0.4e-3$
						(0.6)	(0.14)
Atm Ivol							1.2
							(13.85)
R^2	3.42	4.33	5.78	6.67	7.14	7.18	13.69
N obs	669.00	669.00	630.00	630.00	630.00	630.00	519.00

	More Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.81	−0.8	−0.76	−0.7	−0.66	−0.66	−0.43
	(−13.27)	(−13.43)	(−11.08)	(−10.92)	(−11.17)	(−11.2)	(−8.34)
Momentum		0.07	0.08	0.11	0.1	0.11	0.1
		(1.17)	(2.03)	(3.27)	(3.08)	(3.24)	(4.07)
Beta			0.14	0.11	0.11	0.11	0.04
			(7.3)	(7.01)	(7.03)	(7.06)	(3.72)
Size				−0.11	−0.11	−0.11	−0.01
				(−10.48)	(−10.94)	(−10.93)	(−1.5)
M/B					0.01	0.01	$1.6e-3$
					(4.88)	(4.85)	(1.68)
Idio Skew						−0.01	$-4.3e-3$
						(−2.46)	(−2.03)
Atm Ivol							2.1
							(12)
R^2	4.36	6.1	10.01	16.71	16.85	16.9	24.44
N obs	805.00	805.00	772.00	772.00	772.00	772.00	667.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 50th percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the Slope Skew.

Table V: FM Regression with Nanalyst as Accounting Transparency

	Less Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	-0.18	-0.17	-0.16	-0.1	-0.09	-0.09	-0.08
	(-8.88)	(-10.62)	(-10.33)	(-6.34)	(-5.31)	(-5.59)	(-4.75)
Momentum		0.04	0.04	0.06	0.06	0.06	0.03
		(2.14)	(3.07)	(4.38)	(4.77)	(4.83)	(2.73)
Beta			0.03	0.02	0.02	0.02	$3.9e-3$
			(9.09)	(7.21)	(6.74)	(6.51)	(1.57)
Size				-0.11	-0.11	-0.11	-0.06
				(-17.34)	(-17.73)	(-17.17)	(-11.48)
M/B					$0.1e-3$	$0e-3$	$-2.9e-3$
					(0.08)	$(0e-3)$	(-2.55)
Idio Skew						$2.2e-3$	$4.7e-3$
						(1.27)	(1.94)
Atm Ivol							0.5
							(11.51)
R^2	2.24	4.87	7.98	26.73	28.1	28.51	36.02
N obs	146.00	146.00	138.00	138.00	138.00	138.00	123.00

	More Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	-0.23	-0.23	-0.22	-0.17	-0.17	-0.17	-0.13
	(-10.74)	(-11.93)	(-11.88)	(-12.45)	(-12.12)	(-12.26)	(-12.22)
Momentum		-0.02	-0.01	0.02	0.02	0.02	0.02
		(-0.63)	(-0.64)	(1.61)	(1.52)	(1.46)	(2.55)
Beta			0.06	0.03	0.03	0.03	$3.9e-3$
			(9.08)	(9.24)	(9.17)	(9.11)	(3.08)
Size				-0.1	-0.1	-0.1	-0.06
				(-14.5)	(-14.58)	(-14.59)	(-13.28)
M/B					$0.7e-3$	$0.7e-3$	$-1.8e-3$
					(1.73)	(1.82)	(-7.11)
Idio Skew						$0.4e-3$	$0.5e-3$
						(0.4)	(0.64)
Atm Ivol							0.88
							(12.52)
R^2	3.42	6.06	11.71	40.89	40.99	41.1	51.93
N obs	828.00	828.00	790.00	790.00	790.00	790.00	679.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 50th percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the $Skew^Q$.

Table VI: FM Regression with Disp as Accounting Transparency

	Less Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.25	−0.25	−0.23	−0.16	−0.15	−0.15	−0.12
	(−9.21)	(−9.65)	(−10)	(−8.32)	(−7.14)	(−7.14)	(−9)
Momentum		0.01	0.01	0.03	0.02	0.02	0.02
		(0.36)	(0.94)	(2.42)	(1.91)	(1.89)	(3.1)
Beta			0.03	0.02	0.02	0.02	$1.4e-3$
			(10.83)	(8.78)	(8.62)	(8.53)	(1.01)
Size				−0.07	−0.07	−0.07	−0.04
				(−15.78)	(−15.81)	(−15.71)	(−12.4)
M/B					$2.4e-3$	$2.4e-3$	$-0.8e-3$
					(5.32)	(5.54)	(−2.4)
Idio Skew						$1.3e-3$	$0.7e-3$
						(1.03)	(0.55)
Atm Ivol							0.69
							(14.43)
R^2	7.18	9.06	12.15	31.74	32.22	32.32	44.41
N obs	374.00	374.00	352.00	352.00	352.00	352.00	299.00

	More Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.32	−0.32	−0.31	−0.24	−0.23	−0.23	−0.15
	(−15.81)	(−15.83)	(−12.6)	(−15.63)	(−15.46)	(−15.45)	(−13.2)
Momentum		0.02	0.03	0.03	0.03	0.03	0.02
		(0.79)	(1.79)	(2.08)	(2.15)	(2.11)	(1.65)
Beta			0.06	0.03	0.03	0.03	$4.4e-3$
			(8.35)	(9.09)	(9.21)	(9.18)	(2.79)
Size				−0.11	−0.11	−0.11	−0.07
				(−15.15)	(−15.33)	(−15.37)	(−14.02)
M/B					$-0.3e-3$	$-0.3e-3$	$-1.6e-3$
					(−0.77)	(−0.79)	(−4.55)
Idio Skew						$-0.8e-3$	$0.3e-3$
						(−0.67)	(0.28)
Atm Ivol							0.93
							(10.74)
R^2	3.67	6.11	10.93	43.16	43.25	43.39	52.04
N obs	559.00	558.00	537.00	537.00	537.00	537.00	469.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 50th percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the $Skew^Q$.

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Internet Appendix

Additional Tables

Table VII: FM Regression with Nanalyst as Accounting Transparency

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Less Accounting Transparency							
Leverage	−0.47	−0.48	−0.43	−0.36	−0.32	−0.32	−0.21
	(−5.36)	(−5.5)	(−4.9)	(−3.96)	(−3.52)	(−3.48)	(−2.29)
Momentum		0.01	0.04	0.09	0.07	0.08	0.03
		(0.14)	(1.19)	(2.96)	(2.25)	(2.61)	(0.96)
Beta			0.07	0.06	0.05	0.06	0.02
			(9.94)	(8.11)	(7.82)	(7.94)	(2.88)
Size				−0.12	−0.12	−0.12	$4.7e-3$
				(−7.6)	(−7.51)	(−7.44)	(0.27)
M/B					0.01	0.01	$3.3e-3$
					(7.26)	(7.44)	(1.47)
Idio Skew						−0.02	−0.02
						(−3.03)	(−2.63)
Atm Ivol							1.28
							(12.43)
R^2	2.1	2.89	4.39	6.82	7.4	7.79	12
N obs	218.00	217.00	205.00	205.00	205.00	205.00	175.00
More Accounting Transparency							
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.55	−0.56	−0.55	−0.52	−0.49	−0.49	−0.4
	(−12.4)	(−13.71)	(−10.86)	(−11.53)	(−11.66)	(−11.73)	(−11.89)
Momentum		−0.07	−0.06	0.04	0.03	0.03	0.12
		(−1.14)	(−1.35)	(1.12)	(0.81)	(0.84)	(4.57)
Beta			0.13	0.1	0.1	0.1	0.03
			(7.18)	(6.96)	(6.92)	(6.96)	(3.51)
Size				−0.11	−0.11	−0.11	$-0.1e-3$
				(−9.89)	(−10.05)	(−10.05)	(−0.01)
M/B					0.01	0.01	$-0.9e-3$
					(5.11)	(5.12)	(−1.42)
Idio Skew						$-2e-3$	$-3.5e-3$
						(−1.05)	(−1.9)
Atm Ivol							2.09
							(10.6)
R^2	3.34	5.54	10.48	17.24	17.45	17.5	28.66
N obs	1081.00	1080.00	1031.00	1031.00	1031.00	1031.00	867.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 40th (60th) percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the Slope Skew.

Table VIII: FM Regression with Disp as Accounting Transparency

	Less Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.39	−0.39	−0.36	−0.35	−0.32	−0.32	−0.27
	(−9.58)	(−9.91)	(−9.72)	(−9.91)	(−9.02)	(−9.01)	(−10.53)
Momentum		0.03	0.05	0.05	0.04	0.04	0.06
		(1.25)	(2.13)	(2.37)	(1.67)	(1.63)	(2.65)
Beta			0.05	0.05	0.04	0.04	0.02
			(11.29)	(10.15)	(9.96)	(9.81)	(4.47)
Size				−0.01	−0.01	−0.01	0.05
				(−2.04)	(−2.34)	(−2.37)	(6.57)
M/B					0.01	0.01	$1.8e-3$
					(7.64)	(7.61)	(2.28)
Idio Skew						$0.3e-3$	$-1.8e-3$
						(0.13)	(−0.63)
Atm Ivol							1.14
							(13.15)
R^2	3.33	4.2	5.32	5.99	6.4	6.47	12.75
N obs	518.00	518.00	487.00	487.00	487.00	487.00	399.00

	More Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.83	−0.82	−0.77	−0.72	−0.68	−0.68	−0.43
	(−12.3)	(−12.24)	(−10.45)	(−10.55)	(−10.58)	(−10.63)	(−7.55)
Momentum		0.07	0.08	0.1	0.1	0.1	0.09
		(1.16)	(1.79)	(2.78)	(2.59)	(2.79)	(3.22)
Beta			0.15	0.11	0.11	0.11	0.04
			(7.22)	(6.78)	(6.8)	(6.82)	(3.52)
Size				−0.11	−0.11	−0.11	−0.01
				(−10.22)	(−10.8)	(−10.77)	(−1.32)
M/B					0.01	0.01	$2.1e-3$
					(4.75)	(4.73)	(2.05)
Idio Skew						$-4.9e-3$	$-3.5e-3$
						(−2.14)	(−1.49)
Atm Ivol							2.31
							(11.63)
R^2	4.4	6.17	10.25	17.54	17.7	17.75	25.67
N obs	646.00	646.00	621.00	621.00	621.00	621.00	537.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 40th (60th) percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the Slope Skew.

Table IX: FM Regression with Nanalyst as Accounting Transparency

	Less Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.16	−0.15	−0.14	−0.08	−0.06	−0.06	−0.06
	(−4.89)	(−5.02)	(−5.22)	(−3.27)	(−2.28)	(−2.46)	(−2.22)
Momentum		0.02	0.03	0.04	0.04	0.04	0.02
		(0.98)	(1.25)	(2.12)	(2.46)	(2.5)	(1.23)
Beta			0.03	0.02	0.02	0.02	$1.2e-3$
			(8.26)	(6.63)	(5.94)	(5.27)	(0.33)
Size				−0.11	−0.11	−0.11	−0.07
				(−18.25)	(−18.22)	(−17.61)	(−9.19)
M/B					$0.9e-3$	$0.9e-3$	$-4.4e-3$
					(0.66)	(0.64)	(−3.06)
Idio Skew						$1.1e-3$	$3.6e-3$
						(0.46)	(0.95)
Atm Ivol							0.48
							(9.01)
R^2	1.57	5.07	8.52	28.23	28.66	29.18	36.69
N obs	86.00	86.00	81.00	81.00	81.00	81.00	72.00

	More Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.22	−0.22	−0.21	−0.18	−0.17	−0.17	−0.14
	(−9.78)	(−11.06)	(−11.04)	(−11.9)	(−11.64)	(−11.77)	(−11.79)
Momentum		−0.04	−0.03	0.02	0.01	0.01	0.02
		(−1.25)	(−1.4)	(1.1)	(1.01)	(0.96)	(2.27)
Beta			0.06	0.04	0.04	0.04	$3.8e-3$
			(8.9)	(8.8)	(8.75)	(8.71)	(2.77)
Size				−0.1	−0.1	−0.1	−0.06
				(−14.43)	(−14.52)	(−14.54)	(−13.57)
M/B					$0.6e-3$	$0.7e-3$	$-1.7e-3$
					(1.81)	(1.91)	(−7.12)
Idio Skew						$0.1e-3$	$0.3e-3$
						(0.11)	(0.33)
Atm Ivol							0.93
							(12.18)
R^2	3.2	5.9	11.65	40.75	40.83	40.95	52.21
N obs	753.00	753.00	720.00	720.00	720.00	720.00	618.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 40th (60th) percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the $Skew^Q$.

Table X: FM Regression with Disp as Accounting Transparency

	Less Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.25	−0.24	−0.22	−0.16	−0.14	−0.14	−0.11
	(−8.65)	(−9.02)	(−9.14)	(−7.56)	(−6.43)	(−6.46)	(−8.25)
Momentum		0.01	0.01	0.03	0.02	0.02	0.02
		(0.61)	(1.19)	(2.59)	(1.99)	(1.93)	(2.59)
Beta			0.03	0.02	0.02	0.02	$0.7e-3$
			(10.68)	(8.22)	(8.02)	(7.95)	(0.45)
Size				−0.07	−0.07	−0.07	−0.04
				(−14.93)	(−15)	(−14.89)	(−11.49)
M/B					$2.4e-3$	$2.5e-3$	$-0.6e-3$
					(4.65)	(4.79)	(−1.6)
Idio Skew						$1.3e-3$	$1.1e-3$
						(0.96)	(0.81)
Atm Ivol							0.65
							(14.47)
R^2	7.58	9.29	12.22	30.23	30.8	30.89	42.85
N obs	281.00	281.00	264.00	264.00	264.00	264.00	224.00

	More Accounting Transparency						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	−0.33	−0.33	−0.31	−0.24	−0.24	−0.24	−0.16
	(−15.33)	(−15.28)	(−12.07)	(−16.03)	(−15.69)	(−15.64)	(−13.38)
Momentum		0.03	0.04	0.02	0.02	0.02	0.01
		(0.91)	(1.77)	(1.46)	(1.51)	(1.45)	(0.71)
Beta			0.07	0.03	0.03	0.03	$4.5e-3$
			(8.26)	(8.87)	(9.03)	(9.01)	(2.57)
Size				−0.11	−0.11	−0.11	−0.07
				(−15.05)	(−15.24)	(−15.28)	(−14.1)
M/B					$-0.3e-3$	$-0.3e-3$	$-1.4e-3$
					(−0.66)	(−0.69)	(−3.79)
Idio Skew						$-0.1e-3$	$1e-3$
						(−0.08)	(0.95)
Atm Ivol							0.97
							(9.97)
R^2	3.53	5.88	10.99	44.09	44.16	44.28	52.44
N obs	455.00	455.00	439.00	439.00	439.00	439.00	383.00

Notes: Quarterly regressions are estimated using Fama and Macbeth [1973] from January 1997 to December 2017 on all firms with listed equity options. Panel A (B), each of the 7 FM regression models is estimated using the subset of data each quarter that is below (above) 40th (60th) percentile level of Accounting Transparency. The variable (and sign) of interest is Leverage in regression (1). Control variables are added individually from regressions (2) to (7): Momentum, Beta, Size, M/B, Idio Skew, and Atm Ivol. R-squared (R^2) and quarterly number of observations (N obs) are computed in each regression. t-statistics are computed using Newey and West [1987] standard errors with 3 lags and reported in parentheses below the coefficient. The dependent variable is the $Skew^Q$.

Proofs

We re-write equation $g\left(u|z,y\right)$ in the following form.

$$g\left(u|z,y\right)=\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}} \tag{29}$$

where $\tilde{u}=u-\underline{\nu}$,

$$\begin{aligned}
M_1 &= \underline{\nu}+\frac{(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt))}{(a^2+\sigma^2 t)}\\
M_2 &= \underline{\nu}+\frac{(\tilde{y}\sigma^2 t-a^2(\tilde{z}_0-mt))}{(a^2+\sigma^2 t)}\\
\psi &= \frac{\sigma^2 a^2 t}{(a^2+\sigma^2 t)}\\
L_0 &= \left(1/\sqrt{\alpha/2\pi}\right)\left[\Phi\left(\sqrt{\alpha}\beta\right)-\Phi\left(\sqrt{\alpha}\left(\beta-\frac{2\tilde{z}_0}{\sigma^2 t\alpha}\right)\right)e^{\left(-\frac{\alpha\eta}{2}+\frac{2\tilde{z}_0\left(\tilde{z}_0-\alpha\beta\sigma^2 t\right)}{\sigma^4 t^2\alpha}\right)}\right]\\
L_1 &= e^{\frac{-\left(a^2+\sigma^2 t\right)}{2\sigma^2 a^2 t}\left[\frac{-\left(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt)\right)^2}{(a^2+\sigma^2 t)^2}+\frac{\left(\tilde{y}^2\sigma^2 t+a^2(\tilde{z}_0+mt)^2\right)}{(a^2+\sigma^2 t)}\right]}\\
L_2 &= e^{\frac{-\left(a^2+\sigma^2 t\right)}{2\sigma^2 a^2 t}\left[\frac{-\left(\tilde{y}\sigma^2 t-a^2(\tilde{z}_0-mt)\right)^2}{(a^2+\sigma^2 t)^2}+\frac{\left(\tilde{y}^2\sigma^2 t+a^2(\tilde{z}_0+mt)^2\right)}{(a^2+\sigma^2 t)}\right]}
\end{aligned} \tag{30}$$

We then compute a closed form expression for an option pricing model of Toft and Pryck [1997] in the spirit of Duffie and

Lando [2001] where the firm value is imperfectly observed. We derive the call option pricing model as:

$$\begin{aligned} DL_{CALL} &= \int_{\underline{\nu}}^{\infty} TP_{CALL}\left(e^u, V_B, C, r, \delta, \tau, \sigma_A, T, K, t\right) g\left(u|z,y\right) du \\ &= \int_{\underline{\nu}}^{\infty} e^{u+(-r+m+\sigma^2/2)(T-t)} \left[\Phi\left(-z_1\right) - e^{(2m/\sigma^2+2)(\underline{\nu}-u)}\Phi\left(-z_2\right)\right] g\left(u|z,y\right) du \\ &+ \int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})} \left[\Phi\left(-z_3\right) - e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi\left(-z_4\right)\right] g\left(u|z,y\right) du \\ &- \int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right) \left[\Phi\left(-z_5\right) - e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi\left(-z_6\right)\right] g\left(u|z,y\right) du \end{aligned} \tag{31}$$

where

$$\begin{aligned} -z_1 &= \frac{-y^*+\left(m(T-t)+u+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & -z_2 &= \frac{-y^*+\left(m(T-t)-u+2\underline{\nu}+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\ -z_3 &= \frac{-y^*+\left(m(T-t)+u-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & -z_4 &= \frac{-y^*+\left(m(T-t)-u+2\underline{\nu}-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\ -z_5 &= \frac{-y^*+(m(T-t)+u)}{\sigma\sqrt{T-t}} & -z_6 &= \frac{-y^*+(m(T-t)-u+2\underline{\nu})}{\sigma\sqrt{T-t}} \end{aligned} \tag{32}$$

To compute the integral in equation 23, note that equation 19 is essentially a difference in the Normally distributed random variable probability distribution functions (PDF) being integrated over the TP option pricing formula, which is a function of Normal distributed cumulative distribution functions (CDFS). As such the end product will be an integral, over the range of either $(-\infty, \underline{\nu})$ for a put or $(\underline{\nu}, \infty)$ for a call, of the product of a Normally distributed CDF and the Normally PDF. To compute this integral we make use of the equations $10,010.1$ and $10,010.4$ in Owen [1980] (stated in Lemmas VIII.B and VIII.B respectively) which allows for a closed form expression of the integral. The result itself is then just a function of the Bivariate Normally distributed cumulative probability distribution function.

Let A and B are real valued constants and $Z \sim \mathcal{N}(0,1)$, with probability (cumulative) distribution function denoted $\phi(z)$ $(\Phi(z))$ respectively then

$$\int_{-\infty}^{\underline{\nu}} \Phi(A+Bz)\phi(z)dz = \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, \underline{\nu}; \frac{-B}{\sqrt{1+B^2}}\right) \tag{33}$$

where $\Phi_2\left(z_1, z_2; \rho\right)$ is the cumulative bivariate normal distribution of two joint random variables Z_1 and Z_2 with correlation ρ.

Let A and B are real valued constants and $Z \sim \mathcal{N}(0,1)$, with probability (cumulative) distribution function denoted $\phi(z)$

($\Phi(z)$) respectively then

$$\int_h^k \Phi(A+Bx)\phi(x)dx = \int_{-\infty}^{A/\sqrt{1+B^2}} \phi(x)\Phi\left(k\sqrt{B^2+1}+Bx\right)dx$$
$$-\int_{-\infty}^{A/\sqrt{1+B^2}} \phi(x)\Phi\left(h\sqrt{B^2+1}+Bx\right)dx$$
$$= \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, k; \frac{-B}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \tag{34}$$

Taking $k \longrightarrow \infty$ in equation 54 yields:

$$\int_h^\infty \Phi(A+Bx)\phi(x)dx = \lim_{k\to\infty} \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, k; \frac{-B}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right)$$
$$= \Phi\left(\frac{A}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \tag{35}$$

In order to solve for our option pricing model with incomplete accounting information, given by H_t, in closed form, the function $g\left(u|y, z_0, t\right)$ can be re-written as a difference of Normal distribution probability density functions with different means and variances in the general form (see appendix for details of derivation and notation):

$$g\left(u|z_0, y\right) = \frac{L_1}{L_0} \times e^{\frac{-(u-M_1)^2}{2\psi}} - \frac{L_2}{L_0} \times e^{\frac{-(u-M_2)^2}{2\psi}} \tag{36}$$

Our formula for H_t involves the integral of the product of the TP option pricing formula (denoted $h_t\left(u\right)$) and the density function $g\left(u|y, z_0, t\right)$. Expanding the product results in computing twelve integrals of the form in equation 17 (see Owen [1980]). The resulting sum of the twelve integrals results in a closed for solution, up to Bivariate Normal probability function, of the expression H_t.

Next we apply the framework of Duffie and Lando [2001] to the above call option option pricing formulas above:

$$g\left(u|z,y\right)=\frac{\sqrt{\alpha/(2\pi)}\left[1-e^{\left(\frac{-2\tilde{z}_0\tilde{u}}{\sigma^2 t}\right)}\right]e^{\left(\frac{-(\tilde{y}-\tilde{u})^2}{2a^2}\right)}e^{\left(\frac{-(\tilde{u}-\tilde{z}_0-mt)^2}{2\sigma^2 t}\right)}}{\Phi\left(\sqrt{\alpha}\beta\right)-\Phi\left(\sqrt{\alpha}\left(\beta-\frac{2\tilde{z}_0}{\sigma^2 t\alpha}\right)\right)e^{\left(-\frac{\alpha\eta}{2}+\frac{2\tilde{z}_0\left(\tilde{z}_0-\alpha\beta\sigma^2 t\right)}{\sigma^4 t^2\alpha}\right)}} \tag{37}$$

where

$$\begin{array}{ll}\tilde{z}_0=z_0-\underline{\nu} & \alpha=\frac{\sigma^2 t+a^2}{a^2\sigma^2 t}\\ \tilde{u}=u-\underline{\nu} & \beta=\frac{\sigma^2 t\tilde{y}+a^2(\tilde{z}_0+mt)}{\sigma^2 t+a^2}\\ \tilde{y}=y-\underline{\nu} & \eta=\frac{a^2\sigma^2 t(\tilde{y}-(\tilde{z}_0+mt))^2}{(\sigma^2 t+a^2)^2}\end{array} \tag{38}$$

The denominator of equation 37 is a constant in u since it does not depend on u whereas the numerator of equation 37 can be broken down (working in the exponential term) as:

$$\begin{aligned}
&\sigma^2 t\left(\tilde{y}-\tilde{u}\right)^2+a^2\left(\tilde{u}-\tilde{z}_0-mt\right)^2\\
&=\sigma^2 t\left(\tilde{y}^2-2\tilde{y}\tilde{u}+\tilde{u}^2\right)+a^2\left(\tilde{u}^2-2\tilde{u}\left(\tilde{z}_0+mt\right)+\left(\tilde{z}_0+mt\right)^2\right)\\
&=\left(a^2+\sigma^2 t\right)\tilde{u}^2-2\tilde{u}\left(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt)\right)+\left(\tilde{y}^2\sigma^2 t+a^2\left(\tilde{z}_0+mt\right)^2\right)\\
&=\left(a^2+\sigma^2 t\right)\left[\tilde{u}^2-2\tilde{u}\frac{(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt))}{(a^2+\sigma^2 t)}+\left(\frac{(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt))}{(a^2+\sigma^2 t)}\right)^2\right]-\frac{(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt))^2}{(a^2+\sigma^2 t)}+\left(\tilde{y}^2\sigma^2 t+a^2\left(\tilde{z}_0+mt\right)^2\right)\\
&=\left(a^2+\sigma^2 t\right)\left[\tilde{u}-\frac{(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt))}{(a^2+\sigma^2 t)}\right]^2-\frac{(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt))^2}{(a^2+\sigma^2 t)}+\left(\tilde{y}^2\sigma^2 t+a^2\left(\tilde{z}_0+mt\right)^2\right)
\end{aligned} \tag{39}$$

This results in the simplified expression of the second part of the numerator:

$$\begin{aligned}
&e^{\left(\frac{-\sigma^2 t(\tilde{y}-\tilde{u})^2}{2a^2\sigma^2 t}-\frac{a^2(\tilde{u}-\tilde{z}_0-mt)^2}{2a^2\sigma^2 t}\right)}\\
&=e^{-\frac{\left(a^2+\sigma^2 t\right)\left[\tilde{u}-\frac{\left(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt)\right)}{(a^2+\sigma^2 t)}\right]^2-\frac{\left(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt)\right)^2}{(a^2+\sigma^2 t)}+\left(\tilde{y}^2\sigma^2 t+a^2(\tilde{z}_0+mt)^2\right)}{2a^2\sigma^2 t}}
\end{aligned} \tag{40}$$

We then combine the intermediate step in equation 40 to obtain the first part of the numerator in equation 37 (working on the terms in the exponentials):

$$\begin{aligned}
&4a^2\tilde{z}_0\tilde{u} + \left(a^2+\sigma^2 t\right)\tilde{u}^2 - 2\tilde{u}\left(\tilde{y}\sigma^2 t + a^2(\tilde{z}_0+mt)\right) + \left(\tilde{y}^2\sigma^2 t + a^2\left(\tilde{z}_0+mt\right)^2\right)\\
&= \left(a^2+\sigma^2 t\right)\tilde{u}^2 - 2\tilde{u}\left(\tilde{y}\sigma^2 t + a^2(\tilde{z}_0+mt) - 2a^2\tilde{z}_0\right) + \left(\tilde{y}^2\sigma^2 t + a^2\left(\tilde{z}_0+mt\right)^2\right)\\
&= \left(a^2+\sigma^2 t\right)\tilde{u}^2 - 2\tilde{u}\left(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0-mt)\right) + \left(\tilde{y}^2\sigma^2 t + a^2\left(\tilde{z}_0+mt\right)^2\right)\\
&= \left(a^2+\sigma^2 t\right)\left[\tilde{u}^2 - 2\tilde{u}\frac{(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0-mt))}{(a^2+\sigma^2 t)} + \left(\frac{(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0-mt))}{(a^2+\sigma^2 t)}\right)^2\right] - \frac{(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0-mt))^2}{(\alpha^2+\sigma^2 t)} + \left(\tilde{y}^2\sigma^2 t + \alpha^2\left(\tilde{z}_0+mt\right)^2\right)\\
&= \left(a^2+\sigma^2 t\right)\left[\tilde{u} - \frac{(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0-mt))}{(a^2+\sigma^2 t)}\right]^2 - \frac{(\tilde{y}\sigma^2 t - a^2(\tilde{z}_0-mt))^2}{(a^2+\sigma^2 t)} + \left(\tilde{y}^2\sigma^2 t + a^2\left(\tilde{z}_0+mt\right)^2\right) \qquad (41)
\end{aligned}$$

Which results in the following exponential term:

$$= e^{\frac{-\left(\left(a^2+\sigma^2 t\right)\left[\tilde{u}-\frac{\left(\tilde{y}\sigma^2 t-a^2(\tilde{z}_0-mt)\right)}{(a^2+\sigma^2 t)}\right]^2-\frac{\left(\tilde{y}\sigma^2 t-a^2(\tilde{z}_0-mt)\right)^2}{(a^2+\sigma^2 t)}+\left(\tilde{y}^2\sigma^2 t+a^2(\tilde{z}_0+mt)^2\right)\right)}{2a^2\sigma^2 t}} \qquad (42)$$

We can then re-write the numerator of equation 37 as the difference between equations 40 and 42, hence:

$$\begin{aligned}
\left[1-e^{\left(\frac{-2\tilde{z}_0\tilde{u}}{\sigma^2 t}\right)}\right]e^{\left(\frac{-(\tilde{y}-\tilde{u})^2}{2a^2}\right)}e^{\left(\frac{-(\tilde{u}-\tilde{z}_0-mt)^2}{2\sigma^2 t}\right)} &= e^{-\frac{\left(a^2+\sigma^2 t\right)\left[\tilde{u}-\frac{\left(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt)\right)}{(a^2+\sigma^2 t)}\right]^2-\frac{\left(\tilde{y}\sigma^2 t+a^2(\tilde{z}_0+mt)\right)^2}{(a^2+\sigma^2 t)}+\left(\tilde{y}^2\sigma^2 t+a^2(\tilde{z}_0+mt)^2\right)}{2a^2\sigma^2 t}}\\
&\quad - e^{\frac{-\left(\left(a^2+\sigma^2 t\right)\left[\tilde{u}-\frac{\left(\tilde{y}\sigma^2 t-a^2(\tilde{z}_0-mt)\right)}{(a^2+\sigma^2 t)}\right]^2-\frac{\left(\tilde{y}\sigma^2 t-a^2(\tilde{z}_0-mt)\right)^2}{(a^2+\sigma^2 t)}+\left(\tilde{y}^2\sigma^2 t+a^2(\tilde{z}_0+mt)^2\right)\right)}{2a^2\sigma^2 t}} \qquad (43)
\end{aligned}$$

we can then re-write equation 37 as

$$g\left(u|z,y\right)=\frac{e^{-\frac{\left(\left(a^2+\sigma^2t\right)\left[\tilde{u}-\frac{\left(\tilde{y}\sigma^2t+a^2(\tilde{z}_0+mt)\right)}{(a^2+\sigma^2t)}\right]^2-\frac{\left(\tilde{y}\sigma^2t+a^2(\tilde{z}_0+mt)\right)^2}{(a^2+\sigma^2t)}+\left(\tilde{y}^2\sigma^2t+a^2(\tilde{z}_0+mt)^2\right)\right)}{2a^2\sigma^2t}}}{\Phi\left(\sqrt{\alpha}\beta\right)-\Phi\left(\sqrt{\alpha}\left(\beta-\frac{2\tilde{z}_0}{\sigma^2t\alpha}\right)\right)e^{\left(-\frac{\alpha\eta}{2}+\frac{2\tilde{z}_0(\tilde{z}_0-\alpha\beta\sigma^2t)}{\sigma^4t^2\alpha}\right)}}-\frac{e^{\frac{-\left(\left(a^2+\sigma^2t\right)\left[\tilde{u}-\frac{\left(\tilde{y}\sigma^2t-a^2(\tilde{z}_0-mt)\right)}{(a^2+\sigma^2t)}\right]^2-\frac{\left(\tilde{y}\sigma^2t-a^2(\tilde{z}_0-mt)\right)^2}{(a^2+\sigma^2t)}+\left(\tilde{y}^2\sigma^2t+a^2(\tilde{z}_0+mt)^2\right)\right)}{2a^2\sigma^2t}}}{\Phi\left(\sqrt{\alpha}\beta\right)-\Phi\left(\sqrt{\alpha}\left(\beta-\frac{2\tilde{z}_0}{\sigma^2t\alpha}\right)\right)e^{\left(-\frac{\alpha\eta}{2}+\frac{2\tilde{z}_0(\tilde{z}_0-\alpha\beta\sigma^2t)}{\sigma^4t^2\alpha}\right)}} \tag{44}$$

We then denote the following constants, which are not a function of $\tilde{u}$ to simplify the notation of equation 44

$$L_0=\left(1/\sqrt{\alpha/2\pi}\right)\left[\Phi\left(\sqrt{\alpha}\beta\right)-\Phi\left(\sqrt{\alpha}\left(\beta-\frac{2\tilde{z}_0}{\sigma^2t\alpha}\right)\right)e^{\left(-\frac{\alpha\eta}{2}+\frac{2\tilde{z}_0\left(\tilde{z}_0-\alpha\beta\sigma^2t\right)}{\sigma^4t^2\alpha}\right)}\right] \tag{45}$$

$$L_1=e^{\frac{-\left(a^2+\sigma^2t\right)}{2\sigma^2a^2t}\left[\frac{-\left(\tilde{y}\sigma^2t+a^2(\tilde{z}_0+mt)\right)^2}{(a^2+\sigma^2t)^2}+\frac{\left(\tilde{y}^2\sigma^2t+a^2(\tilde{z}_0+mt)^2\right)}{(a^2+\sigma^2t)}\right]}\qquad L_2=e^{\frac{-\left(a^2+\sigma^2t\right)}{2\sigma^2a^2t}\left[\frac{-\left(\tilde{y}\sigma^2t-a^2(\tilde{z}_0-mt)\right)^2}{(a^2+\sigma^2t)^2}+\frac{\left(\tilde{y}^2\sigma^2t+a^2(\tilde{z}_0+mt)^2\right)}{(a^2+\sigma^2t)}\right]} \tag{46}$$

which results in

$$g\left(u|z,y\right)=\frac{L_1}{L_0}\times e^{\frac{-[u-M_1]^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-[u-M_2]^2}{2\psi}} \tag{47}$$

where $\tilde{u}=u-\underline{\nu}$,

$$\begin{aligned}M_1&=\underline{\nu}+\frac{(\tilde{y}\sigma^2t+a^2(\tilde{z}_0+mt))}{(a^2+\sigma^2t)}\\M_2&=\underline{\nu}+\frac{(\tilde{y}\sigma^2t-a^2(\tilde{z}_0-mt))}{(a^2+\sigma^2t)}\\\psi&=\frac{\sigma^2a^2t}{(a^2+\sigma^2t)}\end{aligned} \tag{48}$$

We then compute, using equation 44 to compute a closed form expression for an option pricing model of Toft and Pryck [1997] in the spirit of Duffie and Lando [2001] where the firm value is imperfectly observed. We derive the call option pricing model as:

$$
\begin{aligned}
DL_{CALL} &= \int_{\underline{\nu}}^{\infty} TP_{CALL}\left(e^u, V_B, C, r, \delta, \tau, \sigma_A, T, K, t\right) g\left(u|z, y\right) du \\
&= \int_{\underline{\nu}}^{\infty} e^{u+(-r+m+\sigma^2/2)(T-t)} \left[\Phi\left(-z_1\right) - e^{(2m/\sigma^2+2)(\underline{\nu}-u)}\Phi\left(-z_2\right)\right] g\left(u|z, y\right) du \\
&+ \int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})} \left[\Phi\left(-z_3\right) - e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi\left(-z_4\right)\right] g\left(u|z, y\right) du \\
&- \int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right) \left[\Phi\left(-z_5\right) - e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi\left(-z_6\right)\right] g\left(u|z, y\right) du
\end{aligned} \tag{49}
$$

where

$$
\begin{aligned}
-z_1 &= \frac{-y^*+\left(m(T-t)+u+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & -z_2 &= \frac{-y^*+\left(m(T-t)-u+2\underline{\nu}+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\
-z_3 &= \frac{-y^*+\left(m(T-t)+u-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & -z_4 &= \frac{-y^*+\left(m(T-t)-u+2\underline{\nu}-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\
-z_5 &= \frac{-y^*+(m(T-t)+u)}{\sigma\sqrt{T-t}} & -z_6 &= \frac{-y^*+(m(T-t)-u+2\underline{\nu})}{\sigma\sqrt{T-t}}
\end{aligned} \tag{50}
$$

To compute the integral in equation 23, note that equation 19 is essentially a difference in the Normally distributed random variable probability distribution functions (PDF) being integrated over the TP option pricing formula, which is a function of Normal distributed cumulative distribution functions (CDFS). As such the end product will be an integral, over the range of either $(-\infty, \underline{\nu})$ for a put or $(\underline{\nu}, \infty)$ for a call, of the product of a Normally distributed CDF and the Normally PDF. To compute this integral we make use of the equations $10,010.1$ and $10,010.4$ in Owen [1980] which allows for a closed form expression of the integral. The result itself is then just a function of the Bivariate Normally distributed cumulative probability distribution function. We provide a description and proof in Lemma VIII.B.

Let A and B are real valued constants and $Z \sim \mathcal{N}(0,1)$, with probability (cumulative) distribution function denoted $\phi(z)$ ($\Phi(z)$) respectively then

$$
\int_{-\infty}^{\underline{\nu}} \Phi(A+Bz)\phi(z)dz = \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, \underline{\nu}; \frac{-B}{\sqrt{1+B^2}}\right) \tag{51}
$$

where $\Phi_2\left(z_1, z_2; \rho\right)$ is the cumulative bivariate normal distribution of two joint random variables Z_1 and Z_2 with correlation ρ.

Proof.

$$
\begin{aligned}
\int_{-\infty}^{\underline{\nu}} \Phi(A+Bz)\phi(z)dz &= \int_{-\infty}^{\underline{\nu}} \int_{-\infty}^{A+Bz} \phi(x)\phi(z)dxdz \\
&= \int_{-\infty}^{\underline{\nu}} \int_{-\infty}^{A+Bz} \frac{e^{\frac{-1(x^2+z^2)}{2}}}{\sqrt{2\pi}\sqrt{2\pi}} dydz \\
&= \int_{-\infty}^{\underline{\nu}} \int_{-\infty}^{A/\sqrt{1+B^2}} \frac{\sqrt{1+B^2} e^{\frac{-1\left((y\sqrt{1+B^2}+Bz)^2+z^2\right)}{2}}}{\sqrt{2\pi}\sqrt{2\pi}} dydz \\
&= \int_{-\infty}^{\underline{\nu}} \int_{-\infty}^{A/\sqrt{1+B^2}} \frac{e^{\frac{-1\left(y^2(1+B^2)+2yzB\sqrt{1+B^2}+B^2z^2+z^2\right)}{2}}}{\sqrt{2\pi}\sqrt{2\pi}\sqrt{1/(1+B^2)}} dydz \\
&= \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, \underline{\nu}; \frac{-B}{\sqrt{1+B^2}}\right) \qquad (52)
\end{aligned}
$$

□

where we let $y = (x - Bz)/\sqrt{1+B^2}$ $(x = y\sqrt{1+B^2} + Bz)$ hence $dy = dx/\sqrt{1+B^2}$ and

$$
\begin{aligned}
y^2(1+B^2) + 2yzB\sqrt{1+B^2} + z^2(1+B^2) &= (1+B^2)\left[y^2 + 2yz\frac{B}{\sqrt{1+B^2}} + z^2\right] \\
&= \frac{1}{1+B^2}\left[y^2 - 2yz\left(\frac{-B}{\sqrt{1+B^2}}\right) + z^2\right] \qquad (53)
\end{aligned}
$$

Let A and B are real valued constants and $Z \sim \mathcal{N}(0,1)$, with probability (cumulative) distribution function denoted $\phi(z)$

($\Phi(z)$) respectively then

$$\begin{aligned}
\int_h^k \Phi(A+Bx)\phi(x)dx &= \int_{-\infty}^{A/\sqrt{1+B^2}} \phi(x)\Phi\left(k\sqrt{B^2+1}+Bx\right)dx \\
&- \int_{-\infty}^{A/\sqrt{1+B^2}} \phi(x)\Phi\left(h\sqrt{B^2+1}+Bx\right)dx \\
&= \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, k; \frac{-B}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \qquad (54)
\end{aligned}$$

Proof.

$$\begin{aligned}
\int_h^k \Phi(A+Bx)\phi(x)dx &= \int_{-\infty}^{k} \Phi(A+Bx)\phi(x)dx - \int_{-\infty}^{h} \Phi(A+Bx)\phi(x)dx \\
&= \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, k; \frac{-B}{1+B^2}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{1+B^2}\right) \qquad (55)
\end{aligned}$$

□

Taking $k \longrightarrow \infty$ in equation 54 yields:

$$\begin{aligned}
\int_h^\infty \Phi(A+Bx)\phi(x)dx &= \lim_{k\to\infty} \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, k; \frac{-B}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \\
&= \Phi\left(\frac{A}{\sqrt{1+B^2}}\right) - \Phi_2\left(\frac{A}{\sqrt{1+B^2}}, h; \frac{-B}{\sqrt{1+B^2}}\right) \qquad (56)
\end{aligned}$$

We next derive the following transformation to evaluate an integral

$$
\begin{aligned}
&\int_{\underline{\nu}}^{\infty} e^{\theta u}\Phi\left(A+Bu\right)\phi(u;m,s^2)du \\
&= \int_{\underline{\nu}}^{\infty} e^{\theta u}\Phi\left(A+Bu\right)\frac{e^{\frac{-(u-m)^2}{2s^2}}}{\sqrt{2\pi s^2}}du \\
&= \int_{\underline{\nu}}^{\infty} \Phi\left(A+Bu\right)\frac{e^{\frac{-(u^2-2um+m^2-2s^2u\theta)}{2s^2}}}{\sqrt{2\pi s^2}}du \\
&= \int_{\underline{\nu}}^{\infty} \Phi\left(A+Bu\right)\frac{e^{\frac{-(u^2-2um+m^2-2s^2u\theta)}{2s^2}}}{\sqrt{2\pi s^2}}du \\
&= e^{\frac{-m^2+(m+s^2\theta)^2}{2s^2}}\int_{\underline{\nu}}^{\infty} \Phi\left(A+Bu\right)\frac{e^{\frac{-(u^2-2u(m+s^2\theta)+(m+s^2\theta)^2)}{2s^2}}}{\sqrt{2\pi s^2}}du \\
&= e^{\frac{-m^2+(m+s^2\theta)^2}{2s^2}}\int_{\underline{\nu}}^{\infty} \Phi\left(A+Bu\right)\frac{e^{\frac{-(u-(m+s^2\theta))^2}{2s^2}}}{\sqrt{2\pi s^2}}du \\
&= e^{\frac{-m^2+(m+s^2\theta)^2}{2s^2}}\int_{\underline{\nu}'}^{\infty} \Phi\left(A+B(sz+(m+s^2\theta))\right)\phi(z)dz \\
&= e^{\theta m+(s\theta)^2/2}\left[\Phi\left(\frac{A'}{\sqrt{1+(Bs)^2}}\right)-\Phi_2\left(\frac{A'}{\sqrt{1+(Bs)^2}},\underline{\nu}';\frac{-Bs}{\sqrt{1+(Bs)^2}}\right)\right]
\end{aligned}
\tag{57}
$$

where we let $z=(u-(m+s^2\theta))/s$ which results in $A'=A+B(m+s^2\theta)$ and $\underline{\nu}'=(\underline{\nu}-(m+s^2\theta))/s$. In the resulting equation 57 we then set $\theta=\{1,-2m/\sigma^2-1,-\gamma,-2m/\sigma^2+\gamma,-2m/\sigma^2\}$ to evaluate some of the integrals in equation 58.

$$
\begin{aligned}
DL_{CALL} = & \int_{\underline{\nu}}^{\infty} e^{u-\delta(T-t)} \left[\Phi\left(-z_1\right) - e^{(2m/\sigma^2+2)(\underline{\nu}-u)} \Phi\left(-z_2\right) \right] g\left(u|z,y\right) du \\
& + \int_{\underline{\nu}}^{\infty} B e^{-\gamma(u-\underline{\nu})} \left[\Phi\left(-z_3\right) - e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)} \Phi\left(-z_4\right) \right] g\left(u|z,y\right) du \\
& - \int_{\underline{\nu}}^{\infty} e^{-r(T-t)} \left(A+K\right) \left[\Phi\left(-z_5\right) - e^{(2m/\sigma^2)(\underline{\nu}-u)} \Phi\left(-z_6\right) \right] g\left(u|z,y\right) du
\end{aligned} \tag{58}
$$

where

$$
\begin{aligned}
-z_1 &= \tfrac{-y^* + \left(m(T-t)+u+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & \quad -z_2 &= \tfrac{-y^* + \left(m(T-t)-u+2\underline{\nu}+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\
-z_3 &= \tfrac{-y^* + \left(m(T-t)+u-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} & \quad -z_4 &= \tfrac{-y^* + \left(m(T-t)-u+2\underline{\nu}-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}} \\
-z_5 &= \tfrac{-y^* + (m(T-t)+u)}{\sigma\sqrt{T-t}} & \quad -z_6 &= \tfrac{-y^* + (m(T-t)-u+2\underline{\nu})}{\sigma\sqrt{T-t}}
\end{aligned} \tag{59}
$$

For evaluating the first integral in equation 58 we make use of the result from equation 57

$$
\begin{aligned}
&\int_{\underline{\nu}}^{\infty} e^{u-\delta(T-t)} \left[\Phi\left(-z_1\right) - e^{(2m/\sigma^2+2)(\underline{\nu}-u)}\Phi\left(-z_2\right)\right] g\left(u|z,y\right) du \\
&= \int_{\underline{\nu}}^{\infty} e^{u-\delta(T-t)} \left[\Phi\left(-z_1\right) - e^{(2m/\sigma^2+2)(\underline{\nu}-u)}\Phi\left(-z_2\right)\right] \left[\frac{L_1}{L_0} \times e^{-\frac{(u-M_1)^2}{2\psi}} - \frac{L_2}{L_0} \times e^{-\frac{(u-M_2)^2}{2\psi}}\right] du \\
&= e^{-\delta(T-t)} \int_{\underline{\nu}}^{\infty} e^{u} \Phi\left(-z_1\right) \left[\frac{L_1}{L_0} \times e^{-\frac{(u-M_1)^2}{2\psi}} - \frac{L_2}{L_0} \times e^{-\frac{(u-M_2)^2}{2\psi}}\right] du \\
&- e^{-\delta(T-t)} \int_{\underline{\nu}}^{\infty} e^{u} e^{(2m/\sigma^2+2)(\underline{\nu}-u)} \Phi\left(-z_2\right) \left[\frac{L_1}{L_0} \times e^{-\frac{(u-M_1)^2}{2\psi}} - \frac{L_2}{L_0} \times e^{-\frac{(u-M_2)^2}{2\psi}}\right] du \\
&= e^{-\delta(T-t)} \int_{\underline{\nu}}^{\infty} e^{u} \Phi\left(-z_1\right) \frac{L_1}{L_0} \times e^{-\frac{(u-M_1)^2}{2\psi}} du - e^{-\delta(T-t)} \int_{\underline{\nu}}^{\infty} e^{u} \Phi\left(-z_1\right) \frac{L_2}{L_0} \times e^{-\frac{(u-M_2)^2}{2\psi}} du \\
&- e^{-\delta(T-t)} \int_{\underline{\nu}}^{\infty} e^{u} e^{(2m/\sigma^2+2)(\underline{\nu}-u)} \Phi\left(-z_2\right) \frac{L_1}{L_0} \times e^{-\frac{(u-M_1)^2}{2\psi}} du \\
&+ e^{-\delta(T-t)} \int_{\underline{\nu}}^{\infty} e^{u} e^{(2m/\sigma^2+2)(\underline{\nu}-u)} \Phi\left(-z_2\right) \frac{L_2}{L_0} \times e^{-\frac{(u-M_2)^2}{2\psi}} du \\
&= e^{-\delta(T-t)} \frac{L_1}{L_0} \times \int_{\underline{\nu}}^{\infty} e^{u} \Phi\left(-z_1\right) e^{-\frac{(u-M_1)^2}{2\psi}} du - e^{-\delta(T-t)} \frac{L_2}{L_0} \times \int_{\underline{\nu}}^{\infty} e^{u} \Phi\left(-z_1\right) e^{-\frac{(u-M_2)^2}{2\psi}} du \\
&- e^{-\delta(T-t)} e^{(2m/\sigma^2+2)(\underline{\nu})} \frac{L_1}{L_0} \times \int_{\underline{\nu}}^{\infty} e^{u(-2m/\sigma^2-1)} \Phi\left(-z_2\right) e^{-\frac{(u-M_1)^2}{2\psi}} du \\
&+ e^{-\delta(T-t)} e^{(2m/\sigma^2+2)\underline{\nu}} \frac{L_2}{L_0} \times \int_{\underline{\nu}}^{\infty} e^{u(-2m/\sigma^2-1)} \Phi\left(-z_2\right) e^{-\frac{(u-M_2)^2}{2\psi}} du
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
&= e^{-\delta(T-t)}\frac{L_1}{L_0}\sqrt{2\pi\psi}e^{M_1+\psi/2}\left[\Phi\left(\frac{\frac{-y^*+(M_1+\psi)+\left(m(T-t)+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right]\\
&- e^{-\delta(T-t)}\frac{L_1}{L_0}\sqrt{2\pi\psi}e^{M_1+\psi/2}\Phi_2\left(\frac{\frac{-y^*+(M_1+\psi)+\left(m(T-t)+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_1+\psi)}{\sqrt{\psi}};\left(\frac{-(\sqrt{\psi})/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right)\\
&- e^{-\delta(T-t)}\frac{L_2}{L_0}\sqrt{2\pi\psi}e^{M_2+\psi/2}\left[\Phi\left(\frac{\frac{-y^*+(M_2+\psi)+\left(m(T-t)+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right]\\
&+ e^{-\delta(T-t)}\frac{L_2}{L_0}\sqrt{2\pi\psi}e^{M_2+\psi/2}\Phi_2\left(\frac{\frac{-y^*+(M_2+\psi)+\left(m(T-t)+\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_2+\psi)}{\sqrt{\psi}};\left(\frac{-\sqrt{\psi}/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right)\\
&- e^{-\delta(T-t)}e^{(2m/\sigma^2+2)\underline{\nu}}\frac{L_1}{L_0}\sqrt{2\pi\psi}e^{M_1(-2m/\sigma^2-1)+\psi(-2m/\sigma^2-1)^2/2}\times\left[\Phi\left(\frac{\frac{-y^*+(M_1+\psi(-2m/\sigma^2-1))+\left(m(T-t)+2\underline{\nu}+\sigma^2(T-t)\right)}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)\right]\\
&+ e^{-\delta(T-t)}e^{(2m/\sigma^2+2)\underline{\nu}}\frac{L_1}{L_0}\sqrt{2\pi\psi}e^{M_1(-2m/\sigma^2-1)+\psi(-2m/\sigma^2-1)^2/2}\\
&\times\Phi_2\left(\frac{\frac{-y^*+(M_1+\psi(-2m/\sigma^2-1))+\left(m(T-t)+2\underline{\nu}+\sigma^2(T-t)\right)}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_1+\psi(-2m/\sigma^2-1))}{\sqrt{\psi}};-\left(\frac{\sqrt{\psi}/(-\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)\right)\\
&+ e^{-\delta(T-t)}e^{(2m/\sigma^2+2)(\underline{\nu})}\frac{L_2}{L_0}\sqrt{2\pi\psi}e^{M_2(-2m/\sigma^2-1)+\psi(-2m/\sigma^2-1)^2/2}\times\left[\Phi\left(\frac{\frac{-y^*+(M_2+\psi(-2m/\sigma^2-1))+\left(m(T-t)+2\underline{\nu}+\sigma^2(T-t)\right)}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)\right]\\
&- e^{-\delta(T-t)}e^{(2m/\sigma^2+2)(\underline{\nu})}\frac{L_2}{L_0}\sqrt{2\pi\psi}e^{M_2(-2m/\sigma^2-1)+\psi(-2m/\sigma^2-1)^2/2}\\
&\times\left[\Phi_2\left(\frac{\frac{-y^*+(M_2+\psi(-2m/\sigma^2-1))+\left(m(T-t)+2\underline{\nu}+\sigma^2(T-t)\right)}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_2+\psi(-2m/\sigma^2-1)^2)}{\sqrt{\psi}};-\left(\frac{-\sqrt{\psi}/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)\right)\right]
\end{aligned}
\tag{61}
$$

For evaluating the second integral in equation 58 we make use of the result from equation 57

$$
\begin{aligned}
&\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}\left[\Phi\left(-z_3\right)-e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi\left(-z_4\right)\right] g\left(u|z,y\right) du \\
&=\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}\left[\Phi\left(-z_3\right)-e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi\left(-z_4\right)\right]\left[\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}}\right] du \\
&=\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}\Phi\left(-z_3\right)\left[\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}}\right] du \\
&-\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi\left(-z_4\right)\left[\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}}\right] du \\
&=\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}\Phi\left(-z_3\right)\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}} du-\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}\Phi\left(-z_3\right)\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}} du \\
&-\frac{L_1}{L_0}\times\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi\left(-z_4\right)e^{\frac{-(u-M_1)^2}{2\psi}} du \\
&+\int_{\underline{\nu}}^{\infty} Be^{-\gamma(u-\underline{\nu})}e^{(2m/\sigma^2-2\gamma)(\underline{\nu}-u)}\Phi\left(-z_4\right)\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}} du \\
&=\frac{L_1}{L_0}Be^{\gamma(\underline{\nu})}\int_{\underline{\nu}}^{\infty} e^{-\gamma(u)}\Phi\left(-z_3\right)e^{\frac{-(u-M_1)^2}{2\psi}} du-\frac{L_2}{L_0}Be^{\gamma(\underline{\nu})}\int_{\underline{\nu}}^{\infty} e^{-\gamma(u)}\Phi\left(-z_3\right)e^{\frac{-(u-M_2)^2}{2\psi}} du \\
&-\frac{L_1}{L_0}Be^{\gamma(\underline{\nu})}e^{(2m/\sigma^2-2\gamma)(\underline{\nu})}\int_{\underline{\nu}}^{\infty} e^{-(2m/\sigma^2-\gamma)(u)}\Phi\left(-z_4\right)e^{\frac{-(u-M_1)^2}{2\psi}} du \\
&+\frac{L_2}{L_0}Be^{\gamma(\underline{\nu})}e^{-(2m/\sigma^2-2\gamma)\underline{\nu}}\int_{\underline{\nu}}^{\infty} e^{-(2m/\sigma^2-\gamma)(u)}\Phi\left(-z_4\right)e^{\frac{-(u-M_2)^2}{2\psi}} du \\
&+\frac{L_1}{L_0}Be^{\gamma(\underline{\nu})}\sqrt{2\pi\psi}e^{(-\gamma)M_1+(-\gamma)^2\psi/2}\times\left[\Phi\left(\frac{\frac{-y^*+(M_1+\psi(-\gamma))+\left(m(T-t)-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right] \\
&-\frac{L_1}{L_0}Be^{\gamma(\underline{\nu})}\sqrt{2\pi\psi}e^{(-\gamma)M_1+(-\gamma)^2\psi/2} \\
&\times\left[\Phi_2\left(\frac{\frac{-y^*+(M_1+\psi(-\gamma))+\left(m(T-t)-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_1+\psi(-\gamma))}{\sqrt{\psi}};\left(\frac{-\sqrt{\psi}/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right)\right] \\
&-\frac{L_2}{L_0}Be^{\gamma(\underline{\nu})}\sqrt{2\pi\psi}e^{M_2(-\gamma)+(-\gamma)^2\psi/2}\times\Phi\left(\frac{\frac{-y^*+M_2+(\psi(-\gamma))+\left(m(T-t)-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right) \\
&+\frac{L_2}{L_0}Be^{\gamma(\underline{\nu})}\sqrt{2\pi\psi}e^{M_2(-\gamma)+(-\gamma)^2\psi/2} \\
&\times\left[\Phi_2\left(\frac{\frac{-y^*+(M_2+\psi(-\gamma))+\left(m(T-t)-\gamma\sigma^2(T-t)\right)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_2+\psi(-\gamma))}{\sqrt{\psi}};\left(\frac{-(\sqrt{\psi})/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right)\right] \\
&-\frac{L_1}{L_0}Be^{\gamma(\underline{\nu})}e^{(2m/\sigma^2-2\gamma)(\underline{\nu})}\sqrt{2\pi\psi}e^{M_1(-2m/\sigma^2+\gamma)+\psi(-2m/\sigma^2+\gamma)^2/2} \\
&\times\left[\Phi\left(\frac{\frac{-y^*+(M_1+(\psi(-2m/\sigma^2+\gamma)))+\left(m(T-t)+2\underline{\nu}-\gamma\sigma^2(T-t)\right)}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)\right]
\end{aligned}
$$

For evaluating the third integral in equation 58 the result of equation 57

$$
\begin{aligned}
&-\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)\left[\Phi\left(-z_5\right)-e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi\left(-z_6\right)\right] g\left(u|z,y\right)du \\
&=-\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)\left[\Phi\left(-z_5\right)-e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi\left(-z_6\right)\right]\left[\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}}\right]du \\
&=-\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)\Phi\left(-z_5\right)\left[\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-(u-M_2)^2}{2\psi}}\right]du \\
&+\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi\left(-z_6\right)\left[\frac{L_1}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}-\frac{L_2}{L_0}\times e^{\frac{-(u-M_1)^2}{2\psi}}\right]du \\
&=-\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)\Phi\left(-z_5\right)\frac{L_1}{L_0}e^{\frac{-(u-M_1)^2}{2\psi}}du+\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)\Phi\left(-z_5\right)\frac{L_2}{L_0}e^{\frac{-(u-M_2)^2}{2\psi}}du \\
&+\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi\left(-z_6\right)\frac{L_1}{L_0}e^{\frac{-(u-M_1)^2}{2\psi}}du-\int_{\underline{\nu}}^{\infty} e^{-r(T-t)}\left(A+K\right)e^{(2m/\sigma^2)(\underline{\nu}-u)}\Phi\left(-z_6\right)\frac{L_2}{L_0}e^{\frac{-(u-M_2)^2}{2\psi}}du \\
&=-e^{-r(T-t)}\left(A+K\right)\frac{L_1}{L_0}\int_{\underline{\nu}}^{\infty}\Phi\left(-z_5\right)e^{\frac{-(u-M_1)^2}{2\psi}}du+e^{-r(T-t)}\left(A+K\right)\frac{L_2}{L_0}\int_{\underline{\nu}}^{\infty}\Phi\left(-z_5\right)e^{\frac{-(u-M_1)^2}{2\psi}}du \\
&+e^{-r(T-t)}\left(A+K\right)\frac{L_1}{L_0}e^{(2m/\sigma^2)(\underline{\nu})}\int_{\underline{\nu}}^{\infty} e^{-(2m/\sigma^2)(u)}\Phi\left(-z_6\right)e^{\frac{-(u-M_1)^2}{2\psi}}du \\
&-e^{-r(T-t)}\left(A+K\right)e^{(2m/\sigma^2)(\underline{\nu})}\frac{L_2}{L_0}\int_{\underline{\nu}}^{\infty} e^{-(2m/\sigma^2)(u)}\Phi\left(-z_6\right)e^{\frac{-(u-M_2)^2}{2\psi}}du
\end{aligned}
\tag{63}
$$

$$
\begin{aligned}
&-e^{-r(T-t)}\left(A+K\right)\frac{L_1}{L_0}\sqrt{2\pi\psi}\left[\Phi\left(\frac{\frac{-y^*+(m(T-t)+M_1)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)-\Phi_2\left(\frac{\frac{-y^*+(m(T-t)+M_1)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-M_1}{\sqrt{\psi}};\left(\frac{-\sqrt{\psi}/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right)\right]\\
&+e^{-r(T-t)}\left(A+K\right)\frac{L_2}{L_0}\sqrt{2\pi\psi}\left[\Phi\left(\frac{\frac{-y^*+(m(T-t)+M_2)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)-\Phi_2\left(\frac{\frac{-y^*+(m(T-t)+M_2)}{\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-M_2}{\sqrt{\psi}};\left(\frac{-\sqrt{\psi}/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}}\right)\right)\right]\\
&+e^{-r(T-t)}\left(A+K\right)\frac{L_1}{L_0}e^{(2m/\sigma^2)(\underline{\nu})}\sqrt{2\pi\psi}e^{M_1(-2m/\sigma^2)+\psi(-2m/\sigma^2)^2/2}\\
&\times\left[\Phi\left(\frac{\frac{-y^*+(M_1+(\psi(-2m/\sigma^2)))+(m(T-t)+2\underline{\nu})}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)-\Phi_2\left(\frac{\frac{-y^*+(M_1+\psi(-2m/\sigma^2)^2)+(m(T-t)+2\underline{\nu})}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_1+(\psi(-2m/\sigma^2)))}{\sqrt{\psi}};-\left(\frac{-\sqrt{\psi}/(-\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)\right)\right]\\
&-e^{-r(T-t)}\left(A+K\right)\frac{L_2}{L_0}e^{(2m/\sigma^2)(\underline{\nu})}\sqrt{2\pi\psi}e^{M_2(-2m/\sigma^2)+\psi(-2m/\sigma^2)^2/2}\\
&\times\left[\Phi\left(\frac{\frac{-y^*+(M_2+(\psi(-2m/\sigma^2)))+(m(T-t)+2\underline{\nu})}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)-\Phi_2\left(\frac{\frac{-y^*+(M_2+\psi(-2m/\sigma^2))+(m(T-t)+2\underline{\nu})}{-\sigma\sqrt{T-t}}}{\sqrt{1+\left(\sqrt{\psi}/(\sigma\sqrt{T-t})\right)^2}},\frac{\underline{\nu}-(M_2+(\psi(-2m/\sigma^2)))}{\sqrt{\psi}};-\left(\frac{-\sqrt{\psi}/(\sigma\sqrt{T-t})}{\sqrt{1+\left(\sqrt{\psi}/(-\sigma\sqrt{T-t})\right)^2}}\right)\right)\right] \qquad (64)
\end{aligned}
$$