Across Industry Allocation Decisions in Private Equity

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Abstract

I study the determinants of across industry fund level portfolio allocation decisions by private equity (PE) firms. I construct a dynamic agency model with an exploration versus exploitation trade off of a PE firm raising capital for subsequent funds. The PE firm can allocate capital to a known industry (exploitation) or explore a new market (exploration). The model features moral hazard between the general and limited partners and learning from past investments by the PE firm. I endogenize the across industry portfolio allocation of the PE firm and the capital allocation of the limited partners. Firms with high opportunity cost of exploration do not explore; firms that choose to explore, base their allocation decision on the cost of managing the funds, opportunity cost of exploration, and their skill to learn from past investments. Exploration and subsequent period investments increase in the severity of moral hazard. Using data from Preqin, I find empirical evidence consistent with the model.

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1 Introduction

In this paper I study the determinants of across industry portfolio allocation of private equity firms. Academic work has identified a considerable heterogeneity in how private equity firms are organised and how the organisational structure is linked to the performance of investments (Gompers, Kovner and Lerner (2009); Metrick and Yasuda, (2010)). Some PE firms seem to specialize in making investments within a particular industry while others take a more generalist approach, diversifying their investments across industries (Gompers, Kovner, Lerner and Scharfstein (2005)). Most academic work has been devoted to examining how the degree of diversification across industries affects performance; however, there is limited research studying the drivers of across industry allocation choice. Why are certain private equity firms more specialized than others? Do agency problem between investors (LPs) and managers (GPs) play a role in the portfolio allocation? Worldwide private equity funds manage more than \$1 trillion of capital. Besides playing an increasingly important role as financial intermediaries, private equity firms (in particular, venture capital firms) have substantial impact on innovation and development of new technologies (Kortum and Lerner (2001); Lerner (2000)). Providing a clear answer to these questions is of primary importance; moral hazard problems typically distort first best outcomes and may reduce allocational efficiency which may impede future innovation.[1]

I study these questions theoretically and empirically. First, I study a dynamic agency problem with an exploration versus exploitation trade off of a private equity firm raising capital for two consecutive funds. The model features moral hazard between the private equity firm and its investors in the spirit of Bolton and Scharfstein (1990). Similarly to Bolton and Scharfstein (1990) the PE firm can divert cash flows from the LP for both fund 1 and fund 2. The threat to terminate financing for the follow-up fund alleviates the moral hazard problem. The private equity firm can allocate each fund's capital to a well known industry (exploitation) or explore a new market. I add learning to the model by assuming that investing a fraction of its first fund's capital in the new market enables the firm to make better investments which raises returns for the follow up fund in the new market. An implicit assumption in my model is that learning from past investments takes place at an industry level and investments in one industry is not informative about investments in other industries (similar as in Sorensen (2008)).[2]. The model solution endogenizes the portfolio allocation for both funds across industries and the capital allocation of the LP across both funds.

The first best outcome is determined by 1) opportunity cost of exploration - how good the private equity firm is at managing investments in their core industry relative to the new market for its first fund, and 2) skill to explore - how good is the private equity firm at learning to manage investments in the new market. In the first best I find that firms that have a very high opportunity cost of exploration choose to specialize, i.e., they do not invest in the outside market. Firms with a

[1] There is a large academic literature studying agency problems in the context of private equity e.g. see Kaplan and Stomberg (2004); Manso (2011) and more recently Ivashina and Lerner (2019) and Maurin, Robinson, Stromberg (2020). To the extent of my knowledge this is the first paper to use a dynamic agency model to study the impact of agency issues on across industry portfolio allocation in the context of private equity.

[2] Goldfarb, Kirsch and Miller (2007) for instance provide empirical evidence in the context of venture capital consistent with learning at the industry level.

low opportunity cost of exploration choose to invest a fraction of fund 1's capital in the new market. The fraction invested in the new market decreases with the opportunity cost of exploration. Among the firms who choose to diversify their fund 1 investments, the ratio of capital allocated to fund 2 relative to fund 1 increases in the skill of exploration and decreases in the opportunity cost of exploration.

To introduce moral hazard in the model, I assume the private equity firm can divert a fraction of its fund returns (in the spirit of Bolton and Scharfstein (1990)). When limited partners have full bargaining power the first best outcome is not achieved. In this case, firms that choose to diversify (firms with low opportunity cost of exploration) allocate a larger fraction of their fund 1 portfolio to the outside market relative to the first best. Furthermore, the ratio of capital allocated to fund 2 relative to fund 1 by the limited partners is higher than in the first best. The intuition for this result is simple: Since limited partners have full bargaining power, in order to curb the moral hazard problem for fund 1, they choose to explore more. This decreases the returns of fund 1, which are not fully pledgeable, but increases the returns of fund 2 (due to learning), and therefore increases the investment ratio between fund 2 and fund 1. Both the investment ratio between fund 2 and fund 1, and the allocation to the outside market increase with the severity of the moral hazard problem.

I test the model predictions empirically, using Preqin's venture capital deal level data I aggregate the number of deals for each fund for each industry. Following prior literature, I compute the success rate for each fund for each industry as the ratio of the number of IPOs or merger exits to the total number of investments in the industry by the fund (Sorensen 2008). I construct proxies for fund skill across each industry by demeaning the fund's success rate by the average success rate across all funds who have invested in the same industry and with vintages in the same year. Since precise information on the size of each deal is not available, I proxy for the fraction allocated to each industry by the ratio of the number of investments in this industry to the overall number of investments by the fund. I assign a core industry to each fund to be the industry where the fund has made the majority of its investments. I match the deal level data to Preqin's fund level data to obtain fund level characteristics. To test the predictions of the model one needs to construct proxies for the opportunity cost of exploration and the skill to explore. Constructing ex-ante proxies for skill is challenging. Instead, I construct proxy for skill based on realized outcomes. The opportunity cost of exploration will be the difference between the fund's demeaned success rate in its core industry relative to the sum of its demeaned success rates in the outside industries. The skill to explore i.e., learning rate will be the difference between fund's 2 success rate in the core industry and fund 1's success rate in the same industry given that there has been a change in the core industry focus between fund 1 and fund 2. My final sample features 1118 matched first and second funds raised by the same firm.

I find support for the predictions of the model. First, the choice to change an industry of focus in their second fund (relative to the first fund) is negatively associated with the measure of opportunity cost of exploration in the first fund as the model predicts. The main predictions of the model are about the interior region i.e., firms who choose to change an industry of focus for their follow up fund. Conditioning on changing the industry of focus, I find support that the more severe the moral hazard problem the higher the ratio of fund 2 to fund 1 investment, and the higher is the allocation fraction outside of the core industry for fund 1.

This paper contributes to the large literature the exploitation versus exploration trade - off in the context of venture capital investments. Manso (2011) studies incentive provision in a learning model similar to the bandit problem. Bergeman and Hege (2005) present theories of staged financing based on the bandit model in the context of VC investments. My paper is closest to Sorensen (2008) who uses the bandit framework to study the exploration - exploitation trade off in the context of VC investments. Sorensen (2008) also provides empirical evidence consistent with learning from past investments. Unlike Sorensen (2008), I study the effects of agency issues interacted with an exploration exploitation trade off. Another point of the departure is that I assume that learning takes place at the level of the private equity fund, instead of analyzing the problem of learning from individual investments.

This paper also contributes to the broader literature on studying the effects of agency issues between GPs and LPs in private equity. Maurin, Robinson and Stromberg (2022) study the effects of moral hazard between the LPs and GPs on fund returns, liquidity and fund persistence. Gryglewicz and Mayer (2021) develop a theory of the three key components through which PE firms can affect target firm performance (i.e., operational, financial and governance engineering) based on an agency conflict between the GPs and LPs. Unlike these papers, I study a model of cash diversion - whereby the GP can divert a certain fraction of fund 1 and fund 2 cashflows from the LP. This diversion can come for example by charging ex-post fees to portfolio companies that are not fully rebated to tbe LP (Phalippou, Rauch, Umber (2018)). To the extent of my knowledge this paper is the first to use a dynamic agency setting to study the determinants of fund level portfolio allocation.

Finally, this paper contributes to the broad literature on dynamic financial contracting. Building on the result of Bolton and Scharfstein (1990), I show that introducing an exploration - exploitation trade off in the context of a dynamic agency problem may increase second period investment provided the discount factor among the contracting parties does not differ a lot (for a discussion on the role of discounting see Biais et al. (2007) and Biais and Landier (2020) - Lemma 3 in particular).

The paper proceeds as follows. Section 2 presents the main theory and model. The main results of the paper are given in this section, in Propositions 4 and 9. Section 3 presents empirical evidence consistent with the predictions of the model.

2 Theoretical model

In this section I describe and solve the theoretical model that motivates the empirical analysis. In subsection 2.1, I describe the players and the timing structure of the model; in subsection 2.2 I describe the nature of the exploration versus exploitation trade-off and the structure of learning featuring in my model. In subsection 2.3, I solve for the first best. The solution of the first best is characterized in proposition 3. I analyze the dynamic agency problem assuming the limited partner has full bargaining power in subsection 2.5. The main results of the paper are given in Propositions

4 and 5. In section 2.6, I study the role of discounting and extend the model assuming that the general partner and the limited partner have different discount factors. The characterization result is given by Propositions 7 and 8. In section 2.7, I study the same problem under the assumption that the general partners have full bargaining power. Section 2.8 concludes and summarizes the main theoretical findings.

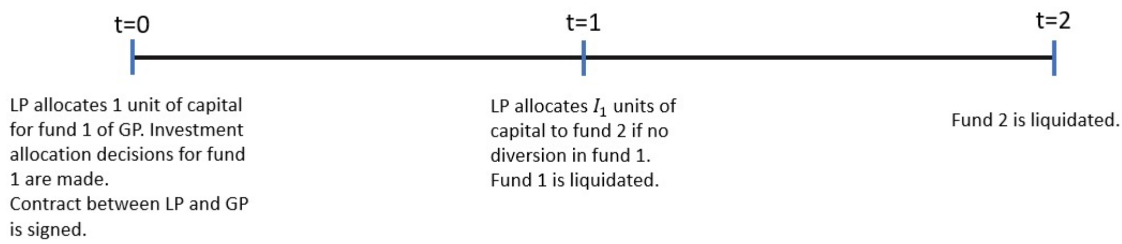
2.1 Players and timing of events

We study the exploration versus exploitation trade off of a private equity firm raising capital for consecutive funds. In the baseline model, a single private equity firm (GP) can raise capital from a single investor (LP). The LP has deep pockets and has a choice to either allocate to the PE firm or invest in a risk-free asset whose return is normalized to 0. In the baseline model we assume that both the GP and LP are risk-neutral and we ignore discounting. The role of discounting is studied in an extension.

There are two periods. At time 0, the LP's supply of capital to fund 1 is normalized to 1. At time 1, the GP liquidates fund 1 and raises I_1 capital for fund 2 (notice that I_1 is to be interpreted as the ratio between fund 2 and fund 1 allocation of the LP). At time 2, fund 2 is liquidated.

In practice, GP's receive fee payments from companies they control which are not fully rebated to the LP (Phalippou, Rauch and Umber (2018)). The creates a potential for diverting a fraction of fund returns by the GP. We model this by assuming assume that at time $t = 1$ and $t = 2$ the GP can divert a fraction of returns of fund 1 and fund 2. In order to mitigate this incentive problem the LP commits to not finance fund 2 if diversion at time $t = 1$ occurs. We assume that the contract between the LP and GP is signed at time $t = 0$ and it is non-negotiable at time $t = 1$. The threat to terminate financing for the subsequent fund ensures that the private equity firm does not divert resources at the expense of investors. The timing and sequence of events is explained in Figure 1. below. In the baseline model we assume constant returns to scale.

Figure 1: Timeline of actions



2.2 Exploration, exploitation and structure of learning

The private equity firm can allocate each fund's capital to a well known industry or explore a new market. By investing an amount of fund 1's capital in the new market the private equity firm may become better at selecting and managing investments in this new market which may increases subsequent returns.

For both fund 1 and fund 2, we assume that the gross return per unit capital invested in the well known industry comes from a distribution with a mean of θ_s . Hence, if the private equity firm decided to allocate a fraction $1-\alpha$ to the well known industry the expected return per unit of capital invested is given by:

$$(1-\alpha)\theta_s. \tag{1}$$

We assume that the quality of investments for fund 1 in the exploratory industry is uniformly distributed around a mean θ_1. We make the following assumption:

$$\theta_s \geq \theta_1 \tag{2}$$

Assumption (2) specifies that for the private equity firm the ex-ante expected return for fund 1 in the known industry is greater than the expected return obtained in the exploratory industry.

If fund 1 invests a fraction of $1-\alpha$ to the known industry and a fraction α in the exploratory industry the total expected return per unit capital for fund 1 is given by:

$$(1-\alpha)\theta_s + \alpha\theta_1 \tag{3}$$

To incorporate learning by doing in the model we assume that by allocating an amount of capital α in the exploratory industry the private equity managers may learn about the quality of the available investments in that industry. We model this by assuming that with a probability ρ the gross return in the exploratory industry is uniformly distributed with a mean given by the following function $\theta_2(\alpha) : [0,1] \to [\theta_1, \theta_2]$. $\theta_2(\alpha)$ is an increasing concave function of α. With probability $1-\rho$ the mean return in the exploratory industry for fund 2 investments does not change and remains at θ_s. The expected return per unit capital in the exploratory industry in fund 2 is thus:

$$(1-\rho)\theta_1 + \rho\theta_2(\alpha). \tag{4}$$

Let the fraction of fund 2's capital allocated to the exploratory industry be β and the fraction allocated to the known industry be $1-\beta$. Assuming that the LP allocates a unit capital to fund 1 (time 0) and I_1 units of capital to fund 2 (time 1) the total expected return on investment of the two funds is given by:

$$\Big(\alpha\theta_1 + (1-\alpha)\theta_s\Big) + I_1\Big(\beta\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) + (1-\beta)\theta_s\Big). \tag{5}$$

The first term in equation (5) is the expected return of fund 1, and the second term is the expected return of fund 2. We assume a convex cost of managing a fund of a given size I given by $C(I) = I + \gamma\frac{I^2}{2}$ (Berk and Green (2004)).

2.3 First best

The social surplus over fund 1 and fund 2 investments is given by:

$$S(\alpha,\beta,I_1) = \Big(\alpha\theta_1 + (1-\alpha)\theta_s\Big) - 1 - \frac{\gamma}{2} + I_1\Big(\beta\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) + (1-\beta)\theta_s\Big) - I_1 - \frac{\gamma I_1^2}{2}. \tag{6}$$

We will assume that allocation to fund 1 in the case of allocating all of fund 1's capital to the known industry is efficient i.e.:

$$\theta_s - 1 - \frac{\gamma}{2} > 0. \tag{7}$$

If condition (7) is not fulfilled then allocating to the risk - free rate dominates.

From expression (6) we can see that for fund 2 allocation we will have a solution with either $\beta = 0$ or $\beta = 1$. We solve for each case separately, the final solution is given in proposition below.

1. **Case** $\beta = 0$: The case $\beta = 0$ obtains if and only if:

$$\theta_s > (1-\rho)\theta_1 + \rho\theta_2(\alpha). \tag{8}$$

In this case $\alpha = 0$ and the unique solution is $(\alpha, \beta) = (0, 0)$. We summarize the result with the following proposition:

Proposition 1 *The first best admits a solution with no exploration ($\alpha = 0$) given by $(\alpha, \beta, I_1) = (0, 0, \frac{\theta_s - 1}{2\gamma})$. The value of the social surplus in this case is given by:*

$$S_1^{FB} = \theta_s - 1 - \frac{\gamma}{2} + \frac{(\theta_s - 1)^2}{2\gamma} \tag{9}$$

2. **Case** $\beta = 1$: : The case $\beta = 1$ obtains if and only if:

$$\theta_s < (1-\rho)\theta_1 + \rho\theta_2(\alpha). \tag{10}$$

In this case we have $\alpha > 0$ and $\beta = 1$. The social surplus to be maximized is given by:

$$S(\alpha, 1, I_1) = \Big(\alpha\theta_1 + (1-\alpha)\theta_s\Big) - 1 - \frac{\gamma}{2} + I_1\Big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\Big) - I_1 - \frac{\gamma I_1^2}{2}. \tag{11}$$

To simplify the analysis we normalize $\theta_1 = 1$ and we assume that $\theta_2(\alpha) = f(\alpha) + \theta_1$ where $f(\alpha)$ is a concave function of α. We define $\eta := \theta_s - \theta_1 = \theta_s - 1$. We summarize the result in this case with the following proposition:

Proposition 2 *The first best admits a solution $(\alpha^{FB}, \beta^{FB}, I_1^{FB})$ with exploration ($\alpha^{FB} > 0$). The solutions satisfy the following first order conditions:*

$$I_1^{FB} = \frac{\rho f(\alpha^{FB})}{\gamma}, \text{ and } f(\alpha^{FB}) f(\alpha^{FB})' = \frac{\gamma\eta}{\rho^2}. \tag{12}$$

The social surplus is given by:

$$S_2^{FB} = \theta_s - 1 - \frac{\gamma}{2} - \alpha^{FB}\eta + \frac{\rho^2 f(\alpha^{FB})^2}{2\gamma} \tag{13}$$

The term $\alpha^{FB}\eta$ in expression (13) is the return sacrificed by exploring in fund 1 and the term $\frac{\rho^2 f(\alpha^{FB})}{2\gamma}$ is the gain for fund 2 obtained by exploring.

2.4 Example - Analytical solution to the first best

To characterize the comparative statics and solve the problem analytically we assume the following form for the leaning function $f(\alpha)$:

$$f(\alpha) = \epsilon\alpha^{\frac{1}{4}}. \tag{14}$$

The following proposition characterizes the solution for the first best given the form for $f(\alpha)$.

Proposition 3 *The first best admits an interior solution with positive exploration* $\alpha^{FB} > 0$ *if the following conditions are satisfied:*

$$0 < \eta < \epsilon\rho, \text{ and } \frac{\eta}{4} < \gamma < 2\eta. \tag{15}$$

In this case α^{FB} *and* I_1^{FB} *are given by:*

$$\alpha^{FB} = \frac{1}{16}\frac{\epsilon^4\rho^4}{\gamma^2\eta^2}\ ,\ I_1^{FB} = \frac{1}{2\gamma}\frac{\epsilon^2\rho^2}{\sqrt{\gamma\eta}}. \tag{16}$$

The social surplus in this case is given by:

$$S^{FB} = \eta - \frac{\gamma}{2} + \frac{1}{16}\frac{\epsilon^4\rho^4}{\gamma^2\eta} \tag{17}$$

If the parameter values lie outside of the region defined by (15) *we obtain a solution with no exploration* $\alpha^{FB} = 0$*. In this case the social surplus is given by:*

$$S^{FB} = \eta - \frac{\gamma}{2} + \frac{\eta^2}{2\gamma} \tag{18}$$

The first best admits an interior solution in the parameter region defined by (15). Here $\eta < \epsilon\rho$, where $\eta = \theta_s - \theta_1$ is the opportunity cost of exploration in the first period. Intuitively, positive exploration obtains at optimum as long as the opportunity cost of exploration is not high. The parameter ρ measures the probability that the exploration will be successful, if ρ is high enough we obtain an interior solution. From the expressions given in (16) it is obvious that of α^{FB} and I_1^{FB} increase in ϵ and ρ and decrease in γ and η. The higher the probability of success the higher the amount allocated to the industry to be explored in fund 1. This subsequently boosts I_1 which is the ratio of fund 2 to fund 1 investments.

2.5 Problem of the Limited Partners

In this section we analyze the problem when the limited partners have full bargaining power. To prevent diversion from the GP for fund 1 and fund 2 the limited partners are going to incentivze the GP with positive transfers. Furthermore we have assumed that the contract between the GP and LP is signed at date $t = 0$ and that the LP commits to not finance fund 2 if diversion for fund 1 occurs. Suppose that the GP has the ability to divert a fraction λ_1 from fund 1 and a fraction λ_2 from fund 2 returns. This diversion may occur for instance by financing projects that yield a private benefit to the GP, but lower overall fund level returns. In this section we are going to focus on the case with positive exploration i.e. $\alpha > 0$ and subsequently derive conditions for the parameter values that guarantee that an interior α is optimal. Denote by t_1 and t_2 the first and second period transfers from the LP to the GP. The IC conditions read:

$$t_2 \geq \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \tag{19}$$

$$t_1 + t_2 \geq \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s) \tag{20}$$

Given that we have assumed that the LP has full bargaining power the LP solves the following optimization problem.

$$\begin{aligned} \max_{I_1,\alpha} \quad & S(I_1,\alpha) - t_1 - t_2 \\ \text{s.t.} \quad & t_2 \geq \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \\ \text{s.t.} \quad & t_1 + t_2 \geq \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s) \end{aligned} \tag{21}$$

Since the optimization function is linear in t_1 and t_2 it is optimal to bind the first period IC constraint (proof in appendix). It will hold that:

$$t_1 + t_2 = \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s). \tag{22}$$

For the moment we solve the problem of the limited partner and subsequently check conditions which guarantee positive first and second period transfers i.e.

$$t_1 = \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s) - \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \geq 0 \tag{23}$$

The maximization problem for the LP becomes:

$$\max_{I_1,\alpha}(\alpha\theta_1 + (1-\alpha)\theta_s)(1-\lambda_1) + I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) - 1 - \gamma\frac{1}{2} - I_1 - \gamma\frac{I_1^2}{2} \tag{24}$$

The fist order conditions for a maximum (the second order conditions are satisfied) are the following:

$$(1-\rho)\theta_1 + \rho\theta_2(\alpha) = 1 + \gamma I_1 \tag{25}$$

$$I_1 = \frac{(1-\lambda_1)(\theta_s - \theta_1)}{\rho\theta_2'(\alpha)} \tag{26}$$

2.5.1 Example - Analytical solution to the LP problem

Using the same simplification as in the first best ($\theta_2(\alpha) = f(\alpha) + \theta_1$ and $\theta_1 = 1$) we can rewrite the first order conditions to the LP problem as follows:

$$I_1 = \frac{\rho f(\alpha)}{\gamma} \tag{27}$$

$$f(\alpha)f'(\alpha) = \frac{(1-\lambda_1)\eta\gamma}{\rho^2} \tag{28}$$

Using the same functional form for the learning function as in the first best ($f(\alpha) = \epsilon\alpha^{\frac{1}{4}}$) we obtain the following:

Proposition 4 *The LP problem admits an interior solution (α^L, I_1^L) with positive exploration ($\alpha^L > 0$) provided the following condition is satisfied:*

$$0 < \eta < \epsilon\rho, \text{ and } \frac{\eta}{4(1-\lambda_1)} < \gamma < 2\eta(1-\lambda_1) + 2\lambda_1. \tag{29}$$

The expression for the solutions is given by:

$$\alpha^L = \frac{\epsilon^4\rho^4}{16\eta^2\gamma^2(1-\lambda_1)^2} \text{ and } I_1^L = \frac{\rho f(\alpha^L)}{\gamma}. \tag{30}$$

The social surplus is given by:

$$S^L = (\theta_s - \alpha^L\eta)(1-\lambda_1) - 1 - \frac{\gamma}{2} + \frac{\rho^4\epsilon^4}{8\gamma^2\eta(1-\lambda_1)} \tag{31}$$

If the parameter values are not in the region specified by (45) there is no exploration i.e. $\alpha^L = 0$, $I_1^L = I_1^{FB}$ and the social surplus is given by:

$$S^L = \theta_s(1-\lambda_1) - 1 - \frac{\gamma}{2} + \frac{\eta^2}{2\gamma} \tag{32}$$

Similar to the first best problem the inequality conditions (45) ensure that we have an interior solution for α. It is also immediate that if $\lambda_1 \geq 0$ and the parameter values lie in this region, we have:

$$\alpha^L \geq \alpha^{FB} \text{ and } I_1^L \geq I_1^{FB} \tag{33}$$

Since λ_1 measures the severity of the moral hazard problem in fund 1, and the LP would like to discourage diversion it is optimal for the LP to keep first period returns low by exploring more. Similarly they disincentivize diversion in fund 1 by increasing the amount invested for fund 2 relative to fund 1. Notice that the overall compensation of the general partners $t_1 + t_2$ depends only on fund 1returns i.e. is given by (22) so to keep the total transfer to the GP low the LP is going to incentivize exploration. Immediately we have the following comparative statics result:

Proposition 5 *α^L and I_1^L increase in ϵ and ρ and decrease in η and γ. α^L and I_1^L increase in λ_1.*

So far in the analysis we have ignored the condition on positive transfers (23). If we constrain t_1 to be positive we obtain the following claim:

Proposition 6 *Under the conditions $t_1 \geq 0$ and $t_2 \geq 0$ the LP problem does admit an interior solution if and only if:*

$$\lambda_2 \leq \frac{\theta_s - \alpha^L \eta}{I_1^L(1 + \rho f(\alpha^L))} \lambda_1 \tag{34}$$

As long as the fund 2 moral hazard is not severe enough it is worthwhile for the LP to encourage the GP to explore, since by exploring fund 2 returns increase. If the moral hazard problem for fund 2 is not very severe then by setting high t_2 the LP satisfies both IC constraints (IC1 binds and IC2 is slack). This effectively means that fund 2 returns are fully pledgeable, whereas only a fraction of fund 1 returns is pledgeable. Since fund 2 returns increase in fund 1 exploration the LP prefers to keep fund 1 returns low and fund 2 returns high. if however the moral hazard for fund 2 is very severe then exploration is not viable since the LP gets a very small fraction of the total fund 2 returns. In the extreme case $\lambda_2 = 1$ we have $\alpha^L = 0$, $I_1^L = 0$.

2.6 The role of discounting

In standard dynamic agency principal agent problems it is typically assumed that the agent has a lower discount factor than the principal. Here we consider an extension to the result in the previous subsection and analyze a model where the limited partners have full bargaining power, a discount factor δ_{LP} and the general partner has a discount factor of δ_{GP}, where $\delta_{GP} < \delta_{LP}$. The IC conditions for the general partner now read:

$$t_2 \geq \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \tag{35}$$

$$t_1 + \delta_{GP} t_2 \geq \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s). \tag{36}$$

Given that we are still working under the assumption that the limited partner has full bargaining power the limited partners maximize:

$$
\begin{aligned}
\max_{I_1,\alpha} \quad & (\alpha\theta_1 + (1-\alpha)\theta_s) + \delta_{LP} I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) - 1 - \gamma\frac{1}{2} - I_1 - \gamma\frac{I_1^2}{2}. - t_1 - \delta_{LP} t_2 \\
\text{s.t.} \quad & t_2 \geq \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \\
\text{s.t.} \quad & t_1 + \delta_{GP} t_2 \geq \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s)
\end{aligned}
\tag{37}
$$

It can be shown that (see Appendix) in problem (37) both IC constraints bind. For the transfers we therefore obtain:

$$
t_2 = \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \tag{38}
$$

$$
t_1 = \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s) - \delta_{GP}\lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \tag{39}
$$

Plugging in the transfers in the maximization problem we obtain that the LP maximizes the following:

$$
\max_{I_1,\alpha}(\alpha\theta_1+(1-\alpha)\theta_s)(1-\lambda_1)+\delta_{LP} I_1\big((1-\rho)\theta_1+\rho\theta_2(\alpha)\big)\big(1-\lambda_2(1-\frac{\delta_{GP}}{\delta_{LP}})\big)-1-\gamma\frac{1}{2}-I_1-\gamma\frac{I_1^2}{2}. \tag{40}
$$

We have the following observations:

- The term $(\alpha\theta_1 + (1-\alpha)\theta_s)(1-\lambda_1)$ is the pledgeable income for fund 1.
- The return for fund 2 is multiplied by the factor $(1 - \lambda_2(1 - \frac{\delta_{GP}}{\rho_{LP}}))$. Unlike the case where the discount factors of the GP and LP are the same, in this case the second period returns are not fully pledgeable. The lower the discount factor of the GP relative to the discount factor of the LP the lower the pledgeable income for fund 2 to the LP.

We adopt the notation from the previous subsection, where $\theta_2(\alpha) = f(\alpha) + \theta_1$, with $\theta_1 = 1$ and $\eta = \theta_s - \theta_1$. We also normalize the discount factor of the LP to 1 i.e. $\delta_{LP} = 1$. The following proposition characterizes the solution.

Proposition 7 *The limited partner problem with discounting admits an interior solution (α^D, I_1^D) determined by the following conditions:*

$$
f(\alpha^D)f(\alpha^D)' = \frac{\gamma\eta(1-\lambda_1)}{\rho^2\big(1-\lambda_2(1-\delta_{GP})\big)^2}, \tag{41}
$$

$$
I^D = \frac{f(\alpha^D)\rho\big(1-\lambda_2(1-\delta_{GP})\big)}{\gamma} \tag{42}
$$

We have seen that in the first best the optimal level of exploration is determined by $f(\alpha^{FB})f(\alpha^{FB})' = \frac{\gamma\eta}{\rho^2}$. Therefore, we obtain the following result that compares the optimal level of exploration under the first best with the level of exploration when the limited partner has full bargaining power:

Proposition 8 *Assume $f(\alpha)f(\alpha)'$ is decreasing. Then $\alpha^D \geq \alpha^{FB}$, provided:*

$$
1 - \lambda_1 \leq (1 - \lambda_2(1 - \delta_{GP}))^2. \tag{43}
$$

In the case where $\lambda_1 = \lambda_2$ the previous condition transforms to:

$$
\lambda_1 \geq \frac{2(1-\delta_{GP}) - 1}{(1-\delta_{GP})^2}, \tag{44}
$$

otherwise $\alpha^D \leq \alpha^{FB}$.

From condition (44) it is obvious that since $\lambda \geq 0$ if $\rho_{GP} > \frac{1}{2}$ then the condition is satisfied and when the limited partner has full bargaining power, exploration for fund 1 is encouraged if the GP is patient enough.

The next proposition provides an analytical solution for the case $f(\alpha) = \epsilon\alpha^{\frac{1}{4}}$.

Proposition 9 *The LP problem admits an interior solution (α^L, I_1^L) with positive exploration ($\alpha^L > 0$) provided the following condition is satisfied:*

$$0 < \eta < \epsilon\rho, \text{ and } \frac{\eta(1-\lambda_2(1-\delta_{GP}))^2}{4(1-\lambda_1)} < \gamma < 2\eta(1-\lambda_1) + 2\lambda_1. \tag{45}$$

The expression for the solutions is given by:

$$\alpha^D = \frac{\epsilon^4\rho^4(1-\lambda_2(1-\delta_{GP}))^2}{16\eta^2\gamma^2(1-\lambda_1)^2} \text{ and } I_1^L = \frac{\rho f(\alpha^L)(1-\lambda_2(1-\delta_{GP}))}{\gamma}. \tag{46}$$

The social surplus is given by:

$$S^L = (\theta_s - \alpha^L\eta)(1-\lambda_1) - 1 - \frac{\gamma}{2} + \frac{\rho^2 f(\alpha)^2(1-\lambda_2(1-\delta_{GP})^2)}{2\gamma} \tag{47}$$

If the parameter values are not in the region specified by (45) there is no exploration i.e. $\alpha^D = 0$, $I_1^D = I_1^{FB} - \frac{\lambda_2(1-\delta_{GP})}{\gamma}$ and the social surplus is given by:

$$S^L = \theta_s(1-\lambda_1) - 1 - \frac{\gamma}{2} + \frac{(\eta - \lambda_2(1-\delta_{GP}))^2}{2\gamma} \tag{48}$$

We finally notice that that discounting relaxes the constraint on positive transfers i.e. $t_1 \geq 0$ and we have a solution with positive exploration $\alpha^D > 0$ if the conditions of proposition 4 hold and the following is satisfied for fund 2 diversion:

$$\lambda_2 \leq \frac{\lambda_1(\theta_s - \alpha^D\eta)}{\delta_{GP} I_1^D(1 + f(\alpha^D))} \tag{49}$$

2.7 Problem of the General Partner

Here we assume that the general partners have the full bargaining power. The general partners are going to maximize the sum of the first and second period transfers $t_1 + t_2$ subject to the participation constraint of the LP and the incentive compatibility conditions. The LP is willing to participate as long as:

$$(\alpha\theta_1 + (1-\alpha)\theta_s) + I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) - -\gamma\frac{1}{2} - I_1 - \gamma\frac{I_1^2}{2} - t_1 - t_2 \geq 0. \tag{50}$$

The general partners solve the following program:

$$\begin{aligned}
&\max_{t_1,t_2} \quad t_1 + t_2 \\
&\text{s.t.} \quad t_2 \geq \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \\
&\text{s.t.} \quad t_1 + t_2 \geq \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s) \\
&\text{s.t.} \quad (\alpha\theta_1 + (1-\alpha)\theta_s) + I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) - -\gamma\frac{1}{2} - I_1 - \gamma\frac{I_1^2}{2} - t_1 - t_2 \geq 0.
\end{aligned} \tag{51}$$

It is easy to show that in the maximization problem given by (51) the participation constraint of the LP binds and the second period transfer condition for the GP is slack i.e. we must have:

$$(\alpha\theta_1 + (1-\alpha)\theta_s) + I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) - -\gamma\frac{1}{2} - I_1 - \gamma\frac{I_1^2}{2} = t_1 + t_2 \tag{52}$$

$$t_2 > \lambda_2 I_1\big((1-\rho)\theta_1 + \rho\theta_2(\alpha)\big) \tag{53}$$

The GP problem then becomes similar to the first best with the additional constraint that $t_1+t_2 \geq \lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s)$.

Proposition 10 *The general partner implements the first best i.e. interior solution for $\alpha^G = \alpha^{FB}$ and $I_1^G = I_1^{FB}$ when the conditions in proposition 1 hold. Additionally the total surplus under the first best has to be higher than $\lambda_1(1+(1-\alpha^{FB})\eta)$ i.e.*

$$\theta_s - 1 - \frac{\gamma}{2} - \alpha^{FB}\eta + \frac{\rho^2 f(\alpha^{FB})^2}{2\gamma} \geq \lambda_1(1+(1-\alpha^{FB})\eta) \tag{54}$$

It is clear from Proposition 6. that if λ_1 is very high it becomes harder for the GP to implement the firs best. If the moral hazard problem is severe enough, in particular for values $\lambda_1 \to 1$ there is no exploration and $\alpha^G = 0$.

2.8 Summary of findings and main predictions

In this section, I studied a dynamic agency model of an exploration - exploitation trade off of a PE firm raising capital for subsequent funds. For both funds the private equity firm can allocate capital between a known industry in its core area of expertise or explore a new industry. The model features learning by doing; by investing a fraction of its fund 1 capital to the new industry the private equity firm learns how to screen, select or manage investments in this industry which increases fund 2 returns for the new industry.

In the frictionless benchmark, the across industry allocation for fund 1 and fund 2 of the private equity firm and the invested amount in the private equity firm for each fund by the limited partners are determined by four parameters:

1. The opportunity cost of exploration (η) - how good the private equity firm is at managing investments in its core industry relative to the new market.

2. Learning rate (ϵ) - how good the private equity firm is at learning to manage screen or select investments in the new market.

3. Probability of successful exploration ρ.

4. Marginal cost of managing a unit of capital (γ).

In the first best firms with very high opportunity cost of exploration optimally choose to not explore i.e. they allocate all of their capital for both fund 1 and fund 2 to their core industry of focus. Firms with moderately high or low opportunity cost of exploration choose to explore for fund 1. The fraction allocated to the new market in fund 1 and the invested amount for fund 2, increase in the learning rate and the probability of successful exploration, and decrease in the opportunity cost of exploration and marginal cost of managing capital.

The presence of moral hazard between the GP and the LP distorts the first best outcome. When the LP has full bargaining power, I show that the presence of moral hazard encourages exploration for fund 1 investments. Since exploration decreases fund 1 returns which are not fully pledgeable and increases fund 2 returns which are fully pledgeable the LP acting as a principal with full bargaining power prefers to explore more. This subsequently increases investments for fund 2 relative to fund 1. The investment ratio between fund 2 and fund 1 and the amount allocated to the new market for fund 1 increase in the severity of the moral hazard problem. This intuition is valid if the GP is patient enough.

The main findings of the model generate predictions in the cross section of PE firms who choose to explore (invest a fraction of their capital in their non-core industry) for their first time fund. Specifically, the two main novel predictions are the following:

1. **Prediction 1**: Firms where the severity of the moral hazard problem is high, are going to allocate a larger fraction of their first fund's capital outside of their core industry.

2. **Prediction 2**: In firms where the severity of the moral hazard problem is high the ratio of fund 2 to fund 1 investment by the limited partners will be higher.

3 Data and Empirical Results

In this section, I provide tests for the main empirical implications of the model as outlined in section 2.8. Section 3.1, describes the data used. Section 3.2, describes the construction of the empirical proxies for the theoretical variables. Section 3.3 presents the main empirical results. The main results and tests for the model are presented in Tables 6 and 7.

3.1 Data

To test the predictions of the model I use Preqin's Venture capital deal, exit and fund level data. Using the deal level data I aggregate the number of investments for each industry classification for each fund. The core industry of each fund is the industry where the fund has made a majority of its investments Table 1 provides a summary of the number of fund in each industry. Matching the deal level data with the exit data on individual investment and fund level, we calculate the number of IPO and Merger exits for each fund for each industry. Lastly, we match the funds in the deal level data to the fund data to determine key characteristics of each fund, such as the fund sequence, fund vintage year, size (in USD) and fund manager characteristics for each fund. Since the main model predictions are about first and second funds we filter on fund sequence to keep only first and second funds. We further match the second funds in a given strategy in a given firm to the previous fund raised using the firm ID and the name of the fund. In the final sample we include funds with vintages between 1995 and 2019. To make sure we have a representative investment sample we include only funds with more than 5 registered investments in the final data. The final sample consists of 1118 matched first and second funds. The industries covered along with the number of funds in each industry are given in Table 1.

3.2 Construction of key variables

To empirically test the main predictions of the model, we need to construct proxies of η i.e. the opportunity cost of exploration for fund 1, and λ_1, the fraction of first period profits the GP can divert and I_1 i.e. the fraction of invested capital in fund 2 to fund 1. In this section we describe how we construct empirical proxies for the main variables featuring in the model.

3.2.1 Opportunity cost of exploration proxy

The academic literature on venture capital typically measures the number of successful exits at a fund level by the number of IPO and Merger exits for the fund's portfolio firms (e.g. see Sorensen 2008). We construct a proxy for η in the following way. For each fund we compute the number of investments and the number of successful exits for each industry the fund has invested in. For fund j, with a vintage year T, the successful exit rate in industry i is given by:

$$s_{ijT} = \frac{S_{ijT}}{N_{ijT}}, \tag{55}$$

where S_{ijT} is the number of IPO or Merger exits for fund j for investments in industry i and N_{ijT} is the number of investments for fund j in industry i. Because IPO and merger rates vary substantially across the business cycle and at an industry level to construct an appropriate measure of skill of fund j in industry i we demean the successful exit rate by the mean successful rate of all fund started in the same vintage year T who have invested in industry i given by:

$$\bar{s}_{iT} = \sum_j \frac{s_{jiT}}{K_T}, \tag{56}$$

where K_T is the number of funds with a vintage year T. The skill of fund j in an industry i is therefore given by:

$$\nu_{ijT} = s_{ijT} - \bar{s}_{iT} \tag{57}$$

We define the core industry of a fund j in vintage year T to be the industry with the maximum number of investments of fund j. The skill in the core industry is labeled by ν^c_{jT}. The skill of the fund outside of the core industry is given by the sum of ν_{ijT} across all industries except the core, fund j has invested in, computed by:

$$\nu^o_{jT} = \sum_{i \neq c} \nu_{ijT}. \tag{58}$$

The opportunity cost of exploration proxy is given by the skill in the core industry negative the skill in the outside of the core industry:

$$\eta_{jT} = \nu^c_{jT} - \nu^o_{jT}. \tag{59}$$

3.2.2 Diversion proxy

Typically, at the time a subsequent fund is started the private equity firm has not liquidated all of its investments of the prior fund. Therefore, at the time of fund raise for a follow up fund, the limited partners have to rely on NAV estimates for the non-exited investments. The NAV estimates may be imprecise or adjusted upwards in order to help the GP raise a follow up fund. The difference between the reported NAV estimate at the time of fund raise and the realized investment value may be quite different (cite academic paper for evidence on this). This generates a potential for diversion by the GP. To proxy for the diversion potential (i.e. empirical counterpart of λ_1) we are going to assume that the bigger the time difference between starting fund 2 and fund 1 is the lower is the diversion potential. The bigger the time difference between starting two consecutive funds the more investments of fund 1 are going to be realized at the time of starting fund 2 which lowers the diversion potential for the GP. We construct the diversion dummy in the following way. For a given private equity firm we compute the difference between the vintage year of fund 2 and fund 1. We assign a value of 1 to fund where this difference is less than 3 years and 0 to funds where this difference is larger than 3 years.

3.2.3 Investment ratio

We compute the investment ratio by dividing the total investment of fund 2 to the total investment of fund 1.

3.3 Empirical results

In this section I discuss the summary statistics of my sample and devise tests for the main implications of my model. To provide evidence for my model I first separate the private equity firms into *switchers* and *non-switchers*. Switchers are firms where the core industry for their first fund is not the same as the core industry for their second fund. For example, a private equity firm that has made a majority of its first fund investments in companies in the Healthcare sector and the majority of the investments for their second fund in Information Technology would be a switcher. As discussed at the end of the theoretical section, the main predictions of the model are about the cross sectional variation in investment and across industry portfolio allocation in the switchers sample.

3.3.1 Summary statistics

Tables 2 and 3 present summary statistics of the main variables for the full sample of first time funds and for the first time funds in the switcher sample. The funds in the switchers sample are bigger and have a lower fraction invested in their primary industry of focus. As predicted by the model the opportunity cost of exploration in the sample of switchers is lower than in the full sample of first time funds, the effect being mainly driven by a better skill of managing investments in industries outside of their core industry of focus.

Tables 4 and 5 present summary statistics of the main variables for the full sample of second funds and for the second funds in the switcher sample. Second funds in the switched sample have a lower opportunity cost of exploration and a lower fraction invested in their core industry of focus.

3.3.2 Main empirical results

To test the predictions of the model regarding first fund investments outside of core industry and ratio of second to first fund investments I run the following specifications:

$$\alpha_{f,1} = \beta_1 Diversion\ dummy_f + \beta_2 \eta_{f,1} + \beta_3 L_f + y_{1,t} + s_{f,1} + I_{f,1} + y_{1,t} \times s_{f,1} \times I_{f,1}. \tag{60}$$

$$\log(\frac{I_{f,2}}{I_{f,1}}) = \beta_1 Diversion\ dummy_f + \beta_2 \eta_{f,1} + \beta_3 L_f + y_{1,t} + s_{f,1} + I_{f,1} + y_{1,t} \times s_{f,1} \times I_{f,1}. \tag{61}$$

The main coefficient of interest is β_1. In specification (60) $\alpha_{f,1}$ is the fraction of investments outside of the core industry for fund 1. In specification (61) $\log(\frac{I_{f,2}}{I_{f,1}})$ is the log ratio of fund 2 to fund 1 investments for firm f. $\eta_{f,1}$ and $L_{f,1}$ are the opportunity cost of exploration for fund 1 in firm f and the learning rate of firm f respectively. $y_{f,1}$, $s_{f,1}$ and $I_{f,1}$ are vintage year, strategy and main industry of focus fixed effects. In specification (60) I include vintage year fixed effect to capture the fact that investment opportunities across industries may vary with time and this may influence the across industry allocation of the private equity firm. I include strategy fixed effects to capture the fact that across industry allocation may vary for different financing stages (for example, seed financing may be more prevalent for IT companies (Ewens, Nanda and Rhodes-Kropf (2018)). I include core industry fixed effects to capture the fact that the fraction allocated to the non-core industry may very dependent of the fund's primary industry of focus. In the most stringent specification (specification (6)) I interact the main industry, strategy and vintage year fixed effects. The main coefficient of interest is identified based on variation across different firms who started their first fund in the same vintage year, with the same strategy and the same core industry of focus. In specification (60) I find a statistically and economically significant effect for the main coefficient of interest. Firms with high level of moral hazard (diversion dummy = 1) allocate around 5% more of their fund 1 investments in industries outside of their core industry of focus. Given that the mean allocation for fund 1 outside of the core industry in the switchers sample is 51% the effect is economically significant.

In specification (61) I find an economically and statistically significant effect for the main coefficient of interest varying between 0.9 and 1.1 depending on the specification. Firms with a high level of moral hazard have around 2.5 higher investments ratio compared to firms where the moral hazard problem is not as severe.

4 Conclusion

In this paper I construct a dynamic agency model to study the determinants of across industry portfolio allocation of private equity firms. The model features an exploration versus exploitation trade off of a private equity firm raising capital for subsequent funds. The treat to terminate financing for the follow up fund by the limited partners alleviates the incentives for the PE firm to divert cash flows away from investors. By encouraging exploration in the first fund limited partners keep the returns for the first fund low and the returns from the second fund high providing additional incentives to the GP to not divert cash from fund 1. This incentive is enhanced by increasing the ratio of fund 2 to fund 1 investments. This incentive scheme works only if the GP is patient enough to care about fund 2 returns.

I test the implications of the model empirically. I construct measures for ex-post skill in the preferred industry of the GP and in outside industries. I test the model implications on a subset of PE firms who have decided to switch industry of focus for their first and second fund. Controlling for various combinations of strategy, industry and vintage year fixed effect I find support for the main predictions of the model. Firms where the potential for diversion is more severe allocate a larger fraction of their fund 1 investments outside of their main industry of focus and have a greater fund 2 relative to fund 1 size.

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Table 1: Number of funds for each industry in final sample

Core industry	First time funds	Second time funds
Real Estate	0	1
Business Services	8	4
Raw materials and natural resources	5	5
Energy and Utilities	15	11
Telecom and Media	10	13
Financial and Insurance services	20	24
Industrials	18	24
Consumer discretionary	71	73
Healthcare	205	209
Information Technology	766	754

Table 2: Summary Statistics for first time funds.

Variables	# Obs	Mean	SD	p25	p50	p75
Skill in Core Industry	1,118	-0.01	0.09	-0.04	-0.03	-0.01
Skill outside of core industry	1,003	-0.03	0.33	-0.15	-0.09	-0.03
Opportunity cost of Exploration	1,003	0.01	0.33	-0.00	0.07	0.14
Fraction invested in Core Industry	1,118	0.63	0.21	0.45	0.61	0.77
Fund Size (Mil)	1,016	106.66	168.75	20.18	50.00	125.00

Table 3: Summary Statistics for first time funds in the switchers sample.

Variables	# Obs	Mean	SD	p25	p50	p75
Skill in Core Industry	317	-0.01	0.11	-0.04	-0.03	-0.02
Skill outside of core industry	313	0.03	0.42	-0.16	-0.09	-0.02
Opportunity cost of Exploration	313	-0.03	0.40	-0.01	0.06	0.14
Fraction invested in Core Industry	317	0.49	0.16	0.38	0.46	0.60
Fund Size (Mil)	289	124.78	225.66	21.00	50.00	130.00

Table 4: Summary Statistics for second time funds

Variables	# Obs	Mean	SD	p25	p50	p75
Skill in Core Industry	1,118	0.00	0.09	-0.04	-0.02	-0.00
Skill outside of Core Industry	1,039	-0.03	0.30	-0.16	-0.09	-0.03 .
Opportunity cost of Exploration	1,039	0.03	0.28	0.01	0.07	0.15 .
Fraction invested in Core Industry	1,118	0.62	0.19	0.47	0.60	0.75
Fund Size (Mil)	1,031	168.32	245.90	40.00	96.43	196.00

Table 5: Summary Statistics for second time funds in the switchers sample

Variables	# Obs	Mean	SD	p25	p50	p75
Skill in Core Industry	317	0.01	0.12	-0.05	-0.02	0.01
Skill outside of Core Industry	316	0.01	0.35	-0.16	-0.09	-0.02
Opportunity cost of Exploration	316	-0.00	0.32	-0.01	0.07	0.15
Fraction invested in Core Industry	317	0.50	0.15	0.40	0.50	0.57
Fund Size (Mil)	294	178.06	282.82	32.70	78.26	200.00

Table 6: **Fund 1 investments outside of core industry**

This table reports the effect of moral hazard on fund 1 investments outside of core industry for firms in the switchers sample. All observations are at the firm-fund 1 level. The dependent variable is the fraction of number of investments of fund 1 invested outside of the fund's core industry. Diversion dummy is an indicator variable that takes on a value 1 for firms that start a subsequent fund less than 3 years after starting their first fund and 0 otherwise. T-statistics values are reported in parentheses.

	(1)	(2)	(3)	(4)	(5)
	Fraction non-core	Fraction non-core	Fraction non-core	Fraction non-core	Fraction non-core
Diversion dummy	0.0448^{**}	0.0480^{**}	0.0518^{**}	0.0569^{***}	0.0590^{**}
	(2.29)	(2.26)	(2.26)	(2.67)	(2.01)
Opportunity cost of exploration	-0.0439^{**}	-0.0468^{**}	-0.0581^{**}	-0.0305	-0.0831^{**}
	(-2.05)	(-2.12)	(-2.23)	(-1.36)	(-2.38)
Learning rate	0.0502	0.0737	0.0700	-0.00203	0.151^{**}
	(1.04)	(1.46)	(1.16)	(-0.04)	(2.02)
Constant	0.500^{***}	0.498^{***}	0.483^{***}	0.498^{***}	0.473^{***}
	(37.62)	(35.29)	(30.90)	(33.99)	(21.95)
Strategy FE	Yes	Yes	Yes	Yes	Yes
Vintage Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
Industry × Strategy FE	No	Yes	No	No	No
Industry × Vintage Year FE	No	No	Yes	No	No
Strategy × Vintage Year FE	No	No	No	Yes	No
Industry × Strategy × Vintage Year FE	No	No	No	No	Yes
Observations	268	259	229	234	155
R^2	0.333	0.402	0.464	0.446	0.565

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: **Ratio of fund 2 to fund 1 investments**

This table reports the effect of moral hazard on the ratio of fund 2 to fund 1 investments in the switchers sample. All observations are at the firm level. The dependent variable is the fraction of number of investments of fund 1 invested outside of the fund's core industry. Diversion dummy is an indicator variable that takes on a value 1 for firms that start a subsequent fund less than 3 years after starting their first fund and 0 otherwise. T-statistics values are reported in parentheses.

	(1)	(2)	(3)	(4)	(5)
	Investment ratio	Investment ratio	Investment ratio	Investment Ratio	Investment ratio
Diversion dummy	0.893***	0.868***	1.044***	0.855***	1.155***
	(3.60)	(3.29)	(3.68)	(2.86)	(2.81)
Opportunity cost of exploration	0.391*	0.365	0.532*	0.286	0.362
	(1.81)	(1.64)	(1.93)	(1.15)	(0.90)
Learning rate	-0.389	-0.204	-0.189	-0.439	-0.0676
	(-0.76)	(-0.38)	(-0.30)	(-0.70)	(-0.08)
Constant	-0.0572	-0.0664	-0.124	-0.0916	-0.260
	(-0.36)	(-0.39)	(-0.66)	(-0.46)	(-0.88)
Strategy FE	Yes	Yes	Yes	Yes	Yes
Vintage Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
Industry × Strategy FE	No	Yes	No	No	No
Industry × Vintage Year FE	No	No	Yes	No	No
Strategy × Vintage Year FE	No	No	No	Yes	No
Industry × Strategy × Vintage Year FE	No	No	No	No	Yes
Observations	225	217	191	192	126
R^2	0.363	0.420	0.539	0.459	0.652

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Appendix

Proofs

Proof of Proposition 1

Substituting $f(\alpha) = \epsilon\alpha^{\frac{1}{4}}$ in the first order condition $f(\alpha)f^{'}(\alpha) = \gamma\eta$ yields:

$$\alpha = \frac{\epsilon^4}{16\eta^2\gamma^2} \tag{62}$$

The conditions for a valid interior are $\alpha \leq 1$ and $f(\alpha) \geq \eta$. Plugging in the solution for α we obtain that for a valid interior solution we must have:

$$16\eta^2\gamma^2 \geq \epsilon^4 \geq 4\eta^3\gamma. \tag{63}$$

Plugging in the solution for α in $I_1 = \frac{f(\alpha)}{\gamma}$ we obtain the expression for I_1. This proves the claim.

Proof of Proposition 2

Proposition 2 obviously holds in the particular case with the α^{FB} given by the expression in Proposition 1. We are going to examine under which conditions we obtain the comparative statics for a particular class of functions. The first order condition for α gives:

$$f(\alpha)f^{'}(\alpha) = \gamma\eta \tag{64}$$

Assume that f is a concave function such that $ff^{'}$ is concave, then from (64) it is clear that α which solves (64) is decreasing in η and γ. The following are sufficient conditions for $ff^{'}$ to be concave given that f is concave. For all $\alpha \in (0,1)$ we must have

$$f^{''}(\alpha)f(\alpha) + (f^{'}(\alpha))^2 \geq 0 \tag{65}$$

$$f^{'''}(\alpha)f(\alpha) + 3f^{'}(\alpha)f^{''}(\alpha) \leq 0 \tag{66}$$

Proof of Proposition 3

The first order conditions yield:

$$f(\alpha)f^{'}(\alpha) = (1-\lambda_1)\eta\gamma \tag{67}$$

Plugging in $f(\alpha) = \epsilon\alpha^{\frac{1}{4}}$ we obtain the interior solution given by proposition 3.

Proof of Proposition 4

In the particular case when $f(\alpha) = \epsilon\alpha^{\frac{1}{4}}$ claim follows by simple differentiation. For the general case the conditions stated in the proof of Proposition 2 are sufficient.

Proof of Proposition 5

In the case of positive transfers we must have:

$$\lambda_1(\alpha\theta_1 + (1-\alpha)\theta_s) - \lambda_2 I_1\theta_2(\alpha) \geq 0. \tag{68}$$

Therefore, α^L is a viable solution in this case only if moral hazard in the second period is not very severe i.e.

$$\lambda_2 \leq \frac{\gamma(1 + \eta(1 - \alpha^L))}{f(\alpha^L) + f(\alpha^L)^2}\lambda_1 \tag{69}$$